Assignment 4 EE320 (Section Aj. Kittichai)

Due on Nov., 26th 2020

Instruction

1) All groups must attempt question 0.
2) Odd-numbered group must attempt all odd-numbered questions.
3) To submit your homework, write your filename as follow **hw4_Group_0x**. One point will be deducted if you don't follow the format of suggested filename.

Question 0: (Three-variable optimization)

Suppose that there are three firms. Each firm produces one single product that is imperfectly substitutable to the other two products, i.e. differentiated products. Each of the firm *strategically* competes with each other, and faces the following market demand equations given by,

$$Q_1 = 80 - 2P_1 + P_{23}$$

$$Q_2 = 80 - 2P_2 + P_{13}$$

$$Q_3 = 80 - 2P_3 + P_{12}$$

where P_{23} is the average of the prices charged by firms 2 and 3, P_{13} is the average of the prices charged by firms 2, P_{12} is the average of the prices charged by firms 1 and 2 [e.g., $P_{12} = 0.5(P_1 + P_2)$]. Suppose that each of the three firms, as indexed by j, has the cost function given by $C^j(Q_j) = c_j Q_j$.

Consider the following problems.

a) Set up the profit function of each individual firm, and derive the profit-maximizing condition of each of the three firms when each of them *optimally* sets for the level of price that maximizes its own profit. (**Hint:** *Derive the best response function of each firm, i.e. optimal contingent plan or reaction function.*)

b) Suppose that Based on (a), what would be the *equilibrium* level of price that each of the firms choose. Calculate the level of profit that each firm yields. (Hint: *Solve for the Bertrand equilibrium with differentiated product*)

c) How does the optimal price vary with respect to the size of marginal cost?

Now suppose that all the three firms agree to operate under a cartel (collusive) agreement. That is, they consider a joint pricing scheme that maximizes the joint profit function of the three firms combined. Consider the following problems.

d) Construct the joint profit function.

e) Given that $c_j = c, \ \forall j.$ Calculate the level of optimal price setting that each of the firm will set under the joint profit function problem. Confirm your result with the second-order derivative test.

f) Is the Cartel agreement self-sustaining? Why?

g) Revisit question (e) and assume that the marginal costs are varied with respect to firms. What would be the implication of the collusive equilibrium?

<u>Question 1:</u>

Given the production function

$$Q = f(K, L) = K^{0.5}L^{0.5}$$

Suppose that the per unit input prices for K and L are \$20 and \$5, respectively.

a. If the producer needs to maintain the output level at $\underline{Q} = 20$ units, what are the values K^* *and* L^* *that minimizes the total cost*, and what is the corresponding minimum cost? Use the bordered Hessian to verify that the second-order sufficient condition is met.
b. Suppose now that the output level is not fixed, but the producer has a budget constraint at \$400. Assume that this producer spends his entire budget on the production. Determine the values K^* *and* L^* *that maximizes the total output.* Also, use the bordered Hessian to verify that the second-order sufficient condition is met.

<u>Question 2:</u>

Pakorn's utility function depends on the consumption of two commodities, *x* and *y*, and it is given by

$$U(x, y) = 2xy$$

Suppose that his income is \$72, and the prices per unit of *x* and *y* are \$4 and \$6, respectively. Assume that Pakorn spends all of his income, and the values of *x* and *y* are both non-zero.

a. Use the Lagrange method to determine the values of x^* and y^* that maximize Pakorn's utility given an income constraint. Verify that the second-order sufficient conditions are satisfied.
b. Determine the maximum utility level and the Lagrange multiplier. Interpret the economic interpretation of the Lagrange multiplier.
c. Suppose that the income is now \$73. Approximate the new maximum utility level.

<u>Question 3:</u>

Consider a two-period consumption model. The consumer's utility is given by:

$$U(C_1, C_2) = \frac{C_1^{1-\theta}}{1-\theta} + \beta\frac{C_2^{1-\theta}}{1-\theta},$$

where C_1 and C_2 are consumption levels in period 1 and period 2, respectively; θ measures the degree of relative risk aversion where $0 < \theta < 1$; and β is the discount factor where $0 < \beta < 1$.

The consumer's budget constraint is given by:

$$C_1 + \frac{C_2}{1+r} = I_1,$$

where r is the interest rate ($0 < r < 1$), and I_1 is the income in period 1.

Answer the following questions.

a) Write down the utility maximization problem subject to the above budget constraint. Use Lagrange method to derive the optimal values of C_1^* and C_2^* in terms of the parameters θ, β, r, and I_1.
b) Based on your answer in (3.1), derive the impact of an increase in the interest rate on the optimal consumption in period 1 ($\frac{\partial C_1^*}{\partial r}$). Interpret its economic meaning.
c) Verify your answer in (a) by using the second-order sufficient condition.

Question 4:

A consumer has the utility function $U = \ln C + \ln(24 - N)$, where C is consumption and N is labor supply. Her budget constraint is $pC = \bar{M} + wN$, where p is the price of the consumption good, w the wage rate, and $\bar{M}$ the consumer's non-wage income.

(a) Formulate the problem of utility maximization subject to the budget constraint, and derive the first-order conditions, using the Lagrange multiplier approach and ignoring the nonnegativity constraints.

(b) Find the demand function $C = C^*(p, w, \bar{M})$ and the labor supply function $N = N^*(p, w, \bar{M})$ (i.e., express C and N in terms of p, w, and $\bar{M}$). Show that N^* and C^* are homogeneous of degree zero in $(p, w, \bar{M})$.

(c) Let $U^* = \ln[C^*(p, w, \bar{M})] + \ln[24 - N^*(p, w, \bar{M})]$. Show that $\partial U^*/\partial \bar{M} > 0$ and $\partial U^*/\partial p < 0$. Show that U^* is concave in $\bar{M}$ and convex in p. What is the relationship between $\partial U^*/\partial \bar{M}$ and the Lagrange multiplier?

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d. Use the Lagrange method to determine the values of x^* and y^* that maximize Pakorn's utility given an income constraint. Verify that the second-order sufficient conditions are satisfied.
e. Determine the maximum utility level and the Lagrange multiplier. Interpret the economic interpretation of the Lagrange multiplier.
f. Suppose that the income is now \$73. Approximate the new maximum utility level.

<u>Question 4:</u> Cost minimization problem

Consider a cost minimization problem where firm chooses for optimal combination of capital (K) and labor (L). Suppose that r and w are the prices per unit of capital and labor, respectively. And assume further that the production technology of this firm is given by $\sqrt{K} + L = Q$. Consider the following problem

a. Solve for the optimal combination of capital and labor.
b. State the condition under which both types of factor inputs are used by firm.
c. Derive the cost function.