EMPIRICAL APPLICATIONS OF SOLOW MODEL: SOLOW MODEL WITH HUMAN CAPITAL

EE 462 Development Macroeconomics

Semester 1/2015

Reference: Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, *107*(2), 407-437.

MRW (1992)

- MRW examines whether the “textbook” Solow model is consistent with the empirical evidence.
- It augments the Solow model by including *human capital accumulation* and capital accumulation, and tests this augmented Solow model.
- It also tests for both unconditional and conditional convergence in per capita income.
- Findings:
  - The prediction of the Solow model are consistent with the data.
  - The cross-country variation in per capita income can best be explained by an augmented Solow model.
  - There is convergence once the differences in saving and population growth rates are controlled for.

"Textbook" Solow Model

- Production function: $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha},\ 0 < \alpha < 1,$

where A = Technology.

- *L* and *A* are assumed to grow exogenously at rates *n* and *g*:

$$L(t) = L(0)e^{nt} \qquad \text{and} \qquad A(t) = A(0)e^{gt}$$

- Define $k = \frac{K}{AL}$ and $y = \frac{Y}{AL}$, where AL is effective units of labor.
- The change in *k* is given by:

$$\dot{k} = sy(t) - (n + g + \delta)k(t)$$

- Hence, the steady-state value k^* is:

$$k^* = [s/(n+g+\delta)]^{1/(1-\alpha)}$$

"Textbook" Solow Model (cont'd)

- Substituting k^*in the production function and taking log yields:

$$\ln\left(\frac{Y(t)}{L(t)}\right) = lnA(0) + gt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta)$$

- Assume that g and δ are constant across countries. (In this paper, it's assume that $\mathrm{g} + \delta = 0.05.$)

- Also, assume that $lnA(0) = \mathrm{a} + \epsilon$, where *a* is a constant, and ϵ is a country-specific shock.

- Hence, log income per capita at a given point time (say t=0) is:

$$\ln\left(\frac{Y}{L}\right) = a + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) \qquad \text{-- (*)}$$

Data and Samples

- Data are from the Real National Accounts constructed by Summers and Heston (1988).
- Data are annual and cover the period 1960-1985.
- n measures the average growth rate of the working-age (15-64) population.
- s is the average share of real investment in real GDP.
- Y/L is real GDP in 1985 divided by the working-age pop.
- There are three samples:
  - 98 non-oil producing countries
  - 75 countries – exclude those with <1m pop or those with data problems
  - 22 OECD countries with pop > 1m

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\overline{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\overline{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

Augmented Solow Model

- Let the production function be:

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta}, 0 < \alpha + \beta < 1,$$

Where H =stock of human capital.

- Let s_k and s_h be the fractions of income invested in physical capital and human capital, respectively.
- The evolution of the economy is determined by:

$$\dot{k} = s_k y(t) - (n + g + \delta)k(t)$$

$$\dot{h} = s_h y(t) - (n + g + \delta)h(t)$$

Where *k = K/AL*, *h = H/AL*, and *y= Y/AL*.

Augmented Solow Model (cont'd)

- Hence, the steady-state levels of K and H are:

$$k^*=\left[\frac{s_k^{1-\beta}s_h^{\beta}}{n+g+\delta}\right]^{1/(1-\alpha-\beta)} \quad \text{and} \quad h^*=\left[\frac{s_k^{\alpha}s_h^{1-\alpha}}{n+g+\delta}\right]^{1/(1-\alpha-\beta)}$$

- Substituting k^* and h^* in the production function yields:

$$\ln\left(\frac{Y}{L}\right) = lnA(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h)$$

- The above equation can be rewritten as:

$$\ln\left(\frac{Y}{L}\right) = lnA(0) + gt + \frac{\alpha}{1-\alpha}\ln(s_k) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \frac{\beta}{1-\alpha}\ln(h^*)$$

Where h* = secondary school enrollment ratio x proportion of labor force of secondary school age

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\overline{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33

TABLE II (Cont'd)

ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\bar{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Endogenous Growth and Convergence

- Goal: reexamine the evidence on convergence of Y/L.
- Previous model assumes that countries in 1985 were in their steady states (or that the deviations from steady state were random). → relax this assumption!
- Let *y** be the steady-state income per capita and *y(t)* be the actual value at time *t*.
- The **speed of convergence** is given by:

$$\frac{dln(y(t))}{dt} = \lambda[\ln(y^*) - \ln(y(t))],$$

Where $\lambda = (n + g + \delta)(1 - \alpha - \beta)$ is the convergence rate.

- Example: If $\alpha = \beta = \frac{1}{3}$ and $n + g + \delta = 0.06$, then $\lambda = 0.02$.

Convergence Rate

- The above equation implies that:

$$\ln\big(y(t)\big) = \big(1 - e^{-\lambda t}\big)ln(y^{*}) + e^{-\lambda t}\ln(y(0)),$$

Where y(0) is income per effective worker at an initial date.

- Subtracting *ln(y(0))* and substituting *y** gives:

$$\ln\big(y(t)\big) - \ln\big(y(0)\big)$$

$$= \big(1 - e^{-\lambda t}\big)\frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \big(1 - e^{-\lambda t}\big)\frac{\beta}{1-\alpha-\beta}\ln(s_h)$$

$$- \big(1 - e^{-\lambda t}\big)\frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) - (1 - e^{-\lambda t})\ln(y(0))$$

TABLE III
Tests for Unconditional Convergence

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	−0.266	0.587	3.69
	(0.380)	(0.433)	(0.68)
ln(Y60)	0.0943	−0.00423	−0.341
	(0.0496)	(0.05484)	(0.079)
$\overline{R}^2$	0.03	−0.01	0.46
s.e.e.	0.44	0.41	0.18
Implied λ	−0.00360	0.00017	0.0167
	(0.00219)	(0.00218)	(0.0023)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960.

TABLE IV
Tests for Conditional Convergence

Dependent variable: log difference GDP per working-age person 1960–1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	1.93	2.23	2.19
	(0.83)	(0.86)	(1.17)
ln(Y60)	−0.141	−0.228	−0.351
	(0.052)	(0.057)	(0.066)
ln(I/GDP)	0.647	0.644	0.392
	(0.087)	(0.104)	(0.176)
$\ln(n + g + \delta)$	−0.299	−0.464	−0.753
	(0.304)	(0.307)	(0.341)
$\bar{R}^2$	0.38	0.35	0.62
s.e.e.	0.35	0.33	0.15
Implied λ	0.00606	0.0104	0.0173
	(0.00182)	(0.0019)	(0.0019)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

TABLE V
Tests for Conditional Convergence

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	3.04	3.69	2.81
	(0.83)	(0.91)	(1.19)
ln(Y60)	−0.289	−0.366	−0.398
	(0.062)	(0.067)	(0.070)
ln(I/GDP)	0.524	0.538	0.335
	(0.087)	(0.102)	(0.174)
$\ln(n + g + \delta)$	−0.505	−0.551	−0.844
	(0.288)	(0.288)	(0.334)
ln(SCHOOL)	0.233	0.271	0.223
	(0.060)	(0.081)	(0.144)
$\overline{R}^2$	0.46	0.43	0.65
s.e.e.	0.33	0.30	0.15
Implied λ	0.0137	0.0182	0.0203
	(0.0019)	(0.0020)	(0.0020)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.