Methodology of Econometrics

1. Statement of theory or hypothesis
2. Specification of mathematical model of the theory
3. Specification of econometric model of theory
4. Obtaining the data
5. Estimation of the parameters of the econometric model
6. Hypothesis testing
7. Forecasting or prediction
8. Using model for control or policy purposes

Econometrics Model for CAPM

Treynor (1961), Sharpe (1964), & Lintner (1965) introduced CAPM, which can stated as an econometric model:

$$r_j - r_f = \left(\sigma_j / \sigma_m\right)\left(r_m - r_f\right)$$

$$r_j - r_f = \beta\left(r_m - r_f\right) + u_t$$

$$Y_t = \beta X_t + u_t$$

According to original CAPM, the regression model is regression through the origin or model without intercept term.

Econometrics Model for CAPM

Jensen (1968) identified the existence of risk premium of portfolio or Jensen's Alpha:

$$r_j - r_f = \alpha_j + \beta\left(r_m - r_f\right) + u_t$$

$$Y_t = \alpha_j + \beta X_t + u_t$$

In this model, β represents relationship of risk-premium of security j and risk-premium of market portfolio. We call this index as Investment Beta.

According to original CAPM, value of α_j should be 0. However, empirical evidences have found the significant α_j.

Econometrics Model for Arbitrage Pricing Theory (APT)

Ross (1976) expanded concept of the CAPM by adding major economic variables into the model. The CAPM changes from simple regression to be multiple regression model.

$$r_j - r_f = \alpha_j + \beta_j \left(r_m - r_f \right) + \sum_{i=1}^{n} \gamma_i x_{it} + u_t$$

where x_i is major economic variables $i = 1, 2, \ldots n$. e.g. interest rate, inflation rate.

Econometrics Model for Fama & French Three-Factor Model

Fama & French (1993) argued that return on portfolio in excess of risk free rate is explained by three factors:

1. Excess return on market portfolio
2. Difference between return on portfolio of small stocks and return on portfolio of large stocks (SMB, small minus big)
3. Difference between return on portfolio of high-book-to-market stocks and return on portfolio of low-book-to market stocks (HML, high minus low).

Econometrics Model for Fama & French Three-Factor Model

Fama & French Three-factor model:

$$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + \beta_{sj}SMB + \beta_{hj}HML + u_t$$

where

SMB is the difference between return on small size stocks portfolio and return on large stocks portfolio.

HML is the difference between return on high growth stocks portfolio and low growth stocks portfolio.

Econometrics Model for Fama & French Three-Factor Model

Fama & French Three-factor model:

$$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + \beta_{sj}SMB + \beta_{hj}HML + u_t$$

where

β_{mj} represents market risk premium.

β_{sj} represents size premium.

β_{hj} represents growth premium.

Econometrics Model for Carhart Four-Factor Model

Carhart (1997) Four-factor model:

$$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + \beta_{sj}SMB + \beta_{hj}HML + \beta_{wj}WML + u_t$$

where

WML is the difference between return on winner stocks portfolio and losser stocks portfolio.

β_{wj} represents momentum premium.

Test CAPM whether there is abnormal return.

To test whether there is abnormal (risk-adjusted) return.

$$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + u_t$$

$$H_0: \ \alpha_j = 0$$

$$t - test = \frac{\hat{\alpha}_j}{SE_{\hat{\alpha}_j}}$$

Test CAPM whether stock j has the same risk as the market

Either t-test or restricted-unrestricted F-test can be applied.

$$H_0: \beta_{mj} = 1 \quad or \quad \beta_{mj} - 1 = 0$$

UR: $$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + u_t$$

R: $$\left(r_j - r_f\right) + \left(r_m - r_f\right) = \alpha_j + u_t$$

$$t - test = \frac{\hat{\beta}_{mj} - 1}{SE_{\hat{\beta}_{mj}}} \quad \text{or} \quad F = \frac{\left(RSS_R - RSS_{UR}\right)/1}{RSS_{UR}/\left(n-2\right)}$$

Test CAPM vs Fama & French Three-Factor Models

To test whether CAPM or Fama & French Three-factor model is more appropriated, restricted-unrestricted F-test can be applied.

$$H_0: \quad \beta_{sj} = \beta_{hj} = 0$$

UR: $$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + \beta_{sj} SMB + \beta_{hj} HML + u_t$$

R: $$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + u_t$$

$$F = \frac{\left(RSS_R - RSS_{UR}\right)/2}{RSS_{UR}/\left(n-4\right)}$$

Test CAPM vs Carhart Four-Factor Models

To test whether CAPM or Carhart Four-factor model is more appropriated, restricted-unrestricted F-test can be applied.

$$H_0: \beta_{sj} = \beta_{hj} = \beta_{wj} = 0$$

UR:

$$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + \beta_{sj}SMB + \beta_{hj}HML + \beta_{wj}WML + u_t$$

R: $$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + u_t$$

$$F = \frac{\left(RSS_R - RSS_{UR}\right)/3}{RSS_{UR}/\left(n-5\right)}$$

Test Fama-French Three-Factor vs Carhart Four-Factor Models

To test whether Fama & French or Carhart is more appropriated, either t-test or restricted-unrestricted F-test can be applied.

$$H_0: \ \beta_{wj} = 0$$

UR:

$$r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + \beta_{sj}SMB + \beta_{hj}HML + \beta_{wj}WML + u_t$$

R: $r_j - r_f = \alpha_j + \beta_{mj}\left(r_m - r_f\right) + \beta_{sj}SMB + \beta_{hj}HML + u_t$

$$t-test = \frac{\hat{\beta}_{wj}}{SE_{\hat{\beta}_{wj}}} \qquad \text{or} \qquad F = \frac{\left(RSS_R - RSS_{UR}\right)/1}{RSS_{UR}/\left(n-5\right)}$$