Discussion handout 3: EE320 Semester 1/2017

Keywords: Derivative; Differential; Single-variable Optimization

Question 1: (The implicit function theorem) A joint-cost function is defined as follow,

$$c + \sqrt{c} = q_A\sqrt{9 + q_B^2}$$

where c is the total cost, q_A is the level of output for product A and q_B is the level of output for product B. Consider the following problems.

1.1 Find the corresponding value of c for $q_A = 6, q_B = 4$.

1.2 Calculate the marginal cost for each of the two products. Evaluate the value of marginal cost where $q_A = 6, q_B = 4$.

1.3 Derive the second-order derivative of cost with respect to q_B. Is the cost function strictly convex in q_B?

Question 2: Given the production function $Q = f(K, L) = A[K^n + L^n]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

2.1) To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

2.2) Under the assumption used in (2.1), show that the production function satisfies the law of diminishing returns.

2.3) Calculate the marginal rate of technical substitution (MRTS) of labor (L) for capital (K).

2.4) Show that MRTS is a decreasing function in L. That is, as labor increases, the value of MRTS decreases.

2.5) Under the condition(s) assumed in (1.1), does the production function have the *global* concave property?

Suppose that $K(t) = \frac{1}{2}t^2 + 2t + 3$ and $L(t) = e^t + 3$, where $t \geq 0$ is the number of periods from now. Consider the following problem

2.6) Show that Q is increasing over time.

2.7) Compute $\frac{\frac{dQ}{Q}}{dt}$ when t = 0, i.e. growth of output in the initial period.

Question 3: Suppose that the preference set of a household can be given by the utility function that takes the form:

$$U(x,y) = \left(x^{\delta} + y^{\delta}\right)^{\frac{1}{\delta}}; \quad 0 < \delta < 1$$

where x and y are the amount of consumption on goods-X and goods-Y, respectively. Given the function, one can derive by the optimization technique that the corresponding optimized utility function (the *indirect utility function*) is given by,

$$v\left(p_x, p_y, M\right) = M\left(p_x^r + p_y^r\right)^{\frac{1}{r}}$$

where M is the level of income, p_x and p_y are the price of goods X and goods Y, respectively. Upon the calculation, we can show that $r = \frac{\delta}{\delta - 1}$. Consider the following problems.

3.1) Derive the expression for the MRS and show that MRS is decreasing in x.

3.2) Explain with economics intuition why the *indirect utility function* is a homogenous function of degree 0.

Now consider when $\delta = \frac{1}{2}$ and M = 100.

3.3) Using the total differential concept, calculate the change in the level of *indirect utility* (v) when the price of x increases from \$10 to \$20, and the price of y decreases from \$20 to \$10. (Hint: if you do not use the total differential method, you will get zero.)