

CHAPTER 14: BOND PRICES AND YIELDS

4. A bond's coupon interest payments and principal repayment are not affected by changes in market rates. Consequently, if market rates increase, bond investors in the secondary markets are not willing to pay as much for a claim on a given bond's fixed interest and principal payments as they would if market rates were lower. This relationship is apparent from the inverse relationship between interest rates and present value. An increase in the discount rate (i.e., the market rate) decreases the present value of the future cash flows.

10.

a.	<u>Zero coupon</u>	<u>8% coupon</u>	<u>10% coupon</u>
Current prices	\$463.19	\$1,000.00	\$1,134.20
b. Price 1 year from now	\$500.25	\$1,000.00	\$1,124.94
Price increase	\$ 37.06	\$ 0.00	- \$ 9.26
Coupon income	\$ 0.00	\$ 80.00	\$100.00
Pre-tax income	\$ 37.06	\$ 80.00	\$ 90.74
Pre-tax rate of return	8.00%	8.00%	8.00%
Taxes*	\$ 11.12	\$ 24.00	\$ 28.15
After-tax income	\$ 25.94	\$ 56.00	\$ 62.59
After-tax rate of return	5.60%	5.60%	5.52%
c. Price 1 year from now	\$543.93	\$1,065.15	\$1,195.46
Price increase	\$ 80.74	\$ 65.15	\$ 61.26
Coupon income	\$ 0.00	\$ 80.00	\$100.00
Pre-tax income	\$ 80.74	\$145.15	\$161.26
Pre-tax rate of return	17.43%	14.52%	14.22%
Taxes**	\$ 19.86	\$ 37.03	\$ 42.25
After-tax income	\$ 60.88	\$108.12	\$119.01
After-tax rate of return	13.14%	10.81%	10.49%

* In computing taxes, we assume that the 10% coupon bond was issued at par and that the decrease in price when the bond is sold at year end is treated as a capital loss and therefore is not treated as an offset to ordinary income.

** In computing taxes for the zero coupon bond, \$37.06 is taxed as ordinary income (see part (b)); the remainder of the price increase is taxed as a capital gain.

11. a. On a financial calculator, enter the following:

$$n = 40; FV = 1000; PV = -950; PMT = 40$$

You will find that the yield to maturity on a semi-annual basis is 4.26%. This implies a bond equivalent yield to maturity equal to: $4.26\% * 2 = 8.52\%$

$$\text{Effective annual yield to maturity} = (1.0426)^2 - 1 = 0.0870 = 8.70\%$$

- 13.

Price	Maturity (years)	Bond equivalent YTM
\$400.00	20.00	4.688%
\$500.00	20.00	3.526%
\$500.00	10.00	7.177%
\$385.54	10.00	10.000%
\$463.19	10.00	8.000%
\$400.00	11.91	8.000%

23. The bond is selling at par value. Its yield to maturity equals the coupon rate, 10%. If the first-year coupon is reinvested at an interest rate of r percent, then total proceeds at the end of the second year will be: $[\$100 * (1 + r)] + \$1,100$

Therefore, realized compound yield to maturity is a function of r , as shown in the following table:

r	Total proceeds	Realized YTM = $\sqrt{\text{Proceeds}/1000} - 1$
8%	\$1,208	$\sqrt{1208/1000} - 1 = 0.0991 = 9.91\%$
10%	\$1,210	$\sqrt{1210/1000} - 1 = 0.1000 = 10.00\%$
12%	\$1,212	$\sqrt{1212/1000} - 1 = 0.1009 = 10.09\%$

b. Since the bond is selling at par, the yield to maturity on a semi-annual basis is the same as the semi-annual coupon rate, i.e., 4%. The bond equivalent yield to maturity is 8%.

$$\text{Effective annual yield to maturity} = (1.04)^2 - 1 = 0.0816 = 8.16\%$$

c. Keeping other inputs unchanged but setting $PV = -1050$, we find a bond equivalent yield to maturity of 7.52%, or 3.76% on a semi-annual basis.

$$\text{Effective annual yield to maturity} = (1.0376)^2 - 1 = 0.0766 = 7.66\%$$

31. a. Initial price $P_0 = \$705.46$ [$n = 20$; $PMT = 50$; $FV = 1000$; $i = 8$]

Next year's price $P_1 = \$793.29$ [$n = 19$; $PMT = 50$; $FV = 1000$; $i = 7$]

$$\text{HPR} = \frac{\$50 + (\$793.29 - \$705.46)}{\$705.46} = 0.1954 = 19.54\%$$

b. Using OID tax rules, the cost basis and imputed interest under the constant yield method are obtained by discounting bond payments at the *original* 8% yield, and simply reducing maturity by one year at a time:

Constant yield prices (compare these to actual prices to compute capital gains):

$$P_0 = \$705.46$$

$$P_1 = \$711.89 \Rightarrow \text{implicit interest over first year} = \$6.43$$

$$P_2 = \$718.84 \Rightarrow \text{implicit interest over second year} = \$6.95$$

Tax on explicit interest plus implicit interest in first year =

$$0.40 * (\$50 + \$6.43) = \$22.57$$

Capital gain in first year = Actual price at 7% YTM – constant yield price =

$$\$793.29 - \$711.89 = \$81.40$$

Tax on capital gain = $0.30 * \$81.40 = \24.42

Total taxes = $\$22.57 + \$24.42 = \$46.99$

$$\text{c. After tax HPR} = \frac{\$50 + (\$793.29 - \$705.46) - \$46.99}{\$705.46} = 0.1288 = 12.88\%$$

d. Value of bond after two years = $\$798.82$ [using $n = 18$; $i = 7\%$]

Reinvested income from the coupon interest payments = $\$50 * 1.03 + \$50 = \$101.50$

Total funds after two years = $\$798.82 + \$101.50 = \$900.32$

Therefore, the investment of \$705.46 grows to \$900.32 in two years:

$$\$705.46 (1 + r)^2 = \$900.32 \Rightarrow r = 0.1297 = 12.97\%$$

e. Coupon interest received in first year:	\$50.00
Less: tax on coupon interest @ 40%:	- 20.00
Less: tax on imputed interest (0.40*□\$6.43):	<u>- 2.57</u>
Net cash flow in first year:	\$27.43

The year-1 cash flow can be invested at an after-tax rate of:

$$3\% * \square(1 - 0.40) = 1.8\%$$

By year 2, this investment will grow to: $\$27.43 \times 1.018 = \27.92

In two years, sell the bond for:	\$798.82	[n = 18; i = 7%]
Less: tax on <i>imputed</i> interest in second year:	- 2.78	[0.40 × \$6.95]
Add: after-tax coupon interest received in second year:	+ 30.00	[\$50 × (1 - 0.40)]
Less: Capital gains tax on (sales price - constant yield value):	- 23.99	[0.30 × (798.82 - 718.84)]
Add: CF from first year's coupon (reinvested):	<u>+ 27.92</u>	[from above]
Total	\$829.97	

$$\$705.46 (1 + r)^2 = \$829.97 \Rightarrow r = 0.0847 = 8.47\%$$

CFA PROBLEMS

1. a. A sinking fund provision requires the early redemption of a bond issue. The provision may be for a specific number of bonds or a percentage of the bond issue over a specified time period. The sinking fund can retire all or a portion of an issue over the life of the issue.

- b. (i) Compared to a bond without a sinking fund, the sinking fund reduces the average life of the overall issue because some of the bonds are retired prior to the stated maturity.

(ii) The company will make the same total principal payments over the life of the issue, although the timing of these payments will be affected. The total interest payments associated with the issue will be reduced given the early redemption of principal.

c. From the investor's point of view, the key reason for demanding a sinking fund is to reduce credit risk. Default risk is reduced by the orderly retirement of the issue.

2. a. (i) Current yield = Coupon/Price = $\$70/\$960 = 0.0729 = 7.29\%$

(ii) YTM = 3.993% semiannually or 7.986% annual bond equivalent yield.
On a financial calculator, enter: $n = 10$; $PV = -960$; $FV = 1000$; $PMT = 35$

Compute the interest rate.

(iii) Realized compound yield is 4.166% (semiannually), or 8.332% annual bond equivalent yield. To obtain this value, first find the future value (FV) of reinvested coupons and principal. There will be six payments of \$35 each, reinvested semiannually at 3% per period. On a financial calculator, enter:

$PV = 0$; $PMT = 35$; $n = 6$; $i = 3\%$. Compute: $FV = 226.39$

Three years from now, the bond will be selling at the par value of \$1,000 because the yield to maturity is forecast to equal the coupon rate. Therefore, total proceeds in three years will be: $\$226.39 + \$1,000 = \$1,226.39$

Then find the rate (y_{realized}) that makes the FV of the purchase price equal to \$1,226.39:

$$\$960 \times (1 + y_{\text{realized}})^6 = \$1,226.39 \Rightarrow y_{\text{realized}} = 4.166\% \text{ (semiannual)}$$

b. Shortcomings of each measure:

(i) Current yield does not account for capital gains or losses on bonds bought at prices other than par value. It also does not account for reinvestment income on coupon payments.

(ii) Yield to maturity assumes the bond is held until maturity and that all coupon income can be reinvested at a rate equal to the yield to maturity.

(iii) Realized compound yield is affected by the forecast of reinvestment rates, holding period, and yield of the bond at the end of the investor's holding period.

CHAPTER 15: THE TERM STRUCTURE OF INTEREST RATES

1. In general, the forward rate can be viewed as the sum of the market's expectation of the future short rate plus a potential risk (or 'liquidity') premium. According to the expectations theory of the term structure of interest rates, the liquidity premium is zero so that the forward rate is equal to the market's expectation of the future short rate. Therefore, the market's expectation of future short

rates (i.e., forward rates) can be derived from the yield curve, and there is no risk premium for longer maturities.

The liquidity preference theory, on the other hand, specifies that the liquidity premium is positive so that the forward rate is greater than the market's expectation of the future short rate. This could result in an upward sloping term structure even if the market does not anticipate an increase in interest rates. The liquidity preference theory is based on the assumption that the financial markets are dominated by short-term investors who demand a premium in order to be induced to invest in long maturity securities.

4. The liquidity theory holds that investors demand a premium to compensate them for interest rate exposure and the premium increases with maturity. Add this premium to a flat curve and the result is an upward sloping yield curve.

7. Maturity	Price	YTM	Forward Rate
1	\$943.40	6.00%	
2	\$898.47	5.50%	$(1.055^2/1.06) - 1 = 5.0\%$
3	\$847.62	5.67%	$(1.0567^3/1.055^2) - 1 = 6.0\%$
4	\$792.16	6.00%	$(1.06^4/1.0567^3) - 1 = 7.0\%$

12. a. The current bond price is:

$$(\$85 \times 0.94340) + (\$85 \times 0.87352) + (\$1,085 \times 0.81637) = \$1,040.20$$

This price implies a yield to maturity of 6.97%, as shown by the following:

$$[\$85 \times \text{Annuity factor } (6.97\%, 3)] + [\$1,000 \times \text{PV factor } (6.97\%, 3)] = \$1,040.17$$

- b. If one year from now $y = 8\%$, then the bond price will be:

$$[\$85 \times \text{Annuity factor } (8\%, 2)] + [\$1,000 \times \text{PV factor } (8\%, 2)] = \$1,008.92$$

The holding period rate of return is:

$$[\$85 + (\$1,008.92 - \$1,040.20)]/\$1,040.20 = 0.0516 = 5.16\%$$

17. a. We obtain forward rates from the following table:

Maturity	YTM	Forward Rate	Price (for parts c, d)
1 year	10%		$\$1,000/1.10 = \909.09
2 years	11%	$(1.11^2/1.10) - 1 = 12.01\%$	$\$1,000/1.11^2 = \811.62
3 years	12%	$(1.12^3/1.11^2) - 1 = 14.03\%$	$\$1,000/1.12^3 = \711.78

- b. We obtain next year's prices and yields by discounting each zero's face value at the forward rates for next year that we derived in part (a):

Maturity	Price	YTM
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1 year	$\$1,000/1.1201 = \892.78	12.01%
2 years	$\$1,000/(1.1201 \times 1.1403) = \782.93	13.02%

Note that this year's upward sloping yield curve implies, according to the expectations hypothesis, a shift upward in next year's curve.

- c. Next year, the 2-year zero will be a 1-year zero, and will therefore sell at a price of:
 $\$1,000/1.1201 = \892.78

Similarly, the current 3-year zero will be a 2-year zero and will sell for: \$782.93

Expected total rate of return:

$$\text{2-year bond: } \frac{\$892.78}{\$811.62} - 1 = 1.1000 - 1 = 10.00\%$$

$$\text{3-year bond: } \frac{\$782.93}{\$711.78} - 1 = 1.1000 - 1 = 10.00\%$$

2. d.

4. The given rates are annual rates, but each period is a half-year. Therefore, the per period spot rates are 2.5% on one-year bonds and 2% on six-month bonds. The semiannual forward rate is obtained by solving for f in the following equation:

$$1 + f = \frac{1.025^2}{1.02} = 1.030$$

This means that the forward rate is $0.030 = 3.0\%$ semiannually, or 6.0% annually.

9. a. Five-year Spot Rate:

$$\$1,000 = \frac{\$70}{(1 + y_1)^1} + \frac{\$70}{(1 + y_2)^2} + \frac{\$70}{(1 + y_3)^3} + \frac{\$70}{(1 + y_4)^4} + \frac{\$1,070}{(1 + y_5)^5}$$

$$\$1,000 = \frac{\$70}{(1.05)} + \frac{\$70}{(1.0521)^2} + \frac{\$70}{(1.0605)^3} + \frac{\$70}{(1.0716)^4} + \frac{\$1,070}{(1 + y_5)^5}$$

$$\$1,000 = \$66.67 + \$63.24 + \$58.69 + \$53.08 + \frac{\$1,070}{(1 + y_5)^5}$$

$$\$758.32 = \frac{\$1,070}{(1 + y_5)^5}$$

$$(1 + y_5)^5 = \frac{\$1,070}{\$758.32} \Rightarrow y_5 = \sqrt[5]{1.411} - 1 = 7.13\%$$

Five-year Forward Rate:

$$\frac{(1.0713)^5}{(1.0716)^4} - 1 = 1.0701 - 1 = 7.01\%$$

- b. The yield to maturity is the single discount rate that equates the present value of a series of cash flows to a current price. It is the internal rate of return.

The short rate for a given interval is the interest rate for that interval available at different points in time.

The spot rate for a given period is the yield to maturity on a zero-coupon bond that matures at the end of the period. A spot rate is the discount rate for each period. Spot rates are used to discount each cash flow of a coupon bond in order to calculate a current price. Spot rates are the rates appropriate for discounting future cash flows of different maturities.

A forward rate is the implicit rate that links any two spot rates. Forward rates are directly related to spot rates, and therefore to yield to maturity. Some would argue (as in the expectations hypothesis) that forward rates are the market expectations of future interest rates. A forward rate represents a break-even rate that links two spot rates. It is important to note that forward rates link spot rates, not yields to maturity.

Yield to maturity is not unique for any particular maturity. In other words, two bonds with the same maturity but different coupon rates may have different yields to maturity. In contrast, spot rates and forward rates for each date are unique.

- c. The 4-year spot rate is 7.16%. Therefore, 7.16% is the theoretical yield to maturity for the zero-coupon U.S. Treasury note. The price of the zero-coupon note discounted at 7.16% is the present value of \$1,000 to be received in 4 years. Using annual compounding:

$$PV = \frac{\$1,000}{(1.0716)^4} = \$758.35$$