BACHELOR of ECONOMICS
International Program

Thammasat University | Thailand

Fixed Income (FN351)

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Fixed Income Securities

Lesson 2: Bond prices, discount factors, and arbitrage

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Road map & key ideas

- Trades and bond trading
- Bond as a portfolio of cash flows
- Cash flows matching
- Relative pricing in bond markets
- Arbitrage
- Arbitrage when there is cost
- Practical considerations

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Types of Orders

- Secondary market for bonds is an over the counter market
- Market for Treasury securities is very liquid (a total of ~ 250 bond types and Trillions of dollars in outstanding bonds)
- Market—executed immediately and investors only specify number of shares
- Price-contingent

  Limit order: Investors specify number of shares and the price to buy or sell at

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Figure 3.5 Price-Contingent Orders

Stop orders

Stop loss: sell immediately if price drops to the stop loss level

Stop buy: do not buy if price increases to a certain level

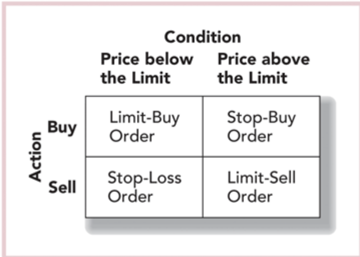

FIGURE 3.5 Price-contingent orders

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Types of Orders

- Short selling is selling securities an investor does not own
- The investor M places a short sell trade with the broker to short sell security XYZ
- The broker finds another account (say J) that has security XYZ then borrows it for M
- M owes J the security and all the cash flow due to the security XYZ
- At a later date M buys XYZ to cover the short sell
- The broker returns security XYZ to J
- M gets sell price-buy price – cash flow due to XYZ during short sell period

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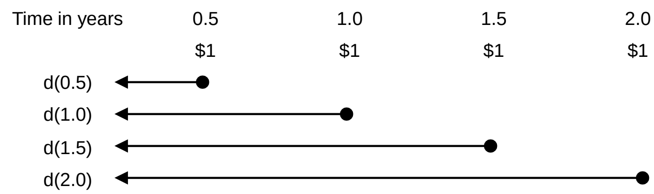
Bond as a portfolio of cash flows

- The discount factor, d(t): Present value of $1 to be received at the end of a period, t.



- The 'market' discount factors can be extracted from STRIPS

Maturity	Price *	t	d(t) **
0.5	97.0870	0.5	0.9709
1.0	93.8970	1.0	0.9390
1.5	90.9090	1.5	0.9091
2.0	87.3360	2.0	0.8734

* Face Value = $100

** d(t) = Price/Face Value

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Concept check

- What should be the market price of a 2-year 10% semi-annual coupon bond face value of 1000? The market price of zero-coupon bonds:

Maturity	Price ($100 Face Value)
0.5	97.087
1.0	93.897
1.5	90.909
2.0	87.336

G) 1242.6

Y) 1054.9

R) 1058

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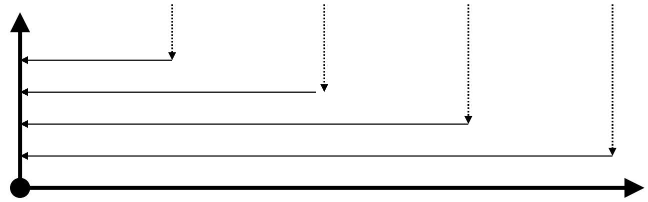
Bond as a portfolio of cash flows

Price of a 6% coupon 2-year bond

Time in years	0.0	0.5	1.0	1.5	2.0
Cash Flow		$30	$30	$30	$1030



$$Price = 30 * d(0.5) + 30 * d(1.0) + 30 * d(1.5) + 1030 * d(2.0)$$

$$Price = 30 * 0.9709 + 30 * 0.9390 + 30 * 0.9091 + 1030 * 0.8734 = 984.2$$

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Replicating portfolio: method 1

Bond prices in the bond market		Maturity (year)	coupor	Price		
	zero coupon	0.5	0	970.87		
	zero coupon	1	0	938.97		
	zero coupon	1.5	0	909.09		
	zero coupon	2	0	873.36		
	6% coupon bon	2	0.06	987		
	10% coupn bon	2	0.1	1057.97		
	6% coupon bon	1	0.06	996.27		
	8% coupon bon	1.5	0.08	983.02		
Time (year)		0.5	1	1.50	2	
Case 1	buy/sell face value					
0.5-year zero	30	30				
1-year zero	30		30			
1.5-year zero	30			30		
2-year zero	1030				1030	
		29.1261	28.17	27.2727	899.6	984.1

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Three ways to construct a replicating portfolio

1. Construct the cash flow of a coupon bond XYZ with a portfolio of zero-coupon bonds
   - In this case, we need a zero coupon bond that the maturity matches each cash flow of XYZ
2. Construct the cash flow of a coupon bond XYZ from 2 bonds
   - In this case, we need two bonds
   - We need a coupon bond with the same maturity and the same frequency of cash flow as XYZ and another zero coupon bond with the same maturity as XYZ

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Replicating portfolio: method 2

Bond prices in the bond market		Maturity (year)	coupor	Price		
	zero coupon	0.5	0	970.87		
	zero coupon	1	0	938.97		
	zero coupon	1.5	0	909.09		
	zero coupon	2	0	873.36		
	6% coupon bond	2	0.06	987		
	10% coupn bond	2	0.1	1057.97		
	6% coupon bond	1	0.06	996.27		
	8% coupon bond	1.5	0.08	983.02		
Case 2	buy/sell face value					
Time (year)		0.5	1	1.50	2	
Cash flow of 6% 2-year coupon bond		30	30	30	1030	
Cash flow of a 10% 2-year coupon bond		50	50	50	1050	
buy 600 face value of 2-year 1(	600	30	30	30	630	
Buy 400 face value of 2 year ze	400				400	
Total cash flow		30	30	30	1030	
Price of portfolio						984.1

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Replicating portfolio: method 2 practice

	Maturity (year)	coupon	Price
zero coupon	0.5	0	970.87
zero coupon	1	0	938.97
zero coupon	1.5	0	909.09
zero coupon	2	0	873.36
6% coupon bond	2	0.06	987
10% coupn bond	2	0.1	1057.97
6% coupon bond	1	0.06	996.27
8% coupon bond	1.5	0.08	1021.85
4% coupon bond	2	0.02	947.21

Use the 4% 2-year coupon bond to replicate the cash flow of the 6% coupon bond

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Replicating portfolio: method 2 practice

Bond prices in the bond market		Maturity (year)	coupon	Price	
	zero coupon	0.5	0	970.87	
	zero coupon	1	0	938.97	
	zero coupon	1.5	0	909.09	
	zero coupon	2	0	873.36	
	6% coupon bond	2	0.06	987	
	10% coupn bond	2	0.1	1057.97	
	6% coupon bond	1	0.06	996.27	
	8% coupon bond	1.5	0.08	983.02	
	4% coupon bond	2	0.02	964.67	

Use the 4% 2-year coupon bond to replicate the cash flow of the 6% coupon bond

Case 2 practice	buy/sell face value					
Time (year)		0.5	1	1.50	2	
Cash flow of 6% 2-year coupon bond		30	30	30	1030	
Cash flow of a 10% 2-year coupon bond		20	20	20	1020	
buy 30/20=1.5 * face value of 2-year 4% counpon	1500	30	30	30	1530	
Short sell 500 face value of 2 year zero	-500				-500	
Total cash flow		30	30	30	1030	
Price of portfolio						984.1

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Replicating portfolio: method 3

3. Construct the cash flow of a coupon bond XYZ with other coupon bonds
   - In this case, we need a coupon bonds that the maturity matches each cash flow of XYZ
   - Match the last cash flow first and then the one before last and keep on matching backwards until the first cash flow
   - Fi is the face value of the bond maturing in period i what we need to construct the replicating portfolio
   - Mi is the last cash flow of the bond that has the maturity equal to the cash flow we want to match
   - Ci is cash flow in period i

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Replicating portfolio: method 3

From previous example, of replicating the cash flow of a 6% coupon 2-year bond using the following coupon bonds

	Maturity (year)	coupor	Price
zero coupon	0.5	0	970.87
10% coupn bond	2	0.1	1057.97
6% coupon bond	1	0.06	996.27
8% coupon bond	1.5	0.08	983.02

F1*0+F2*0+F3*0+F4*1050 = 1030 → F4 = 0.98095

F1*0+F2*0+F3*1040+0.9809* 50=30→ F3= -0.018315

F1*0+F2*1030-0.018315*40+0.9809* 50=30→

F2= -0.01778

F1*1000 -0.01778 *30-0.018315*40+0.9809* 50=30→

F1= -0.01778

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Replicating portfolio: method 3

Case 3		buy/sell face value					
Time (year)			0.5	1	1.50	2	
Cash flow of 6% 2-year coupon bond			30	30	30	1030	
2-year 10% coupon bond	0.980952381	980.952381	49.04761905	49.05	49.04762	1030	
cash flow of port folio							
1.5-year 8% coupon bond	-0.018315018	-18.31501832	-0.732600733	-0.733	-19.04762		
cash flow of port folio					30		
1-year 6% coupon bond	-0.017781571	-17.78157118	-0.533447135	-18.32			
cash flow of port folio				30			
0.5 zero coupon bond	-0.017781571	-17.78157118	-17.78157118				
cash flow of port folio			30				
							984.1

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Replicating portfolio: method 3

In general set up and solve from the last cash (n) flow to the first cash flow

F1*0+F2*0+F3*0+…….+Fn*Mn = Cn → Fn =?

F1*0+F2*0+F3*Mn-1+Fn* Mn=Cn-1→ Fn-1 =?

:

:

:

F1*0+F2*M2+ ……..Fn*Mn=C2→ F2=?

F1*M1+F2*M2+ ……..Fn*Mn=C1

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Relative pricing in bond markets

Are these prices consistent?

1. One dollar a year from now for $0.80 today
2. One dollar 2 years from now for $0.70 today
3. Two dollars a year from now for $1.6 today
4. One dollar one year from now + one dollar two years from now for $1.5 today.

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Relative pricing of bonds

- 2-year 12% coupon bond price = 1092.1
- 2-year 6% coupon bond price = 984.2
- 2-year zero coupon bond price = 873.4

- Is there anything wrong with the bond prices?

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Relative pricing of bonds

- ❑ 2-year 12% coupon bond price = 1092.1
- ❑ 2-year 6% coupon bond price = 984.2
- ❑ 2-year zero coupon bond price = 873.4

- ❑ Is there anything wrong with the bond prices?

  Yes, the price of the 2-year 12% coupon bond is cheap relative to the 2-year 6% coupon bond and the 2-year zero coupon bond.

- ❑ How to spot relative mispricing and take advantage of it?

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Relative pricing in bond markets

The law of one price:

Two portfolios, A and B, that have identical cash flows must have the same price (they have the same risk).

- If portfolio A and B have the same cash flows and the price of A is higher than the price of B, how do we take advantage of this mispricing?
- What if the price of A is lower than B?

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Relative pricing in bond markets

The law of one price:

Two portfolios, A and B, that have identical cash flows must have the same price (they have the same risk).

- If portfolio A and B have the same cash flows and the price of A is higher than the price of B, how do we take advantage of this mispricing?

Sell A and buy B

- What if the price of A is lower than B?

Sell B and buy A

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Arbitrage

- An arbitrage strategy is a **trading strategy** that results in either of the following
  1. Positive cash flow today and non-negative cash flow in the future
  2. Zero investment today and non-negative cash flow in the future with at least one state with a positive cash flow

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Arbitrage

- How to check for arbitrage in the market
  1. Pick a bond.
  2. Construct a replicating portfolio for that bond
  3. Compare prices of the replicating portfolio and the bond

Bond	Maturity (Years)	Price/1000
Zero	0.5	970.87
Zero	1.0	938.97
Zero	1.5	909.09
Zero	2.0	873.36
6% Coupon Bond	2.0	987.0

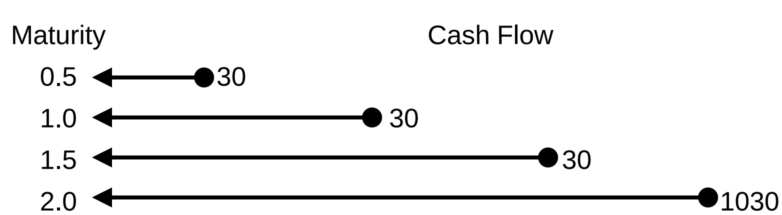
Arbitrage

Example

- Cash flow: 6% coupon, 2-year bond, face value = $1000

Time	0.5	1.0	1.5	2.0
Cash Flow	30	30	30	1000+30

- Portfolio of zero-coupon bonds or STRIPS



Arbitrage

- Replicating portfolio
  1. 0.5 zero face value , price = 970.9* = 29.127
  2. 1.0 zero face value , price = 939* = 28.17
  3. 1.5 zero face value , price = 909.1* = 27.273
  4. 2.0 zero face value , price = 873.4* = 899.602

  Total portfolio price = 984.2

- Coupon bond price = 987.0 > 984.2 = portfolio price.
- What to do?
  – Buy replicating portfolio and sell coupon

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Arbitrage

1. Cash flows if the securities are held until maturity

		Years			
Security	Today	0.5	1.0	1.5	2
Portfolio	-984.2	30	30	30	1030
6% Bond	987.0	-30	-30	-30	-1030
Total	2.8	0	0	0	0

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Arbitrage

2. As more arbitrage transactions take place, there are going to be many sell trades for the coupon bond. The price of 6% coupon bond will decrease until it equals the price of the replicating portfolio.

In this case the market corrects before maturity

What can you do?

- Sell portfolio of zeros and used proceeds to close out short sell position of the coupon bond. Future cash flow = 0.

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Concept check

- Is there an arbitrage opportunity in the following market?

2-year 12% coupon bond price = 1092.1

2-year 6% coupon bond price = 984.2

2-year zero coupon bond price = 873.4

Which portfolio do we form to check?

G) 2-year 6% & 2-year zero = 2-year 12%

Y) 2-year 12% & 2-year zero = 2-year 6%

R) Either

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Applications of cash flow matching

- Arbitrage
- Construct a portfolio investment in bonds for a particular income demand
  - Example: It can be used to answer the following question

    How to invest today to match the cash flow needs in the future
- Pricing other coupon bonds for issuing

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Arbitrage when there is cost

GOVT. BOND & NOTES

RATE	MATURITY MO/YR	BID	ASKED	CHG.	ASKED YLD.
5½	Jul 01n	100:00	100:02	....	1.68
6⅝	Jul 01n	100:00	100:02	– 1	2.76

- Bid price is the price a dealer/broker/market maker is willing to buy the security
- Asked price is the price a dealer/broker/market maker is willing to sell the security
- Asked price is higher or equal to bid price
- Bid-asked spread is the compensation for the market maker for the operation of making markets, hold inventory, and cost of asymmetry of information.

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Arbitrage when there is cost

- If the cash flow of portfolio A and B are identical
- There is an arbitrage opportunity if:
  1. The Asked price of A is lower than the Bid price of B, then buy A and sell B.

     Or

  2. The Asked price of B is lower than the Bid price of A, then buy B sell A.
- If it is possible to sell or buy within the Bid/Asked spread, then arbitrage opportunity exists if investors can sell A at a higher price than they can buy B (or vise versa).

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Arbitrage when there is cost

1. Start with one asset, A
2. Create an equivalent asset, B, using other securities
3. Check if Asked price of A is lower than Bid price of B
   - If so, buy A and sell B.
4. Check if Asked price of B is lower than Bid price of A
   - If so, buy B and sell A
5. Repeat for all possible combinations of equivalent assets.

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Arbitrage when there is cost

- Is there an arbitrage opportunity? If so, how do you set up a trading strategy to capitalize on this mispricing?
- Securities traded in the market:

Rate	Mat. (years)	Bid Price	Ask Price
8%	1.5	100	101.02
0	0.5	97.08	97.1
0	1	93.08	93.1
0	1.5	90.08	90.1

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Arbitrage when there is cost

- We can create the cash flow of the coupon bond using zeros. Let A be the coupon bond and B be the portfolio of zeros.
- Bid price of A: 100.00
- Asked price of portfolio B:
  1. 0.5 zero face value 4, price = 97.1*0.04=3.884
  2. 1.0 zero face value 4, price= 93.1*0.04=3.724
  3. 1.5 zero face value 104, price = 90.1*1.04=93.704
  – Total portfolio = 101.312
- Is bid price of A > asked price of B?

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Arbitrage when there is cost

- Asked price of A: 101.02

- Bid price of portfolio B:
  1. 0.5 zero face value 4, price =97.08*0.04=3.8832
  2. 1.0 zero face value 4, price=93.08*0.04=3.7232
  3. 1.5 zero face value 104, price = 90.08*1.04=93.6832
  – Total portfolio sold = 101.2896

- Bid price of B > Asked price of A?
  – Yes.
- What to do?

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Arbitrage when there is cost

- Cash flow if the securities are held until maturity

Security	Today	Years 0.5	1.0	1.5
Buy A	-101.02	4	4	104
Sell B	101.2896	-4	-4	-104
Total	.2696	0	0	0

- Results in no future liabilities but current gains of $0.2696 per $100 face value of the coupon bond.

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Arbitrage when there is cost

- An alternative arbitrage strategy

		Years		
Security	Today	0.5	1.0	1.5
Buy 1.002667 of A**	-101.2896	4.011	4.011	104.277
Sell B	101.2896	-4	-4	-104
Total	0	0.0110	0.0110	0.277

** 1.002667 = 101.2896/101.02

- Results in no cash flow today but <u>certain</u> positive cash flow in the future

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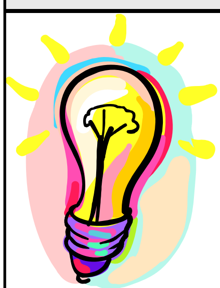
Concept check



Between two securities, it is normal to find arbitrage opportunity both ways when you

1. Check if Asked price of A is lower than Bid price of B
2. Check if Asked price of B is lower than Bid price of A? and Why?

G) True

Y) False

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Arbitrage and dominant securities

- Dominant securities: Securities that are in all respects superior (e.g., higher promised cash flow, lower risk)
- Can we find/create a synthetic security that dominates other securities but has a lower price?
- Check if the Asked price of the dominant security < Bid price of the dominated security
- If yes, then buy the dominant security and sell the dominated one.

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Arbitrage and dominant securities

- A putable bond gives the holder the right to sell the bond back to the issuer at a predetermined price during the put period.
- Compare a simple bond and a putable bond. Which is the dominant security?
- Putable bond.
- Price of putable bond = price of simple bond + put price

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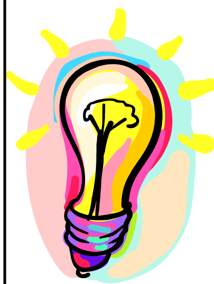
Concept check



The bond market:	Bid	Asked
6-year 10% coupon bond	102:0	102:1
6-year 10% coupon bond putable in the last two years	101:8	101:9

Is there an arbitrage opportunity in this market?

G) No

Y) Yes. Buy putable bond and sell straight bond

R) Yes. Buy straight bond and sell putable bond

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Arbitrage and dominant securities

The bond market:	Bid	Asked
6-year 10% coupon bond	102:0	102:1
6-year 10% coupon bond putable in the last two years	101:8	101:9

Is there an arbitrage opportunity in this market?

Yes. Putable bond is the dominant security, but we can buy it at 101:9 < 102:0 what we can sell the straight bond which is a dominated security

Buy putable bond and sell straight bond.

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Arbitrage and dominant securities

The bond market	Bid	Asked
6-year 10% coupon bond	102:0	102:1
6-year 8% coupon bond putable in the last 2 years	95:01	95:02
6-year zero	71:00	71:01

- Is there an arbitrage opportunity in this market? If so, how do we capitalize on it?

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Arbitrage and dominant securities

- Create a 6-year 8% coupon bond from 6-year 10% coupon bond and a 6-year zero

Cash Flow at:	0.5	1.0	………	6.0 years
10% Bond	5	5		105
8% Bond	4	4		104
Portfolio B: buy 0.8 of 10% and buy .2 of zero				
Buy 0.8 of 10% Bond	4	4		84
Buy 0.2 of Zero	0	0		20
Portfolio B	4	4		104

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Arbitrage and dominant securities

- What is the Asked price of the putable bond?

  95+2/32 =95.0625
- What is the Bid price of Portfolio B?

	Bid	Asked
6-year 10% coupon bond	102:0	102:1
6-year zero	71:00	71:01

- The Bid price is (102)*0.8 + (71)*0.2 = 95.8
- Buy putable bond and sell portfolio B.

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Arbitrage and dominant securities: example

The bond market:	Bid	Ask
6-year 10% coupon bond putable in last two years	102:0	102:1
6-year 8% coupon bond	96:01	96:02
6-year zero	71:00	71:01
4-year zero	80:02	80:04

- Is there an arbitrage in this market? If so, how do we capitalize on it?

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Arbitrage and dominant securities: example

- Create a 6-year 10% coupon bond from 6-year 8% coupon bond and a 6-year zero and find what is the ask price

Cash Flow at:	0.5	1.0	………	6.0 years
10% Bond	5	5		105
8% Bond	4	4		104

Portfolio B: buy 1.25 of 8% and sell .25 of zero

Buy 1.25 of 8% Bond	5	5		130
Sell 0.25 of Zero	0	0		-25
Portfolio B	5	5		105

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Arbitrage and dominant securities: example

- What is the Bid price of Portfolio B?

	Bid	Asked
6-year 8% coupon bond	96:01	96:02
6-year zero	71:00	71:01

- The bid price of Portfolio B:
  (96+1/32)*1.25 - (71+1/32)*0.25 = 102.28
- What is the ask price of putable bond?
  102:1=102+1/32 =102.03125
- What to do?
  Buy putable bond and sell portfolio B.

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Arbitrage and dominant securities

Future obligations

Case 1. If the market corrects itself before maturity, then putable bond price >= portfolio.

- Liquidate positions by selling putable bond and closing out the short position on B. Future cash flow >=0.

Case 2. If the market does not correct and interest rates are low such that it is never optimal to put back the bond, then hold both positions to maturity. Cash flows cancel out. Future obligation =0.

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Arbitrage and dominant securities

Future obligations

Case 3. If markets does not correct but interest rates increase. Interest rate increases both the straight bond price and the putable bond price decreases. Put the bond back when the put price or the exercise price = the price of straight bond. Use the proceeds from put price to buy back straight bond and close out the short position. Future obligation = 0.

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Other practical considerations

- ❑ Liquidity and market depth
- ❑ Short selling may not be possible or costly
- ❑ Can you borrow money? At what cost?
- ❑ Transaction costs

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Summary

- ❑ Trades and bond trading
- ❑ Bond as a portfolio of cash flows
- ❑ Cash flows matching
- ❑ Relative pricing in bond markets
- ❑ Arbitrage
- ❑ Arbitrage when there is cost
- ❑ Practical considerations

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