Characteristics of Discrete Dependent Data

<u>Categorical Data</u>

- <u>Binary-choice Data</u>
- <u>Multi-choice Data</u>
  - Unordered Data
  - Ordered Data
  - Sequential Data

<u>Non-categorical Data</u>

- Counted Number

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Characteristics of Discrete Dependent Data

Categorical Data

- **Binary-choice Data**

1. Binary Data ⟶ 1. Logit & Probit

- **Multi-choice Data**

2. Multinomial Data ⟶ 2. MN Logit & Probit
3. Multivariate Data ⟶ 3. Multivariate Probit
4. Ordered Data ⟶ 4. OLogit & Probit

Non-categorical Data

5. Count Number ⟶ 5. Poisson Model

Multinomial Data

Dependent variable has a finite number of possible outcomes.

Individuals can choose among more than two choices.

Individual can choose only one choice.

Multinomial Model

Let m be number of alternatives.

Response (dependent) variable $y_i = j$ is nominal variable, where $j = 1, 2, \ldots, m$.

Then, model for stochastic utilities is given by:

$$U_i^j = u_{ij} + \varepsilon_{ij} = x_{ij}'\beta + \varepsilon_{ij}$$

where:

x_{ij} is vector of explanatory variables that apply for individual i and alternative j.

$x_{ij} - x_{ih}$ measures the differences between alternative j and alternative h – e.g. differences in interest rates between strategy j and h.

Choice Model and Log-likelihood

Assume maximize utility.

$$p_{ij} = \Pr[y_i = j] = \Pr[u_{ij} + \varepsilon_{ij} > u_{ih} + \varepsilon_{ih} \;\; for\ all\ h \neq j]$$

Assume individuals make *independent* choice, the log-likelihood function can be stated as:

$$\log L = \sum_{i=1}^{n} \sum_{j=1}^{m} y_{ij} \log p_{ij} = \sum_{i=1}^{n} \log p_{iy_i}$$

where:

$y_{ij} = 1$ if $y_i = j$ and $y_{ij} = 0$ otherwise.

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Multinomial Probit Model

Assume joint normal distribution for the model.

$$\begin{pmatrix} \varepsilon_{i1} \\ \vdots \\ \varepsilon_{im} \end{pmatrix} \sim NID(0, V)$$

The model is multinomial probit model.

Multinomial Logit Model

Assume that the error terms are independently and identically distributed.

Multinomial Logit: $$p_{ij} = \frac{e^{x'_{ij}\beta}}{\sum_{h=1}^{m} e^{x'_{ih}\beta}}$$

Marginal Effects

Marginal effects of the explanatory variables on the choice probabilities:

Multinomial Logit:

$$\frac{\partial \Pr_{CL}[y_i = j]}{\partial x_{ij}} = p_{ij}\left(1 - p_{ij}\right)\beta$$

Ordered Response Model

In case that choices are ordered, ordered probit or logit model can be applied.

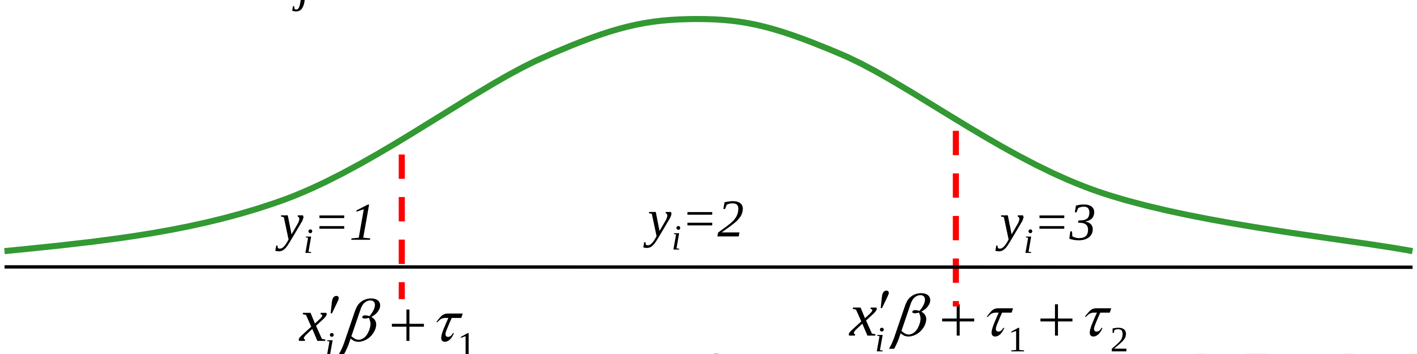
$$y_i^* = x_i'\beta + \varepsilon_i, \quad E(\varepsilon_i) = 0$$

$$y_i = 1 \quad if\ -\infty < y_i^* \le \tau_1 + x_i'\beta,$$

$$y_i = j \quad if \sum_{k=1}^{j-1} \tau_k + x_i'\beta < y_i^* \le \sum_{k=1}^{j} \tau_k + x_i'\beta,\ j = 2, \ldots, m-1,$$

$$y_i = m \quad if \sum_{k=1}^{m-1} \tau_k + x_i'\beta < y_i^* \le \infty$$

where τ_j = Threshold value, $j=1,2,\ldots, m$



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Multivariate Probit Model

Dependent variable has more than two choices.

Individual can choose more than one choice.

The model assumes multivariate distribution.

$$y_1^* = \beta_1' x_1 + \varepsilon_1, \qquad y_1 = 1 \quad if \quad y_1^* > 0, \; 0 \quad otherwise$$

$$y_2^* = \beta_2' x_2 + \varepsilon_2, \qquad y_2 = 1 \quad if \quad y_2^* > 0, \; 0 \quad otherwise$$

$$E\left[\varepsilon_1 \middle| x_1, x_2\right] = E\left[\varepsilon_2 \middle| x_1, x_2\right] = 0$$

$$Var\left[\varepsilon_1 \middle| x_1, x_2\right] = Var\left[\varepsilon_2 \middle| x_1, x_2\right] = 1$$

$$Cov\left[\varepsilon_1, \varepsilon_2 \middle| x_1, x_2\right] = \rho$$