Mainstream: Stocks and CAPM

A&D Ch.2

Ch.5 of Microeconomics by Pindyck&Rubinfeld

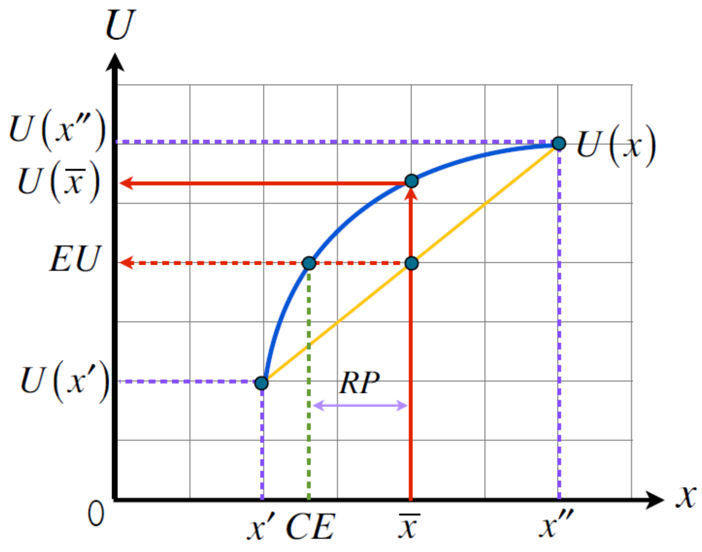
Ch.13 in Intermediate microeconomics by Varian

Assets

- **Asset** Something that provides a flow of money or services to its owner.
- **Risky asset** Asset that provides an uncertain flow of money or services to its owner.
- **Riskless (or risk-free) asset** Asset that provides a flow of money or services that is known with certainty.
- **Portfolio** A grouping of assets such as stocks, bonds, commodities, currencies and cash equivalents, as well as their fund counterparts, including mutual, exchange-traded and closed funds.

- We can use prospect to represent risky situation resulting from investing in a risky asset or investing in a portfolio involving risky assets.

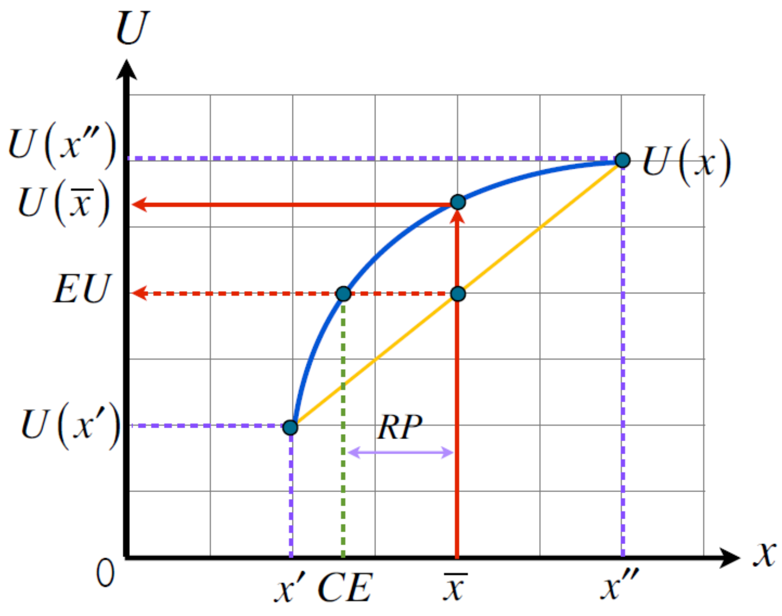
Previously we have known that …

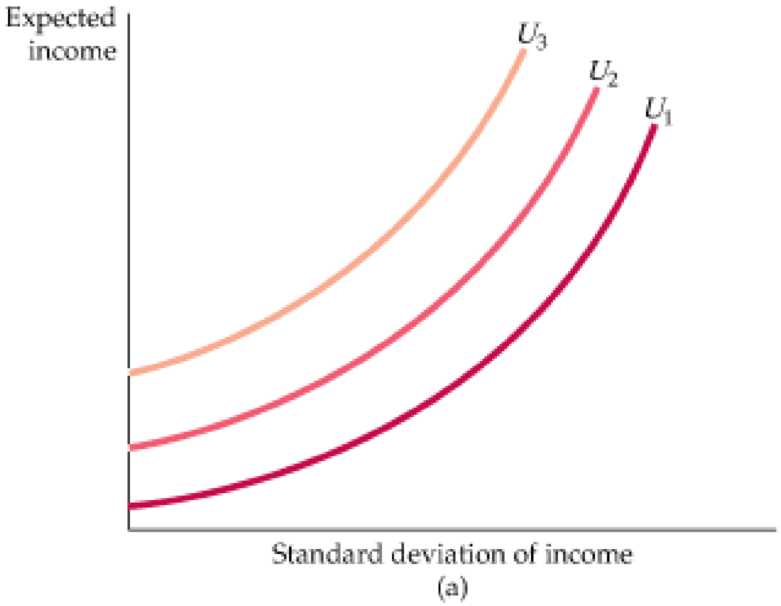


- Economists deal with risk using Expected Utility Theory. A person uses expected utility to choose among various prospects.
- For a Bernoulli utility function $u(x) = \frac{x^{1-A}}{1-A}$, the parameter A represents the coefficient of relative risk aversion.
- If A increases,
  - The function will be more curved
  - The person becomes more risk-averse
  - For a given prospect, certainty equivalent will be smaller
  - For a given prospect, risk premium will be larger.

For a risk-averse person

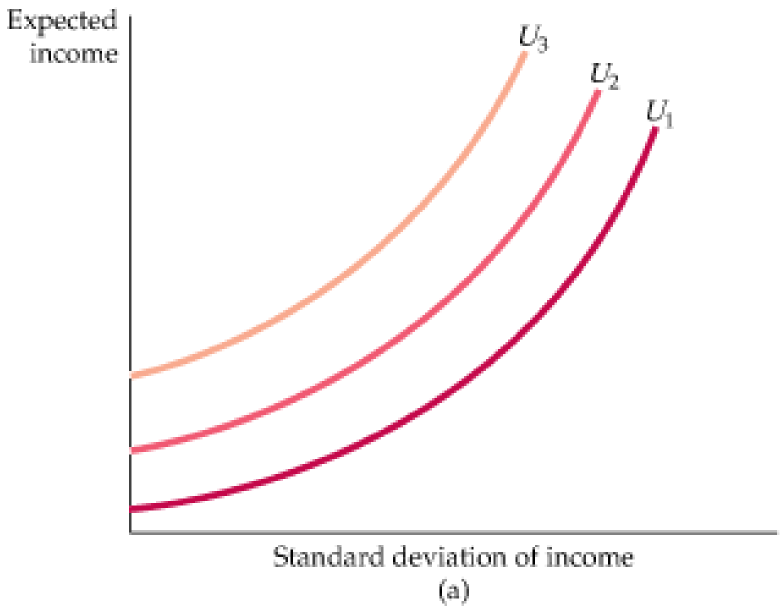
- **Expected return** is a good (increasing expected utility) and **risk** is a bad (reducing expected utility).
- We can convert the left-hand side diagram into the right-hand side diagram.

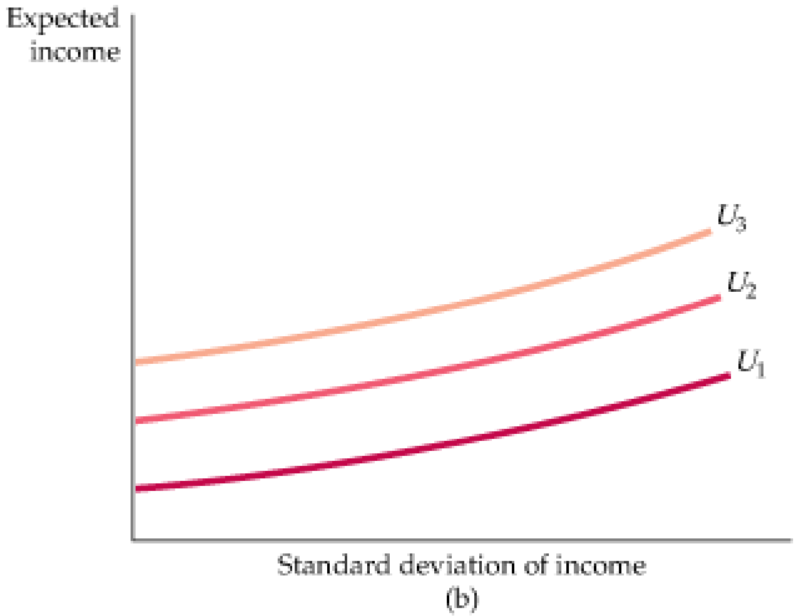




For a risk-averse person

- With higher value of parameter A, the indifferent curves of the person become more steep upward sloping (Left-hand side diagram).
- With lower value of A, the indifferent curves become relatively flat (Right-hand side diagram).





Budget Line: The Case of 2 Assets

- Assume there are only 2 assets available, which are a risk-free asset and a risky asset.
- Assume there are only 2 states of the world, which are good (1) and bad (2) states.
- Let R_f represent return on risk-free asset
  R_m represent expected return on risky asset
  b represent weight on risky asset
  O_s represent return on risky asset in state s
  p_s represent probability that state s will occur.
- Expected return of the portfolio investing in the 2 assets, R_p, can be written as

$$R_p = bR_m + (1 - b)R_f. \quad (1)$$

Budget Line: The Case of 2 Assets

- Variance of the portfolio, σ_p^2, can be calculated from

$$\sigma_p^2 = (bO_1 + (1-b)R_f - R_p)^2 p_1 + (bO_2 + (1-b)R_f - R_p)^2 p_2.$$

- Substituting R_p from equation (1) into the above equation, we get

$$\sigma_p^2 = b^2 \sigma_m^2.$$

- The standard deviation of the portfolio, σ_p, then can be expressed as

$$\sigma_p = b\sigma_m. \qquad (2)$$

Budget Line: The Case of 2 Assets

- In summary, the portfolio investing in the 2 assets can be characterized from

$$R_p = bR_m + (1 - b)R_f. \quad (1)$$

$$\sigma_p = b\sigma_m. \quad (2)$$

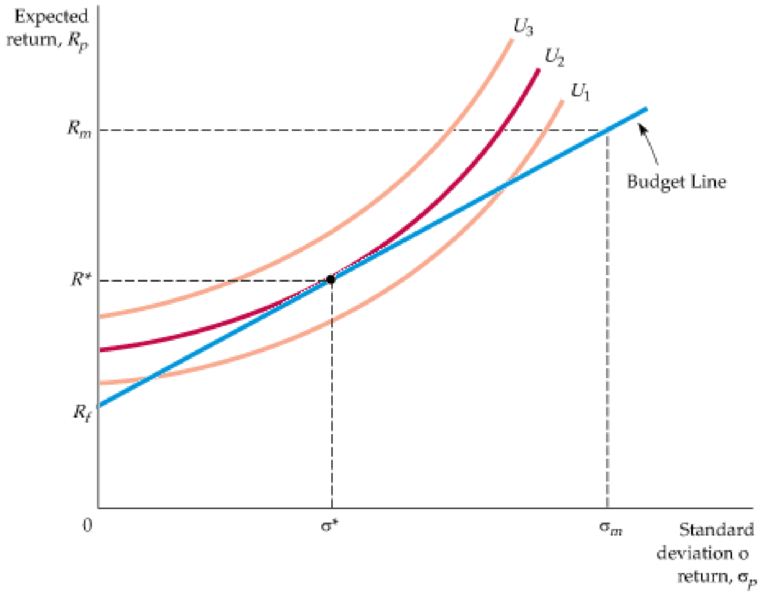
  Observe here that the expected return and risk of the portfolio depend on the weight b.

- Substituting b from equation (2) into equation (1), we get

$$R_p = R_f + \frac{(R_m - R_f)}{\sigma_m}\sigma_p. \quad (3)$$

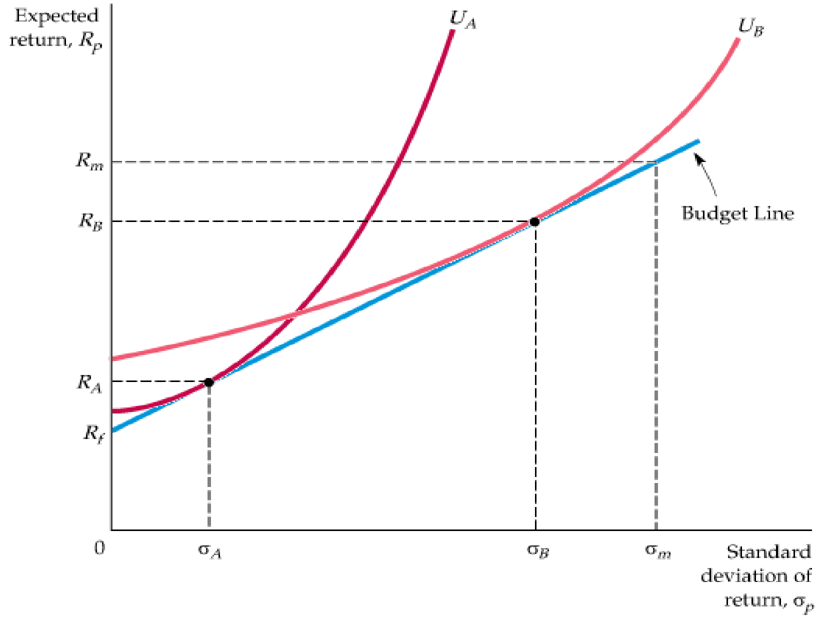
- The expression of the slope in equation (3), which is $\frac{\Delta R_p}{\Delta \sigma_p} = \frac{(R_m - R_f)}{\sigma_m}$, represents the ***price of risk*** in the market. This tells how much the increase in expected return to compensate for one unit increase in the standard deviation of the portfolio.

Equilibrium Point: The Case of 2 Assets



- The left-hand side diagram combines indifferent curves of an investor with the budget line.
- Equilibrium occurs at (σ^*, R^*), where the investor maximizes his utility, represented by the indifferent curve U_2, subject to the budget constraint.
- Note that the price of risk here is $\frac{\Delta R_p}{\Delta \sigma_p} = \frac{(R_m - R_f)}{\sigma_m}$, which will be the same as the slope of the indifferent curve U_2 at the equilibrium point.
- The weight b^*, which is consistent with (σ^*, R^*), can be calculated from (1) or (2).

Equilibrium Point: The Case of 2 Assets



- With different level of risk-aversion, equilibrium points will be different.
- The more risk-averse investor will invest in a portfolio with lower level of risk and lower expected return (σ_A, R_A), while the less risk-averse investor will invest in a portfolio with higher risk and higher return (σ_B, R_B).

Budget Line: The Case of Many Risky Assets

- Now, consider the case that there are many risky assets available for the investor to pick into his portfolio.
- In this case, the sum of standard deviations of risky assets cannot represent the level of risk of the portfolio.
- Recall the case of a shop selling air-conditioner below. Combining two risky returns results in a completely risk-free outcome.

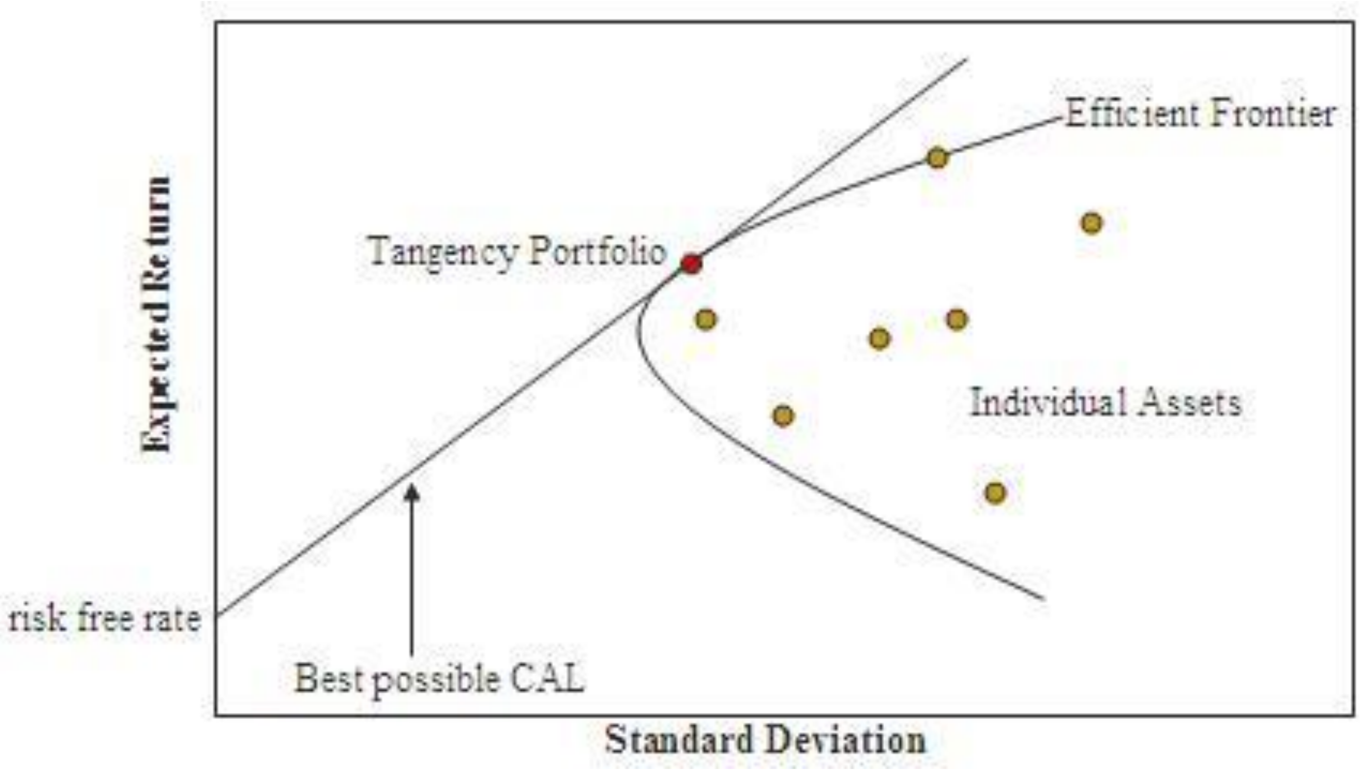
TABLE 5.5 Income from Sales of Appliances ($)

	Hot Weather	Cold Weather
Air conditioner sales	30,000	12,000
Heater sales	12,000	30,000

Budget Line: The Case of Many Risky Assets

- In theory, we can consider all combinations of risky securities and collect the pairs of (standard deviation, expected return) from these combinations.
- It can be shown that a graph depicting all these pairs would be like a bullet-shaped area in the figure in the next slide. The boundary to the left of the area is called the ***efficient frontier****,* which represents the set of portfolios that minimizes risk for a given expected return.
- As more assets are added to the portfolio, the investor can eliminate more risk. We refer to the risk that can be eliminated as ***diversifiable (or nonsystematic) risk***. The risk we cannot eliminate is called ***nondiversifiable (or systematic) risk.***
- Adding the risk-free asset into the framework, investors can maximize expected return for a given level of risk by combining risk-free asset with a portfolio of risky assets.

Budget Line: The Case of Many Risky Assets



- The ***capital market line*** **(or capital allocation line; CAL)** is the line representing the best combinations of risk-free asset with the portfolio of risky assets.
- The tangency portfolio is the market portfolio, which includes all risky assets weighted by their values in the market, denoted by m.
- Equilibrium can turn back to the case of 2 assets we consider before.

Capital Asset Pricing Model (CAPM)

- Under the case of many risky assets, it will be much more convenient to calculate a statistic that reflects the correlation of individual asset's return to the overall market's return. This statistic is called ***beta***, denoted by β.
- Beta of an asset i can be expressed as

$$\beta_i = \frac{COV(R_i, R_m)}{VAR(R_m)} = \rho_{i,m} \frac{\sigma_i}{\sigma_m} \quad (4)$$

  Where $\rho_{i,m}$ represents the coefficient of correlation of asset i's return and market's return.
- It can be said roughly that beta reflects the relative level of riskiness of an asset in comparison with the level of riskiness of the overall market.

Capital Asset Pricing Model (CAPM)

- Recall that the price of risk is $\frac{\Delta R_p}{\Delta \sigma_p} = \frac{(R_m - R_f)}{\sigma_m}$. In this case, the pair (σ_m, R_m) reflects risk and expected return of the market portfolio.
- From equation (4), we can calculate an indicator of the level of riskiness of asset i as $\beta_i \sigma_m$.
- We can calculate the required premium from the investor for investing in an asset i by multiplying the above indicator with the price of risk. That is

$$\text{Risk compensation} \quad = \beta_i \sigma_m \frac{R_m - R_f}{\sigma_m}$$
$$= \beta_i \left(R_m - R_f\right).$$

Capital Asset Pricing Model (CAPM)

- At equilibrium, risky assets i and j, with β_i and β_j, respectively, should provide the same risk-adjusted returns. That is

$$R_i - \beta_i(R_m - R_f) = R_j - \beta_j(R_m - R_f).$$

- This should be true for the risk-free asset as well. However, investors do not require any compensation of risk from the risk-free asset. Hence,

$$R_i - \beta_i(R_m - R_f) = R_f \qquad (5)$$

- Rearranging equation (5), we get

$$R_i = R_f + \beta_i(R_m - R_f), \qquad (6)$$

which shows the required return for a risky asset i. This is central to CAPM.

Capital Asset Pricing Model (CAPM)

- From (6), investors require return from investing in a risky asset i equal to the risk-free return plus with risk premium of the asset.
- The risk premium of the asset can be calculated from the **equity premium (or market risk premium)**, $(R_m - R_f)$, multiply with beta of the asset, β_i.
- If β_i is higher than one, investors require compensation higher than the equity premium. If β_i is lower than one, investors require less.
- β_i can be negative. The investment in this risky asset is like purchasing an insurance.