1. Two individuals agree at date 0 to a forward contract that matures at date 2. The contract is written on an underlying asset that pays a dividend at date 1 equal to D_1. Let f_2 be the date 2 random payoff (profit) to the individual who is the long party in the forward contract. Also let m_{0i} be the stochastic discount factor over the period from dates 0 to i where $i = 1, 2$, and let $E_0[\cdot]$ be the expectations operator at date 0. What is the value of $E_0[m_{02} f_2]$? Explain your answer.

let S_1 = underlying asset at date i

D_0 = date 0

$$S_0 = E_0[m_{01} D_1] + E_0[m_{02} S_2] = D_0 + E_0[m_{02} S_2]$$

payoff $f_2 = S_2 - F_{02}$

$$E_0[m_{02} f_2] = E_0[m_{02}(S_2 - F_{02})] = E[m_{02} S_2] - E[m_{02} F_{02}]$$

So, $E[m_{02} f_2] = E[m_{02} S_2] - E[m_{02} F_{02}]$

$$= S_0 - D_0 - R_f^{-2} F_{02}$$

$\therefore$ arbitrage absense : $F_{02} = R_f^2 (S_0 - D_0)$

$$\therefore E_0(m_{02} f_2) = 0 \ \#$$

2. Assume that there is an economy populated by infinitely-lived representative individuals who maximize the lifetime utility function

$$E_0 \left[\sum_{t=0}^{\infty} -\delta^t e^{-ac_t} \right]$$

where c_t is consumption at date t and $a > 0$, $0 < \delta < 1$. The economy is a Lucas (1978) endowment economy having multiple risky assets paying date t dividends that total d_t per capita. Write down an expression for the equilibrium per capita price of the market portfolio in terms of the assets' future dividends.

$$P_0 = E_0 \left[\sum_{t=1}^{\infty} \frac{u_c(c_t, t)}{u_c(c_0, 0)} d_t \right] \quad ; \quad u(c_t, t) = -\delta^t e^{ac_t}$$

$$= a\delta^t e^{-ac_t} \quad ; \quad c_t = d_t$$

$$P_0 = E_0 \left[\sum_{t=1}^{\infty} \frac{u_c(c_t, t)}{u_c(c_0, 0)} d_t \right] = E_0 \left[\sum_{t=1}^{\infty} \delta^t e^{-a(d_t - d_+)} d_t \right]$$

3. For the Lucas model with labor income, show that assumptions (6.25) and (6.26) lead to the pricing relationship (6.27) and (6.28).

$$P_t = E_t\left[\sum_{t=1}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t}\right)^{\gamma-1} d_{t+j}\right]$$

$$P_t/d_t = E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)\ln(C_{t+j}/C_t) + \ln(d_{t+j}/d_t)}\right]$$

$$\ln(C_{t+j}/C_t) = j u_c + \sigma_c \sum_{j=1}^{j} n_{t+j}$$

$$\ln(D_{t+j}/D_t) = j u_d + \sigma \sum_{j=1}^{j} \varepsilon_{t+j}$$

$$P_t/d_t = E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)(j u_c + \sigma_c \sum_{j=1}^{j} n_{t+3}) + j u_d + \sigma_d \sum_{j=1}^{j} \varepsilon_{t+3}}\right]$$

$$= E_t\left[\sum_{j=1}^{\infty} \delta^j e^{[(\gamma-1)u_c + u_c] + \sum_{j=1}^{j}[(\gamma-1)\sigma_d n_{d+j} + \sigma_d \varepsilon_{t+j}]}\right]$$

$$= \sum_{j=1}^{\infty} \delta^j e^{j[(\gamma-1)u_c + \sigma_d]} e^{\frac{j}{2}[(1-\gamma)^2\sigma_c^2 + \sigma_d^2 - 2(1-\gamma)\sigma_c\sigma_d\rho]}$$

$$= \frac{1}{1-\delta e^{-(1-\gamma)u_c + u_d + \frac{1}{2}((1-\gamma)^2\sigma_c^2 + \sigma_d^2) - 2(1-\gamma)\sigma_c\sigma_d\rho}} - 1$$

$$P_t = d_t \frac{\delta e^a}{1-\delta e^a} \qquad \text{where } a = u_d - (1-\gamma)u_c + \frac{1}{2}\left[(1-\gamma)^2\sigma_c^2 + \sigma_d^2\right] - (1-\gamma)\rho\sigma_c\sigma_d$$

4

a) check $E_t(b_{t+1}) = R_f b_t$

$$E_t[b_{t+1}] = \frac{R_f}{q_c} b_c q_c + E_t[e_{t+1}] q_c + (1-q_c) E[z_{t+1}] = R_f b_t$$ ✓ Valid

b) 4.b Suppose that p_t is the price of a barrel of oil. If $p_t \geq p_{solar}$, then solar energy, which is in perfectly elastic supply, becomes an economically efficient perfect substitute for oil. Can a rational speculative bubble exist for the price of oil? Explain why or why not.

Since $E_t[b_{t+1}] = R_f b_t$

$$\lim_{j \to \infty} E_t[b_{t+i}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

we need to consider only bubbles with $b_t > 0$

it is not rational expectation of oil price for an upper bound.

c) 4.c Suppose p_t is the price of a bond that matures at date $T < \infty$. In this context, the d_t for $t \leq T$ denotes the bond's coupon and principal payments. Can a rational speculative bubble exist for the price of this bond? Explain why or why not.

Since bond's price must be $P_f = d_t$ at maturity and zero after date T. Its price cannot rationally be to increase infinitely. The only rational price is $P_t = P_t^*$ #

5

Asset prices could not have form of $P_t = f_t + b_t$ with $b \neq 0$

because at date T, $P_T = f_T = d_T$ = asset's final dividend payment

and $b_T = 0$ so, $E_{T-1}[b_T] = E_{T-1}[0] = \delta^{-1} b_{T-1}$