Chapter 11

Optimal Portfolio Choice and the Capital Asset Pricing Model

11-1. You are considering how to invest part of your retirement savings. You have decided to put $200,000 into three stocks: 50% of the money in GoldFinger (currently $25/share), 25% of the money in Moosehead (currently $80/share), and the remainder in Venture Associates (currently $2/share). If GoldFinger stock goes up to $30/share, Moosehead stock drops to $60/share, and Venture Associates stock rises to $3 per share,

a. What is the new value of the portfolio?

b. What return did the portfolio earn?

c. If you don't buy or sell shares after the price change, what are your new portfolio weights?

a. Let n_i be the number of share in stock I, then

$$n_G = \frac{200,000 \times 0.5}{25} = 4,000$$

$$n_M = \frac{200,000 \times 0.25}{80} = 625$$

$$n_V = \frac{200,000 \times 0.25}{2} = 25,000.$$

The new value of the portfolio is

$$p = 30n_G + 60n_M + 3n_v$$
$$= \$232,500.$$

b. $\text{Return} = \dfrac{232,500}{200,000} - 1 = 16.25\%$

c. The portfolio weights are the fraction of value invested in each stock.

GoldFinger: $\dfrac{n_G \times 30}{232,500} = 51.61\%$

Moosehead: $\dfrac{n_M \times 60}{232,500} = 16.13\%$

Venture: $\dfrac{n_V \times 3}{232,500} = 32.26\%$

11-2. You own three stocks: 600 shares of Apple Computer, 10,000 shares of Cisco Systems, and 5000 shares of Colgate-Palmolive. The current share prices and expected returns of Apple, Cisco, and Colgate-Palmolive are, respectively, $500, $20, $100 and 12%, 10%, 8%.

a. What are the portfolio weights of the three stocks in your portfolio?

b. What is the expected return of your portfolio?

c. Suppose the price of Apple stock goes up by $25, Cisco rises by $5, and Colgate-Palmolive falls by $13. What are the new portfolio weights?

d. Assuming the stocks' expected returns remain the same, what is the expected return of the portfolio at the new prices?

				Value	a.	b.	New Price	New Value	c.	d.
Apple	600	500	12	300000	0.30	3.6	525	315000	0.315	3.78
Cisco	10000	20	10	200000	0.20	2	25	250000	0.25	2.5
Colgate	5000	100	8	500000	0.50	4	87	435000	0.435	3.48
		Total		1000000		9.6				9.76

11-3. Consider a world that only consists of the three stocks shown in the following table:

Stock	Total Number of Shares Outstanding	Current Price per Share	Expected Return
First Bank	100 Million	$100	18%
Fast Mover	50 Million	$120	12%
Funny Bone	200 Million	$30	15%

a. Calculate the total value of all shares outstanding currently.

b. What fraction of the total value outstanding does each stock make up?

c. You hold the market portfolio, that is, you have picked portfolio weights equal to the answer to part b (that is, each stock's weight is equal to its contribution to the fraction of the total value of all stocks). What is the expected return of your portfolio?

Stock	Total Number of Shares Outstanding	Current Price per Share	Expected Return	Value	b.	
First Bank	100.00	$100	18%	$10,000.00	0.454545455	8.18%
Fast Mover	50.00	$120	12%	$6,000.00	0.272727273	3.27%
Funny Bone	200.00	$30	15%	$6,000.00	0.272727273	4.09%
			a. (in millions)	$22,000.00	c.	15.55%

11-4. There are two ways to calculate the expected return of a portfolio: either calculate the expected return using the value and dividend stream of the portfolio as a whole, or calculate the weighted average of the expected returns of the individual stocks that make up the portfolio. Which return is higher?

Both calculations of expected return of a portfolio give the same answer.

11-5. Using the data in the following table, estimate (a) the average return and volatility for each stock, (b) the covariance between the stocks, and (c) the correlation between these two stocks.

Year	2004	2005	2006	2007	2008	2009
Stock A	–10%	20%	5%	–5%	2%	9%
Stock B	21%	7%	30%	–3%	–8%	25%

a. $$\overline{R}_A = \frac{-10+20+5-5+2+9}{6} = 3.5\%$$

$$\overline{R}_B = \frac{21+30+7-3-8+25}{6}$$

$$= 12\%$$

$$\text{Variance of A} = \frac{1}{5}\begin{bmatrix} (-0.1-0.035)^2 + \\ (0.2-0.08)^2 + (0.05-0.035)^2 + \\ (-0.05-0.035)^2 + (0.02-0.035)^2 \\ +(0.09-0.035)^2 \end{bmatrix}$$

$$= 0.01123$$

$$\text{Volatility of A} = SD(R_A) = \sqrt{\text{Variance of A}} = \sqrt{.01123} = 10.60\%$$

$$\text{Variance of B} = \frac{1}{5}\begin{bmatrix} (0.21-0.12)^2 + (0.3-0.12)^2 + \\ (0.07-0.12)^2 + (-0.03-0.12)^2 + \\ (-0.08-0.12)^2 + (0.25-0.12)^2 \end{bmatrix}$$

$$= 0.02448$$

$$\text{Volatility of B} = SD(R_B) = \sqrt{\text{Variance of B}} = \sqrt{.02448} = 15.65\%$$

b. $$\text{Covariance} = \frac{1}{5}\begin{bmatrix} (-0.1-0.035)(0.21-0.12) + \\ (0.2-0.035)(0.3-0.12) + \\ (0.05-0.035)(0.07-0.12) + \\ (-0.05-0.035)(-0.03-0.12) + \\ (0.02-0.035)(-0.08-0.12) + \\ (0.09-0.035)(0.25-0.12) \end{bmatrix}$$

$$= 0.104\%$$

c. $$\text{Correlation} = \frac{\text{Covariance}}{\text{SD}(\text{R}_\text{A})\text{SD}(\text{R}_\text{B})}$$

$$= 6.27\%$$

11-6. Use the data in Problem 5, consider a portfolio that maintains a 50% weight on stock A and a 50% weight on stock B.

a. What is the return each year of this portfolio?

b. Based on your results from part a, compute the average return and volatility of the portfolio.

c. **Show that (i) the average return of the portfolio is equal to the average of the average returns of the two stocks, and (ii) the volatility of the portfolio equals the same result as from the calculation in Eq. 11.9.**

d. **Explain why the portfolio has a lower volatility than the average volatility of the two stocks.**

a, b, and c. See table below.

Year	2004	2005	2006	2007	2008	2009
A&B	5.5%	13.5%	17.5%	-4.0%	-3.0%	17.0%
Ave	7.75%					
Vol	9.72%					

d. The portfolio has a lower volatility than the average volatility of the two stocks because some of the idiosyncratic risk of the stocks in the portfolio is diversified away.

11-7. Using your estimates from Problem 5, calculate the volatility (standard deviation) of a portfolio that is 70% invested in stock A and 30% invested in stock B.

$$\text{Variance} = (0.7)^2\, 0.106^2 + (0.3)^2\, 0.1565^2 + 2(0.7)(0.3)(0.0627)(0.106)(0.1565)^{.5} =$$

$$\text{Standard Deviation} = \sqrt{} = 9.02\%$$

11-8. Using the data from Table 11.3, what is the covariance between the stocks of Alaska Air and Southwest Airlines?

$$\text{covariance} = \text{con} \times SD(R_D) \times SD(R_{AA}) = 0.30 \times 0.38 \times 0.31 = 0.03534$$

11-9. Suppose two stocks have a correlation of 1. If the first stock has an above average return this year, what is the probability that the second stock will have an above average return?

Because the correlation is perfect, they move together (always), so the probability is 1.

11-10. Arbor Systems and Gencore stocks both have a volatility of 40%. Compute the volatility of a portfolio with 50% invested in each stock if the correlation between the stocks is (a) + 1, (b) 0.50, (c) 0, (d) −0.50, and (e) −1.0. In which cases is the volatility lower than that of the original stocks?

stock vol	40%
corr	50-50 Port
1	40.0%
0.5	34.6%
0	28.3%
-0.5	20.0%
-1	0.0%

Volatility of portfolio is less if the correlation is < 1.

11-11. Suppose Wesley Publishing's stock has a volatility of 60%, while Addison Printing's stock has a volatility of 30%. If the correlation between these stocks is 25%, what is the volatility of the following portfolios of Addison and Wesley: (a) 100% Addison, (b) 75% Addison and 25% Wesley, and (c) 50% Addison and 50% Wesley.

	Vol	Corr
Wesley	60%	25%
Addison	30%	
Portfolio		
x_A	x_W	Vol
100%	0%	30.00%
75%	25%	30.00%
50%	50%	36.74%

11-12. Suppose Avon and Nova stocks have volatilities of 50% and 25%, respectively, and they are perfectly negatively correlated. What portfolio of these two stocks has zero risk?

Avon has twice the risk, so the portfolio needs twice as much weight on Nova => 2/3 Avon, 1/3 Nova.

11-13. Suppose Tex stock has a volatility of 40%, and Mex stock has a volatility of 20%. If Tex and Mex are uncorrelated,

a. What portfolio of the two stocks has the same volatility as Mex alone?

b. What portfolio of the two stocks has the smallest possible volatility?

	Vol	Corr
Tex	40%	0%
Mex	20%	
Portfolio		
x_tex	x_mex	Vol
0%	100%	20.00%
10%	90%	18.44%
20%	80%	17.89%
30%	70%	18.44%
40%	60%	20.00%
50%	50%	22.36%
60%	40%	25.30%
70%	30%	28.64%
80%	20%	32.25%
90%	10%	36.06%
100%	0%	40.00%

11-14. Using the data in Table 11.1,

a. Compute the annual returns for a portfolio with 25% invested in North Air, 25% invested in West Air, and 50% invested in Tex Oil.

b. What is the lowest annual return for your portfolio in part a? How does it compare with the lowest annual return of the individual stocks or portfolios in Table 11.1

a.

Year	North Air	West Air	Tex Oil	N+W	W+T	Portfolio
2007	21%	9%	-2%	15.0%	3.5%	6.50%
2008	30%	21%	-5%	25.5%	8.0%	10.25%
2009	7%	7%	9%	7.0%	8.0%	8.00%
2010	-5%	-2%	21%	-3.5%	9.5%	8.75%
2011	-2%	-5%	30%	-3.5%	12.5%	13.25%
2012	9%	30%	7%	19.5%	18.5%	13.25%

b. The portfolio computed in part a had its lowest annual return in 2007 (6.5%). This is higher than each individual stock and the other portfolios in Table 11.1 as well.

11-15. Using the data from Table 11.3, what is volatility of an equally weighted portfolio of Microsoft, Alaska Air, and Ford Motor stock?

29.3%

var-cov			
MSFT	0.1225	0.03192	0.050085
AA	0.03192	0.1444	0.03021
Ford	0.050085	0.03021	0.2809

ave var	0.1826
ave cov	0.037405
volatility	0.292922
	0.292922

11-16. Suppose that the average stock has a volatility of 50%, and that the correlation between pairs of stocks is 20%. Estimate the volatility of an equally weighted portfolio with (a) 1 stock, (b) 30 stocks, (c) 1000 stocks.

Vol	50%
Var	0.25
Corr	20%
Covar	0.05
N	Vol
1	50.0%
30	23.8%
1000	22.4%

11-17. What is the volatility (standard deviation) of an equally weighted portfolio of stocks within an industry in which the stocks have a volatility of 50% and a correlation of 40% as the portfolio becomes arbitrarily large?

$\sqrt{\text{ave cov}} = (0.5 \times 0.5 \times 0.4)^{0.5} = 31.62\%$

11-18. Consider an equally weighted portfolio of stocks in which each stock has a volatility of 40%, and the correlation between each pair of stocks is 20%.

a. What is the volatility of the portfolio as the number of stocks becomes arbitrarily large?

b. What is the average correlation of each stock with this large portfolio?

a. Ave Covar = 40% × 40% × 20%=0.032

Limit Vol = $(.032)^{0.5}$ = 17.89%

b. From Eq. 11.13

Corr = SD(Rp)/SD(Ri) = 17.89%/40% = 44.72%

11-19. Stock A has a volatility of 65% and a correlation of 10% with your current portfolio. Stock B has a volatility of 30% and a correlation of 25% with your current portfolio. You currently hold both stocks. Which will increase the volatility of your portfolio: (i) selling a small amount of stock B and investing the proceeds in stock A, or (ii) selling a small amount of stock A and investing the proceeds in stock B?

From Eq. 11.13, marginal contribution to risk is SD(Ri) × Corr(Ri,Rp)

For A: 65% × 10% = 6.5%;

For B: 30% × 25% = 7.5%.

So, volatility increases if we sell A and add B.

11-20. You currently hold a portfolio of three stocks, Delta, Gamma, and Omega. Delta has a volatility of 60%, Gamma has a volatility of 30%, and Omega has a volatility of 20%. Suppose you invest 50% of your money in Delta, and 25% each in Gamma and Omega.

a. What is the highest possible volatility of your portfolio?

b. If your portfolio has the volatility in (a), what can you conclude about the correlation between Delta and Omega?

a. Max vol = weighted average = 0.5(60%) + 0.25(30%) + 0.25(20%) = 42.5%

b. Correlation = 1 (otherwise there would be some diversification)

11-21. Suppose Ford Motor stock has an expected return of 20% and a volatility of 40%, and Molson Coors Brewing has an expected return of 10% and a volatility of 30%. If the two stocks are uncorrelated,

a. What is the expected return and volatility of an equally weighted portfolio of the two stocks?

b. Given your answer to (a), is investing all of your money in Molson Coors stock an efficient portfolio of these two stocks?

c. Is investing all of your money in Ford Motor an efficient portfolio of these two stocks?

a.

	A	B	Corr
ER	20%	10%	
Vol	40%	30%	0%
XA	XB	Vol	ER
50%	50%	25.0%	15.0%

b. No, dominated by 50-50 portfolio.

c. Yes, not dominated.

11-22. Suppose Intel's stock has an expected return of 26% and a volatility of 50%, while Coca-Cola's has an expected return of 6% and volatility of 25%. If these two stocks were perfectly negatively correlated (i.e., their correlation coefficient is −1),

a. Calculate the portfolio weights that remove all risk.

b. If there are no arbitrage opportunities, what is the risk-free rate of interest in this economy?

a. If the two stocks are perfectly correlated negatively, they fluctuate due to the same risks, but in opposite directions. Because Intel is twice as volatile as Coke, we will need to hold twice as much Coke stock as Intel in order to offset Intel's risk. That is, our portfolio should be 2/3 Coke and 1/3 Intel.

We can check this using Eq. 11.9.

$$\begin{aligned} Var(R_P) &= (2/3)^2 SD(R_{Coke})^2 + (1/3)^2 SD(R_{Intel})^2 + 2(2/3)(1/3)\mathrm{Corr}(R_{Coke}, R_{Intel})SD(R_{Coke})SD(R_{Intel}) \\ &= (2/3)^2(0.25^2) + (1/3)^2(0.50^2) + 2(2/3)(1/3)(-1)(0.25)(0.50) \\ &= 0 \end{aligned}$$

b. From Eq. 11.3, the expect return of the portfolio is

$$\begin{aligned} E[R_P] &= (2/3)E[R_{Coke}] + (1/3)E[R_{Intel}] \\ &= (2/3)6\% + (1/3)26\% \\ &= 12.67\%. \end{aligned}$$

Because this portfolio has no risk, the risk-free interest rate must also be 12.67%.

For Problems 23-26, suppose Johnson & Johnson and the Walgreen Company have expected returns and volatilities shown below, with a correlation of 22%.

	***E*[*R*]**	***SD*[*R*]**
Johnson & Johnson	7%	16%
Walgreen Company	10%	20%

11-23. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that is equally invested in Johnson & Johnson's and Walgreen's stock.

In this case, the portfolio weights are $x_j = x_w = 0.50$. From Eq. 11.3,

$$\begin{aligned} E[R_P] &= x_j E[R_j] + x_w E[R_w] \\ &= 0.50(7\%) + 0.50(10\%) \\ &= 8.5\%. \end{aligned}$$

We can use Eq. 11.9.

$$\begin{aligned} SD(R_P) &= \sqrt{x_j^{\,2} SD(R_j)^2 + x_w^{\,2} SD(R_w)^2 + 2x_j x_w Corr(R_j, R_w) SD(R_j) SD(R_w)} \\ &= \sqrt{0.50^2(0.16^2) + 0.50^2(0.20)^2 + 2(0.50)(0.50)(0.22)(0.16)(0.20)} \\ &= 14.1\% \end{aligned}$$

11-24. For the portfolio in Problem 23, if the correlation between Johnson & Johnson's and Walgreen's stock were to increase,

a. Would the expected return of the portfolio rise or fall?

b. Would the volatility of the portfolio rise or fall?

a. The expected return would remain constant, assuming only the correlation changes, $0.5 \times 0.07 + 0.5 \times 0.10 = 0.085$.

b. The volatility of the portfolio would increase (due to the correlation term in the equation for the volatility of a portfolio).

11-25. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that consists of a long position of \$10,000 in Johnson & Johnson and a short position of \$2000 in Walgreen's.

In this case, the total investment is \$10,000 – 2,000 = \$8,000, so the portfolio weights are x_j = 10,000/8,000 = 1.25, x_w = –2,000/8,000 = –0.25. From Eq. 11.3,

$$\begin{aligned} E[R_P] &= x_j E[R_j] + x_w E[R_w] \\ &= 1.25(7\%) - 0.25(10\%) \\ &= 6.25\%. \end{aligned}$$

We can use Eq. 11.9,

$$\begin{aligned} SD(R_P) &= \sqrt{x_j^{\,2} SD(R_j)^2 + x_w^{\,2} SD(R_w)^2 + 2x_j x_w Corr(R_j, R_w) SD(R_j) SD(R_w)} \\ &= \sqrt{1.25^2(0.16^2) + (-0.25)^2(0.20)^2 + 2(1.25)(-0.25)0(.22)(0.16)(0.20)} \\ &= 19.5\%. \end{aligned}$$

11-26. Using the same data as for Problem 23, calculate the expected return and the volatility (standard deviation) of a portfolio consisting of Johnson & Johnson's and Walgreen's stocks using a wide range of portfolio weights. Plot the expected return as a function of the portfolio volatility. Using your graph, identify the range of Johnson & Johnson's portfolio weights that yield efficient combinations of the two stocks, rounded to the nearest percentage point.

The set of efficient portfolios is approximately those portfolios with no more than 65% invested in J&J (this is the portfolio with the lowest possible volatility).

x(J&J)	x(Walgreen)	SD	ER
-50%	150%	29.30%	11.50%
-40%	140%	27.32%	11.20%
-30%	130%	25.38%	10.90%
-20%	120%	23.50%	10.60%
-10%	110%	21.70%	10.30%
0%	100%	20.00%	10.00%
10%	90%	18.42%	9.70%
20%	80%	16.99%	9.40%
30%	70%	15.77%	9.10%
40%	60%	14.79%	8.80%
50%	50%	14.11%	8.50%
60%	40%	13.78%	8.20%
65%	**35%**	**13.75%**	**8.05%**
70%	30%	13.82%	7.90%
80%	20%	14.23%	7.60%
90%	10%	14.97%	7.30%
100%	0%	16.00%	7.00%
110%	-10%	17.27%	6.70%
120%	-20%	18.73%	6.40%
130%	-30%	20.34%	6.10%
140%	-40%	22.07%	5.80%
150%	-50%	23.88%	5.50%

11-27. A hedge fund has created a portfolio using just two stocks. It has shorted $35,000,000 worth of Oracle stock and has purchased $85,000,000 of Intel stock. The correlation between Oracle's and Intel's returns is 0.65. The expected returns and standard deviations of the two stocks are given in the table below:

	Expected Return	Standard Deviation
Oracle	12.00%	45.00%
Intel	14.50%	40.00%

a. What is the expected return of the hedge fund's portfolio?

b. What is the standard deviation of the hedge fund's portfolio?

a. The total value of the portfolio is $50m = (–$35 + $85). This means that the weight on Oracle is –70% and the weight on Intel is 170%. The expected return is

$$\text{Expected return} = -0.7 \times 12\% + 1.7 \times 14.5\%$$
$$= 16.25\%.$$

b.

$$\text{Variance} = (-0.7)^2 \times (0.45)^2 + (1.7)^2 \times (0.40)^2$$
$$+ 2 \times (-0.7) \times (1.7) \times 0.65 \times 0.45 \times 0.40$$
$$= 0.283165$$

Std dev = $(0.283165)^{0.5}$ = 53.2%

11-28. Consider the portfolio in Problem 27. Suppose the correlation between Intel and Oracle's stock increases, but nothing else changes. Would the portfolio be more or less risky with this change?

An increase in the correlation would increase the variance of the portfolio; meanwhile, the expected return of the portfolio would remain constant. The riskiness of the portfolio would increase.

11-29. Fred holds a portfolio with a 30% volatility. He decides to short sell a small amount of stock with a 40% volatility and use the proceeds to invest more in his portfolio. If this transaction reduces the risk of his portfolio, what is the minimum possible correlation between the stock he shorted and his original portfolio?

From Eq. 11.13, for a small transaction size, short selling A and investing in P changes risk according to SD(Rp) – SD(Ra)Corr(Ra,Rp).

We gain the risk of the portfolio and lose the risk A has in common with the portfolio. For this to be negative, we must have

SD(Rp)/SD(Ra) < Corr(Ra,Rp)

or

Corr > 30%/40% = 75%.

11-30. Suppose Target's stock has an expected return of 20% and a volatility of 40%, Hershey's stock has an expected return of 12% and a volatility of 30%, and these two stocks are uncorrelated.

a. What is the expected return and volatility of an equally weighted portfolio of the two stocks?

Consider a new stock with an expected return of 16% and a volatility of 30%. Suppose this new stock is uncorrelated with Target's and Hershey's stock.

b. Is holding this stock alone attractive compared to holding the portfolio in (a)?

c. Can you improve upon your portfolio in (a) by adding this new stock to your portfolio? Explain.

a.

	A	B	Corr
ER	20%	12%	
Vol	40%	30%	0%
XA	XB	Vol	ER
50%	50%	25.0%	16.0%

b. No, it has the same expected return with higher volatility.

c. Yes, adding this stock and reducing weight on the others will reduce risk while leaving the expected return unchanged.

11-31. You have \$10,000 to invest. You decide to invest \$20,000 in Google and short sell \$10,000 worth of Yahoo! Google's expected return is 15% with a volatility of 30% and Yahoo!'s expected return is 12% with a volatility of 25%. The stocks have a correlation of 0.9. What is the expected return and volatility of the portfolio?

Expected return = 18%

Volatility= $\sqrt{x^2 0.3^2 + y^2 0.25^2 + 2xy0.9 \times 0.3 \times 0.25} = 39.05\%$

11-32. You expect HGH stock to have a 20% return next year and a 30% volatility. You have \$25,000 to invest, but plan to invest a total of \$50,000 in HGH, raising the additional \$25,000 by shorting *either* KBH or LWI stock. Both KBH and LWI have an expected return of 10% and a volatility of 20%. If KBH has a correlation of +0.5 with HGH, and LWI has a correlation of −0.50 with HGH, which stock should you short?

Either strategy has expected return of 2(20%) – 1(10%) = 30%.

But the portfolio has lower volatility if correlation is +0.5; because you are shorting a POSITIVE correlation, it leads to lower risk.

+2 HGH – KBH volatility = 52.9%

+2 HGH – LWI volatility = 72.1%

11-33. Suppose you have \$100,000 in cash, and you decide to borrow another \$15,000 at a 4% interest rate to invest in the stock market. You invest the entire \$115,000 in a portfolio J with a 15% expected return and a 25% volatility.

a. What is the expected return and volatility (standard deviation) of your investment?

b. What is your realized return if J goes up 25% over the year?

c. What return do you realize if J falls by 20% over the year?

a. $x = \dfrac{115{,}000}{100{,}000} = 1.15$

$E[R] = r_f + x\left(E\left[R_j\right] - r\right) = 4\% + 1.15(11\%) = 16.65\%$

$\text{Volatility} = x\ SD\left[R_j\right] = 1.15\ 25\% = 28.75\%$

b. $R = \dfrac{115{,}000(1.25) - 15{,}000(1.04)}{100{,}000} - 1 = 28.15\%$

c. $R = \dfrac{115{,}000(0.80) - 15{,}000(1.04)}{100{,}000} - 1 = -23.6\%$

11-34. You have \$100,000 to invest. You choose to put \$150,000 into the market by borrowing \$50,000.

a. If the risk-free interest rate is 5% and the market expected return is 10%, what is the expected return of your investment?

b. If the market volatility is 15%, what is the volatility of your investment?

a. Er = 5% + 1.5 × (10% – 5%) = 12.5%

b. Vol = 1.5 × 15% = 22.5%

11-35. You currently have \$100,000 invested in a portfolio that has an expected return of 12% and a volatility of 8%. Suppose the risk-free rate is 5%, and there is another portfolio that has an expected return of 20% and a volatility of 12%.

a. What portfolio has a higher expected return than your portfolio but with the same volatility?

b. What portfolio has a lower volatility than your portfolio but with the same expected return?

Invest an amount x in the other portfolio and the expected return and volatility are

$$E[R_x] = r_f + x(E[R_O] - r_f) = 5\% + x(20\% - 5\%)$$
$$SD(R_x) = x\ SD(R_O) = x(12\%).$$

a. So to maintain the volatility at 8%, $x = 8\%/12\% = 2/3$, you should invest \$66,667 in the other portfolio and the remaining \$33,333 in the risk-free investment. Your expected return will then be 15%.

b. Alternatively, to keep the expected return equal to the current value of 12%, x must satisfy 5% + x(15%) = 12%, so x = 46.667%. Now you should invest \$46,667 in the other portfolio and \$53,333 in the risk-free investment, lowering your volatility to 5.6%

11-36. Assume the risk-free rate is 4%. You are a financial advisor, and must choose *one* of the funds below to recommend to each of your clients. Whichever fund you recommend, your clients will then combine it with risk-free borrowing and lending depending on their desired level of risk.

	Expected Return	Volatility
Fund A	10%	10%
Fund B	15%	22%
Fund C	6%	2%

Which fund would you recommend without knowing your client's risk preference?

Sharpe ratios of A, B, and C are 0.6, .5, and 1, so you would choose C; it is the best choice no matter what your clients' risk preferences are.

11-37. Assume all investors want to hold a portfolio that, for a given level of volatility, has the maximum possible expected return. Explain why, when a risk-free asset exists, all investors will choose to hold the same portfolio of risky stocks.

Investors who want to maximize their expected return for a given level of volatility will pick portfolios that maximize their Sharpe ratio. The set of portfolios that do this is a combination of a risk-free asset—a single portfolio of risk assets—the tangential portfolio.

11-38. In addition to risk-free securities, you are currently invested in the Tanglewood Fund, a broad-based fund of stocks and other securities with an expected return of 12% and a volatility of 25%. Currently, the risk-free rate of interest is 4%. Your broker suggests that you add a venture capital fund to your current portfolio. The venture capital fund has an expected return of 20%, a volatility of 80%, and a correlation of 0.2 with the Tanglewood Fund. Calculate the required return and use it to decide whether you should add the venture capital fund to your portfolio.

$$\text{Required Return} = 4\% + 80\%(.2) \times \frac{(21\% - 14\%)}{20\%} = 10.4\%$$

You should add some of the venture fund to your portfolio because it has an expected return that exceeds the required return.

11-39. You have noticed a market investment opportunity that, given your current portfolio, has an expected return that exceeds your required return. What can you conclude about your current portfolio?

Your current portfolio is not efficient.

11-40. The Optima Mutual Fund has an expected return of 20% and a volatility of 20%. Optima claims that no other portfolio offers a higher Sharpe ratio. Suppose this claim is true, and the risk-free interest rate is 5%.

a. What is Optima's Sharpe Ratio?

b. If eBay's stock has a volatility of 40% and an expected return of 11%, what must be its correlation with the Optima Fund?

c. If the SubOptima Fund has a correlation of 80% with the Optima Fund what is the Sharpe ratio of the SubOptima Fund?

a. Optima's Sharpe Ratio = (20% – 5%)/20% = 0.75

b. Using equation 11.18: eBay's Sharpe Ratio = (11% – 5%)/40% = 0.15, It's correlation must be 0.15/0.75 = 0.2

c. Using equation 11.18: SubOptima's Sharpe Ratio = 0.8 × 0.75 = 0.6

11-41. You are currently only invested in the Natasha Fund (aside from risk-free securities). It has an expected return of 14% with a volatility of 20%. Currently, the risk-free rate of interest is 3.8%. Your broker suggests that you add Hannah Corporation to your portfolio. Hannah Corporation has an expected return of 20%, a volatility of 60%, and a correlation of 0 with the Natasha Fund.

a. Is your broker right?

b. You follow your broker's advice and make a substantial investment in Hannah stock so that, considering only your risky investments, 60% is in the Natasha Fund and 40% is in Hannah stock. When you tell your finance professor about your investment, he says that you made a mistake and should reduce your investment in Hannah. Is your finance professor right?

c. You decide to follow your finance professor's advice and reduce your exposure to Hannah. Now Hannah represents 15% of your risky portfolio, with the rest in the Natasha fund. Is this the correct amount of Hannah stock to hold?

	Initial Portfolio	60-40 Portfolio	85-15 Portfolio
Natasha Fund			
Expected Return	0.14	0.14	0.14
Volatility	0.2	0.2	0.2

Hannah Stock			
Expected Return	0.2	0.2	0.2
Volatility	0.6	0.6	0.6
Risk-Free Rate	0.038	0.038	0.038
Portfolio weight in Hannah	0	0.4	0.15
Expected Return of Portfolio	0.14	0.164	0.149
Volatility of Portfolio	0.2	0.268328157	0.192353841
Beta	0	2	1.459459459
Required Return	0.038	0.29	0.2

a) Yes, because the expected return of Hannah stock exceeds the required return.
b) Yes, because the expected return of Hannah stock is less than the required return.
c) Yes, because now the required and expected return are the same.

11-42. Calculate the Sharpe ratio of each of the three portfolios in Problem 39. What portfolio weight in Hannah stock maximizes the Sharpe ratio?

	Initial Portfolio	60-40 Portfolio	85-15 Portfolio
Natasha Fund			
Expected Return	0.14	0.14	0.14
Volatility	0.2	0.2	0.2
Hannah Stock			
Expected Return	0.2	0.2	0.2
Volatility	0.6	0.6	0.6
Risk-Free Rate	0.038	0.038	0.038
Portfolio weight in Hannah	0	0.4	0.15
Expected Return of Portfolio	0.14	0.164	0.149
Volatility of Portfolio	0.2	0.268328157	0.192353841
Sharpe Ratio	0.51	0.469574275	0.577061522
	0.09002	0.097166359	0.102053829

The Sharpe Ratio is maximized at 15% in Hannah Stock.

11-43. Returning to Problem 38, assume you follow your broker's advice and put 50% of your money in the venture fund.

a. What is the Sharpe ratio of the Tanglewood Fund?

b. What is the Sharpe ratio of your new portfolio?

c. What is the optimal fraction of your wealth to invest in the venture fund? (*Hint:*Use Excel and round your answer to two decimal places.)

a. 0.32 b. 0.271 c. 13%

	Initial Portfolio	50-50 Split	
Tanglewood Fund			
Expected Return	0.12	0.12	
Volatility	0.25	0.25	0.25
Venture Fund			
Expected Return	0.2	0.2	
Volatility	0.8	0.8	
Risk-Free Rate	0.04	0.04	
Portfolio weight in Hannah	0	0.5	
Expected Return of Portfolio	0.12	0.16	
Volatility of Portfolio	0.25	0.44229515	0.25
Sharpe Ratio	0.32	0.271312041	
	0.0656	0.072557445	

The Sharpe Ratio is maximized at 12% in the venture fund.

Weight in venture fund	Expected Return	Volatility	Sharpe Ratio	
0	0.12	0.25	0.32	Part a
0.01	0.1208	0.249223293	0.324207256	
0.02	0.1216	0.248694592	0.328113287	
0.03	0.1224	0.248415479	0.331702358	
0.04	0.1232	0.248386795	0.334961446	
0.05	0.124	0.248608628	0.337880469	
0.06	0.1248	0.249080308	0.340452445	
0.07	0.1256	0.24980042	0.342673563	
0.08	0.1264	0.250766824	0.344543184	
0.09	0.1272	0.251976685	0.346063763	
0.1	0.128	0.253426518	0.347240694	
0.11	0.1288	0.25511223	0.348082097	
0.12	0.1296	0.257029181	0.34859855	
0.13	0.1304	0.25917224	0.348802788	Part c
0.14	0.1312	0.261535848	0.348709366	
0.15	0.132	0.264114085	0.348334319	
0.16	0.1328	0.266900731	0.347694814	
0.17	0.1336	0.269889329	0.346808821	
0.18	0.1344	0.27307325	0.34569479	
0.19	0.1352	0.276445745	0.34437137	
0.2	0.136	0.28	0.342857143	
0.21	0.1368	0.283729184	0.341170403	
0.22	0.1376	0.287626494	0.339328963	
0.23	0.1384	0.29168519	0.337350004	
0.24	0.1392	0.295898631	0.335249945	
0.25	0.14	0.300260304	0.333044358	
0.26	0.1408	0.304763843	0.330747896	
0.27	0.1416	0.309403054	0.328374263	
0.28	0.1424	0.314171927	0.325936187	
0.29	0.1432	0.319064649	0.323445422	

Weight in venture fund	Expected Return	Volatility	Sharpe Ratio	
0.3	0.144	0.324075608	0.320912766	
0.31	0.1448	0.329199408	0.318348082	
0.32	0.1456	0.33443086	0.315760334	
0.33	0.1464	0.339764992	0.313157631	
0.34	0.1472	0.345197045	0.31054727	
0.35	0.148	0.350722469	0.307935789	
0.36	0.1488	0.356336919	0.305329013	
0.37	0.1496	0.362036255	0.302732112	
0.38	0.1504	0.36781653	0.300149642	
0.39	0.1512	0.373673989	0.297585605	
0.4	0.152	0.379605058	0.295043487	
0.41	0.1528	0.385606341	0.29252631	
0.42	0.1536	0.39167461	0.290036671	
0.43	0.1544	0.3978068	0.287576784	
0.44	0.1552	0.404	0.285148515	
0.45	0.156	0.410251447	0.282753421	
0.46	0.1568	0.416558519	0.280392777	
0.47	0.1576	0.422918727	0.278067611	
0.48	0.1584	0.42932971	0.275778725	
0.49	0.1592	0.435789227	0.273526725	
0.5	0.16	0.44229515	0.271312041	Part b
0.51	0.1608	0.448845463	0.269134947	
0.52	0.1616	0.45543825	0.26699558	
0.53	0.1624	0.462071694	0.264893958	
0.54	0.1632	0.468744067	0.262829994	
0.55	0.164	0.475453731	0.260803506	
0.56	0.1648	0.482199129	0.258814238	
0.57	0.1656	0.488978783	0.256861861	
0.58	0.1664	0.495791287	0.254945989	
0.59	0.1672	0.502635305	0.253066187	
0.6	0.168	0.509509568	0.251221975	
0.61	0.1688	0.516412868	0.24941284	
0.62	0.1696	0.523344055	0.247638239	
0.63	0.1704	0.530302037	0.245897604	
0.64	0.1712	0.537285771	0.244190349	
0.65	0.172	0.544294268	0.242515874	
0.66	0.1728	0.551326582	0.240873566	
0.67	0.1736	0.558381814	0.239262807	
0.68	0.1744	0.565459106	0.237682971	
0.69	0.1752	0.572557639	0.236133431	

11-44. When the CAPM correctly prices risk, the market portfolio is an efficient portfolio. Explain why.

All investors will want to maximize their Sharpe ratios by picking efficient portfolios. When a riskless asset exists this means that all investors will pick the same efficient portfolio, and because the sum of all investors' portfolios is the market portfolio this efficient portfolio must be the market portfolio.

11-45. A big pharmaceutical company, DRIg, has just announced a potential cure for cancer. The stock price increased from $5 to $100 in one day. A friend calls to tell you that he owns DRIg. You proudly reply that you do too. Since you have been friends for some time, you know that he holds

the market, as do you, and so you both are invested in this stock. Both of you care only about expected return and volatility. The risk-free rate is 3%, quoted as an APR based on a 365-day year. DRIg made up 0.2% of the market portfolio before the news announcement.

a. On the announcement, your overall wealth went up by 1% (assume all other price changes canceled out so that without DRIg, the market return would have been zero). How is your wealth invested?

b. Your friend's wealth went up by 2%. How is he invested?

a. 26.16% is in the market; the rest in the risk-free asset.

b. 52.53% is in the market; the rest in the risk-free asset.

11-46. Your investment portfolio consists of $15,000 invested in only one stock—Microsoft. Suppose the risk-free rate is 5%, Microsoft stock has an expected return of 12% and a volatility of 40%, and the market portfolio has an expected return of 10% and a volatility of 18%. Under the CAPM assumptions,

a. What alternative investment has the lowest possible volatility while having the same expected return as Microsoft? What is the volatility of this investment?

b. What investment has the highest possible expected return while having the same volatility as Microsoft? What is the expected return of this investment?

a. Under the CAPM assumptions, the market is efficient; that is, a leveraged position in the market has the highest expected return of any portfolio for a given volatility and the lowest volatility for a given expected return. By holding a leveraged position in the market portfolio, you can achieve an expected return of

$$E\left[R_p\right] = r_f + x\left(E[R_m] - r_f\right) = 5\% + x \times 5\% .$$

Setting this equal to 12% gives $12 = 5 + 5x \Rightarrow x = 1.4$.

So the portfolio with the lowest volatility that has the same return as Microsoft has $\$15,000 \times 1.4 = \$21,000$ in the market portfolio, and borrows $\$21,000 - \$15,000 = \$6,000$; that is, –$6,000 in the force asset.

$$SD\left(R_p\right) = xSD\left[R_m\right] = 1.4 \times 18 = 25.2\%$$

Note that this is considerably lower than Microsoft's volatility.

b. A leveraged portion in the market has volatility η

$$SD\left(R_p\right) = xSD\left(R_m\right) = x \times 18\%.$$

Setting this equal to the volatility of Microsoft gives

$$40\% = x \times 18\%$$

$$x = \frac{40}{18} = 2.222.$$

So the portfolio with the highest expected return that has the same volatility as Microsoft has $\$15,000 \times 2.2 = \$33,000$ in the market portfolio, and borrows $33,000 - 15,000 = \$18,333.33$, that is –$18,333.33 in the in force asset.

$$E\left[R_p\right] = r_f + x\left(E\left[R_m\right] - r_f\right) = 5\% + 2.222 \times 5\% = 16.11\%$$

Note that this is considerably higher than Microsoft's expected return.

11-47. **Suppose you group all the stocks in the world into two mutually exclusive portfolios (each stock is in only one portfolio): growth stocks and value stocks. Suppose the two portfolios have equal size (in terms of total value), a correlation of 0.5, and the following characteristics:**

	Expected Return	Volatility
Value Stocks	13%	12%
Growth Stocks	17%	25%

The risk-free rate is 2%.

a. **What is the expected return and volatility of the market portfolio (which is a 50–50 combination of the two portfolios)?**

b. **Does the CAPM hold in this economy? (*Hint:* Is the market portfolio efficient?)**

a. Erm = 15%, vol = 16.3%

b. Value stocks have a higher Sharpe ratio than the market, so the market is not efficient.

11-48. **Suppose the risk-free return is 4% and the market portfolio has an expected return of 10% and a volatility of 16%. Merck & Co. (Ticker: MRK) stock has a 20% volatility and a correlation with the market of 0.06.**

a. **What is Merck's beta with respect to the market?**

b. **Under the CAPM assumptions, what is its expected return?**

a. $\beta_{MRK} = 0.06 \times \dfrac{0.2}{0.16} = 0.075$

b. $E\left[R_{MRK}\right] = 0.04 + 0.075\left(0.1 - 0.04\right) = 4.45\%$

11-49. **Consider a portfolio consisting of the following three stocks:**

	Portfolio Weight	Volatility	Correlation with the Market Portfolio
HEC Corp	0.25	12%	0.4
Green Midget	0.35	25%	0.6
AliveAndWell	0.4	13%	0.5

The volatility of the market portfolio is 10% and it has an expected return of 8%. The risk-free rate is 3%.

a. **Compute the beta and expected return of each stock.**

b. **Using your answer from part a, calculate the expected return of the portfolio.**

c. **What is the beta of the portfolio?**

d. **Using your answer from part c, calculate the expected return of the portfolio and verify that it matches your answer to part b.**

	Portfolio Weight	Volatility	Correlation with the Market Portfolio	Beta (Part a answer)	Expected Return (Part a answer)
HEC Corp	0.25	12%	0.4	0.48	5.4
Green Midget	0.35	25%	0.6	1.5	10.5
Alive and Well	0.4	13%	0.5	0.65	6.25
				Part c answer:	Part b answer:
			Portfolio	0.905	7.525
					Part d answer:
		Expected Return calculated from portfolio			7.525

11-50. Suppose Autodesk stock has a beta of 2.16, whereas Costco stock has a beta of 0.69. If the risk-free interest rate is 4% and the expected return of the market portfolio is 10%, what is the expected return of a portfolio that consists of 60% Autodesk stock and 40% Costco stock, according to the CAPM?

$$\beta = (0.6)(2.16) + (0.4)(0.69) = 1.572$$

$$E[R] = 4 + (1.572)(10 - 4) \ = 13.432\%$$

11-51. What is the risk premium of a zero-beta stock? Does this mean you can lower the volatility of a portfolio without changing the expected return by substituting out any zero-beta stock in a portfolio and replacing it with the risk-free asset?

Risk premium = 0. It is uncorrelated with the market, so there is no incremental risk from adding it to your portfolio.

Note also that since the stock is positively correlated with itself (which is part of the market), to have zero beta it must be negatively correlated with the other stocks. Thus, it offsets risk that other stocks have. Thus, taking it out will not reduce risk.