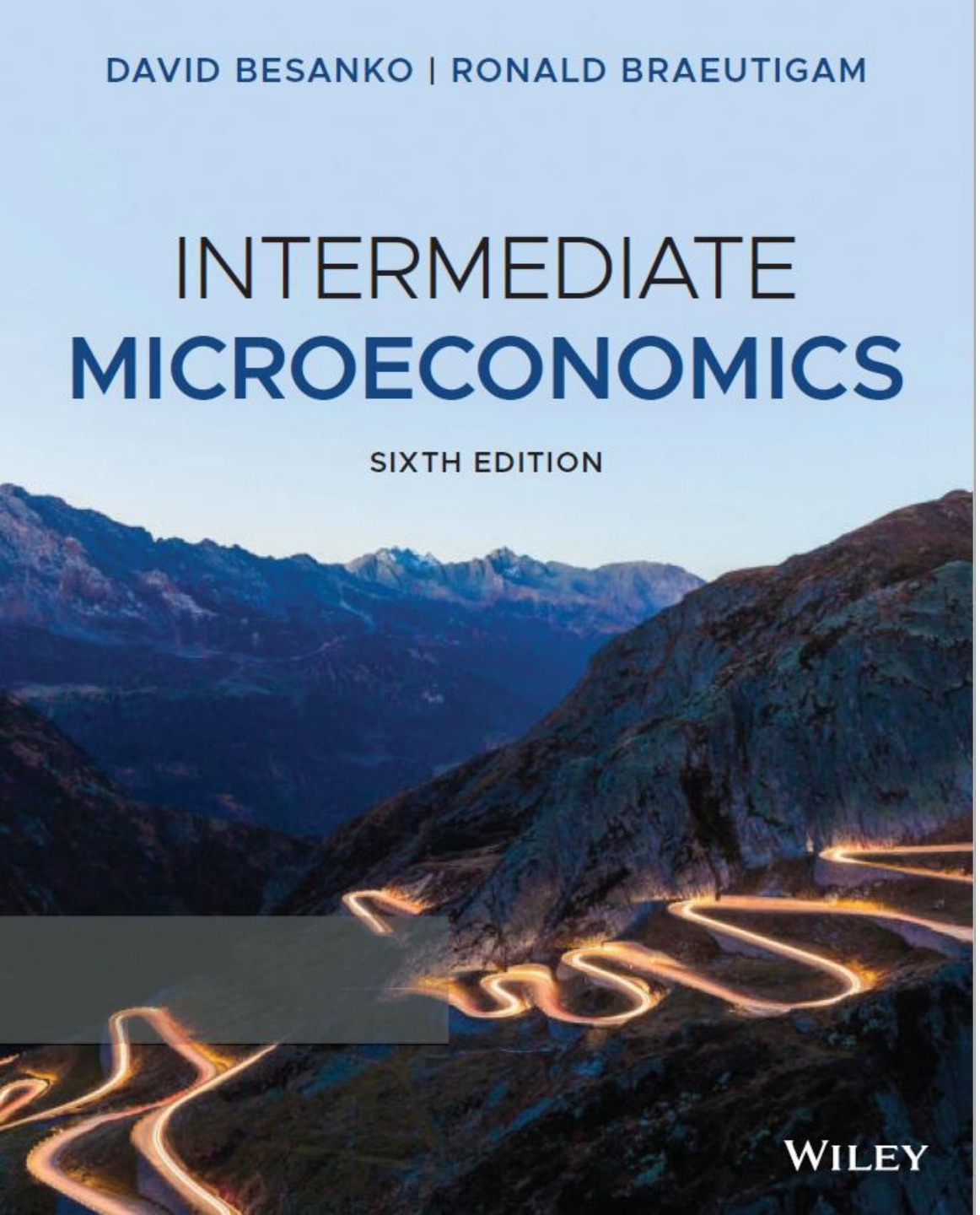


Chapter 7

Costs and Cost Minimization

Chapter Seven Overview

- Cost Concepts for Decision Making
- The Cost-Minimization Problem
- Comparative Statics Analysis of the Cost-Minimization Problem
- Short-Run Cost Minimization
- Minimizing Long-Run Costs Using Lagrange Multipliers
- Advanced Topics in Cost Minimization

Terms

- Explicit costs: Costs that involve a direct monetary outlay
- Implicit costs: Costs that do not involve outlays of cash
- The relevant concept of cost is opportunity cost
  - The value of a resource in its best alternative use
- The only alternative we consider is the best alternative
- Economic costs: Sum of a firm's explicit costs and implicit costs
- Accounting costs: Total of a firm's explicit costs

Sunk Costs

- Sunk costs are costs that must be incurred no matter what the decision
- These costs are not part of opportunity costs
- Non-sunk costs are costs that must be incurred only if a particular decision is made
- Example: Bowling Ball Factory
  - It costs $5M to build and has no alternative uses
  - $5M is not sunk cost for the decision of whether or not to build the factory
  - $5M is sunk cost for deciding whether to operate or shut down the factory

Cost Minimization

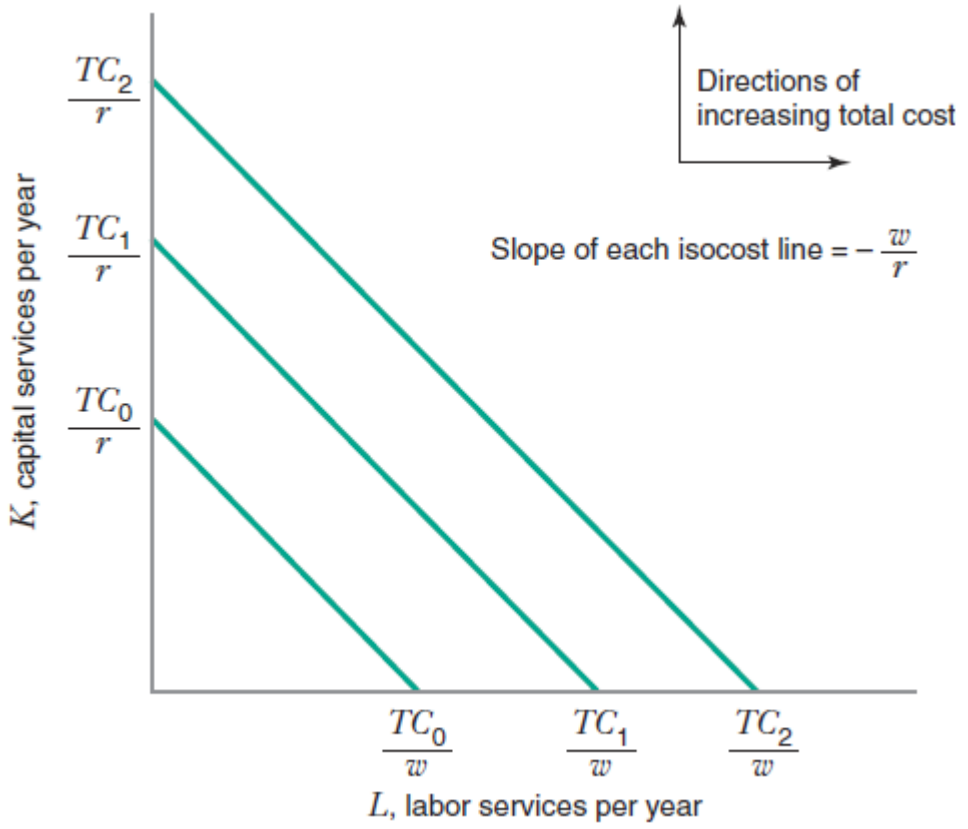
- Cost minimization problem: Finding the input combination that minimizes a firm's total cost of producing a particular level of output
- Cost minimization firm: A firm that seeks to minimize the cost of producing a given amount of output.
- Long run: A period of time when the quantities of all of the firm's input can vary.
- Short run: A period of time when at least one of its inputs' quantities is fixed

Long-Run Cost Minimization

- Minimize the firm's costs, subject to a firm producing a given amount of output
- Cost to the Firm
  - TC = Total Cost
  - w = wage rate
  - L = Quantity of Labor
  - r = price per unit of capital services
  - K = Quantity of Capital
- $TC = wL + rK$

Isocost Line

- The set of combinations of labor and capital that yield the same total cost for the firm
- w = $10/hour
- r = $20/hour
- TC = $1 million
  - $1 mil = $10L + $20K
  - K = $1 mil/20-(10/20)L
- More generally, $K = \frac{TC}{r - (w/r)L}$



Cost-Minimization Inputs

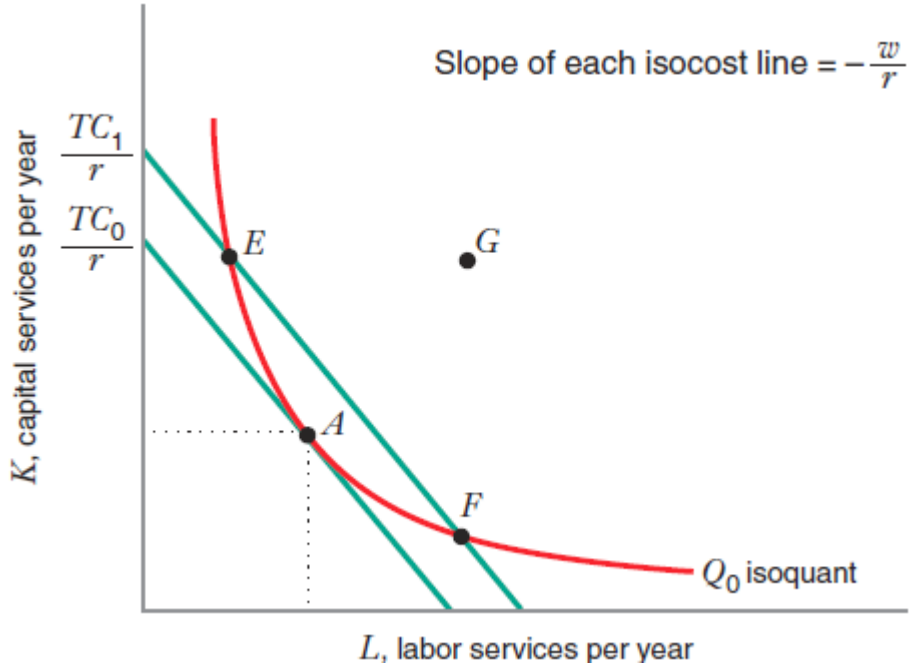
- Suppose that a firm's owners wish to minimize costs
- Let the desired output be Q_0
- Technology: $Q = f(L,K)$
- Owner's problem: min $TC = rK + wL$
- K,L
- Subject to $Q_0 = f(L,K)$
- $TC = rK + wL$ or $K = TC/r – (w/r)L$ is the isocost line

Cost-Minimization Inputs Continued

- Cost minimization subject to satisfaction of the isoquant equation $Q_0 = f(L,K)$
  - Note: analogous to expenditure minimization for the consumer
- Tangency Condition:
  - $MRTS_{L,K} = -MP_L/MP_K = -w/r$ (or) $MP_L/w = MP_K/r$
  - Constraint: $Q_0 = f(K,L)$

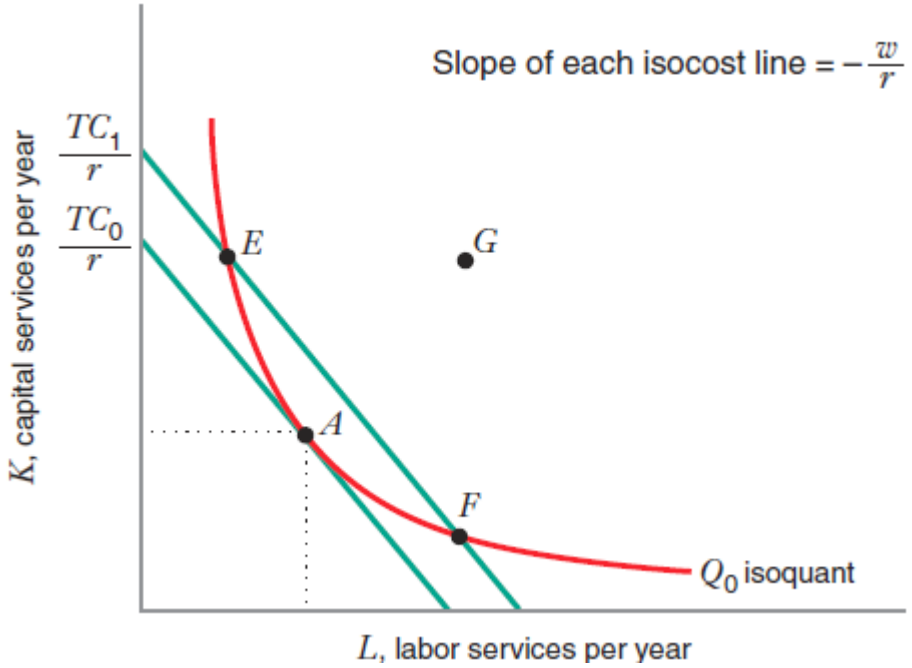
Cost-Minimizing Input Combination

- The cost-minimizing input combination occurs at point A
- Point G is technically inefficient
- Points E and F are technically efficient, but they do not minimize cost



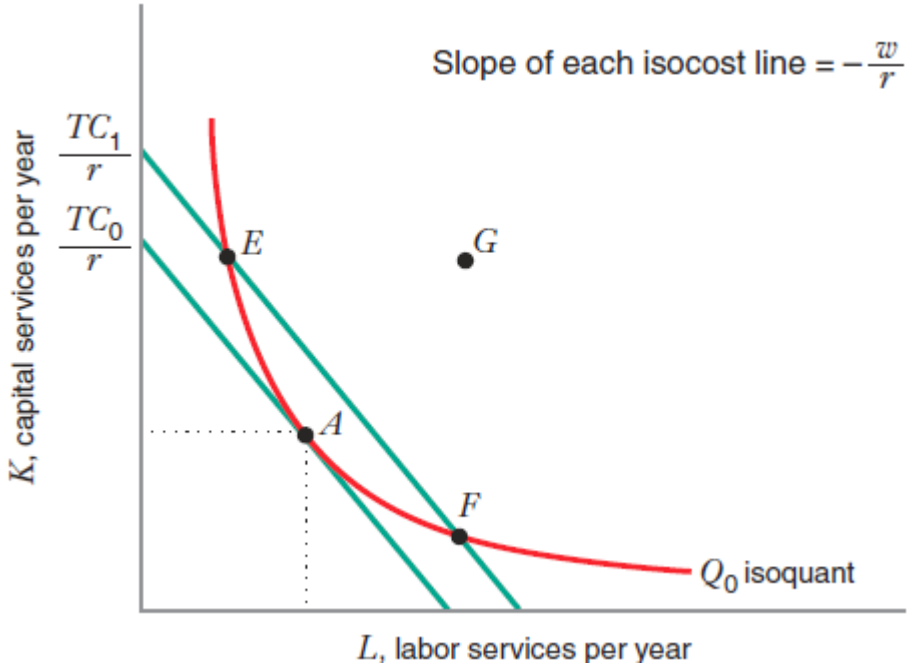
Solution to Cost Minimization

- Slope of isoquant = slope of isocost line
  - $MRTS_{L,K} = \frac{w}{r}$ or $\frac{MP_L}{MP_K} = \frac{w}{r}$
- Ratio of marginal products = ratio of input prices
  - $\frac{MP_L}{w} = \frac{MP_K}{r}$
- At point E, $\frac{MP_L}{MP_K} > \frac{w}{r}$ or $\frac{MP_L}{w} > \frac{MP_K}{r}$
  - This implies the firm could spend an additional dollar on labor and save more than a dollar by reducing its employment of capital and keep output constant



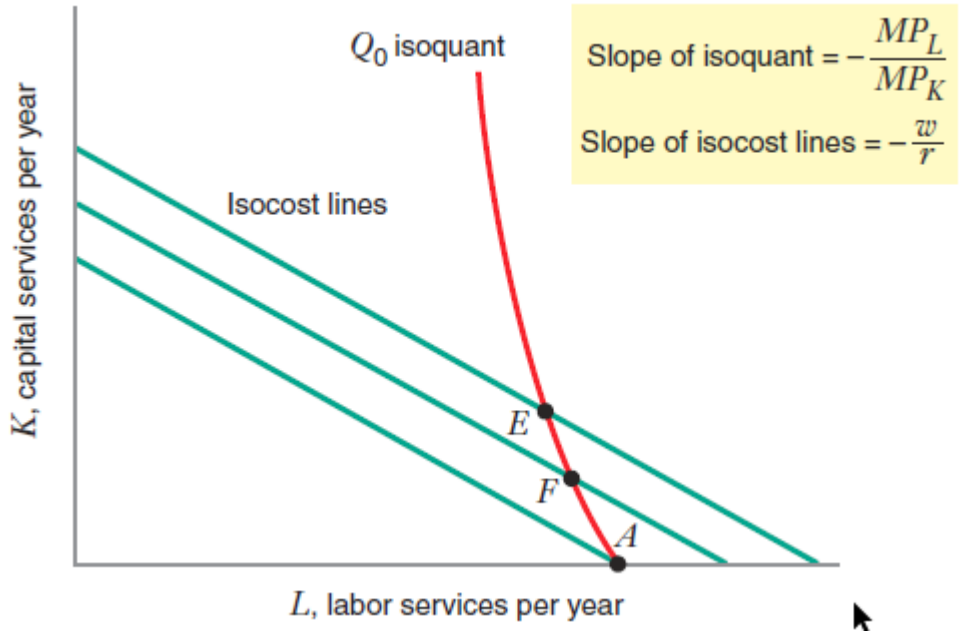
Solution to Cost Minimization Continued

- At point E, $\frac{MP_L}{MP_K} > \frac{w}{r}$ or $\frac{MP_L}{w} > \frac{MP_K}{r}$
  - This implies the firm could spend an additional dollar on labor and save more than a dollar by reducing its employment of capital and keep output constant
- At point F, $\frac{MP_L}{MP_K} < \frac{w}{r}$ or $\frac{MP_L}{w} < \frac{MP_K}{r}$
  - This implies the firm could spend an additional dollar on capital and save more than a dollar by reducing its employment of labor and keep output constant



Corner Solution

- The cost-minimizing input combination for producing Q_0 units of output occurs at point A where the firms uses no capital
- At this corner point the isocost line is flatter than the isoquant
- $-\left(\frac{MP_L}{MP_K}\right) < -\frac{w}{r} \rightarrow \frac{MP_L}{w} > \frac{MP_K}{r}$

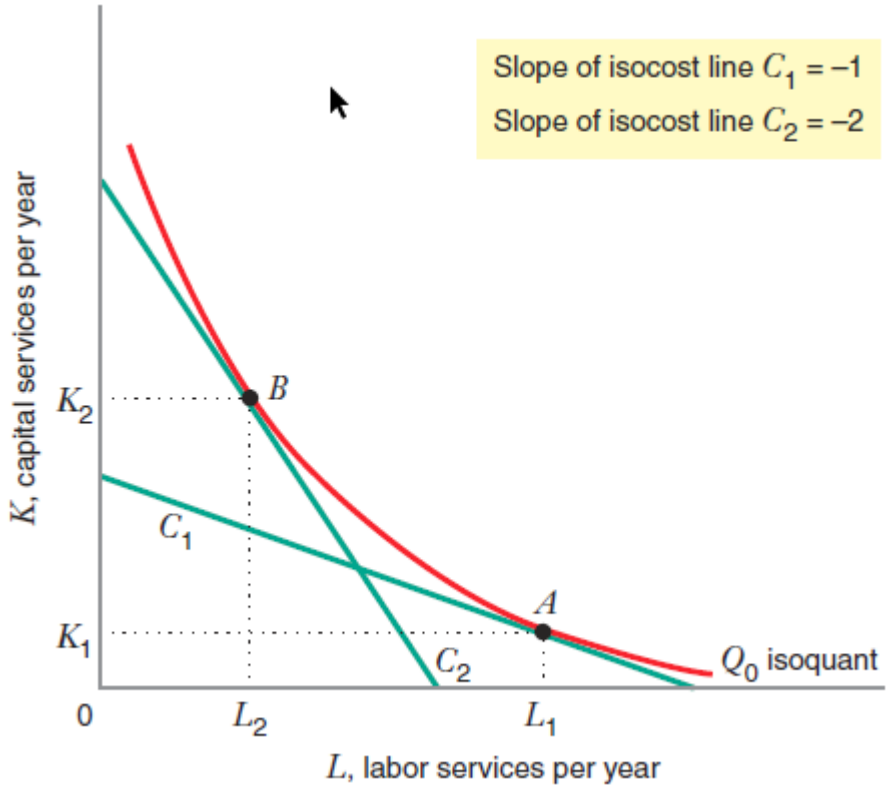


Comparative Statics

- A change in the relative price of inputs changes the slope of the isocost line
- All else equal, an increase in w must decrease the cost minimizing quantity of labor and increase the cost minimizing quantity of capital with diminishing $MRTS_{L,K}$
- All else equal, an increase in r must decrease the cost minimizing quantity of capital and increase the cost minimizing quantity of labor

Comparative Statics Analysis of Cost-Minimization Problem

- Price of capital r = 1
- Quantity of output Q_0 is constant
- When price of labor w = 1 the isocost line is C1, optimal point A
- When price of labor w = 2 isocost line is C2, optimal point B

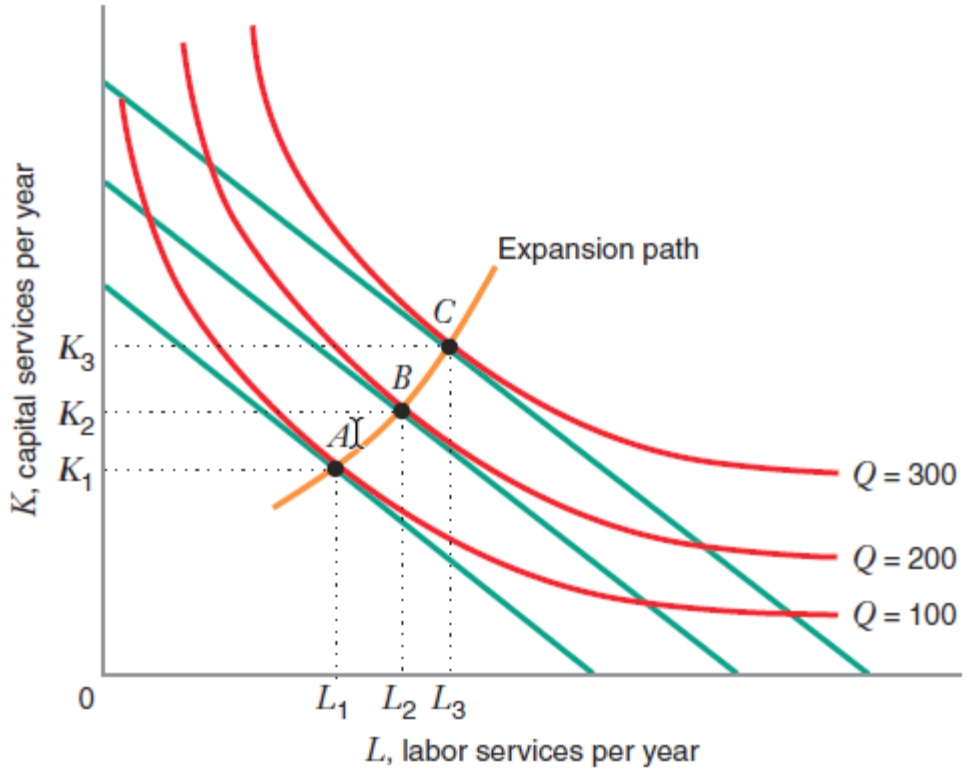


Some Key Definitions

- An increase in Q_0 moves the isoquant northeast
- Expansion path: A line that connects the cost-minimizing input combinations as the quantity of output, Q, varies, holding input prices constant
- Normal inputs: An input whose cost-minimizing quantity increases as the firm produces more output
- Inferior input: An input whose cost-minimizing quantity decreases as the firm produces more output

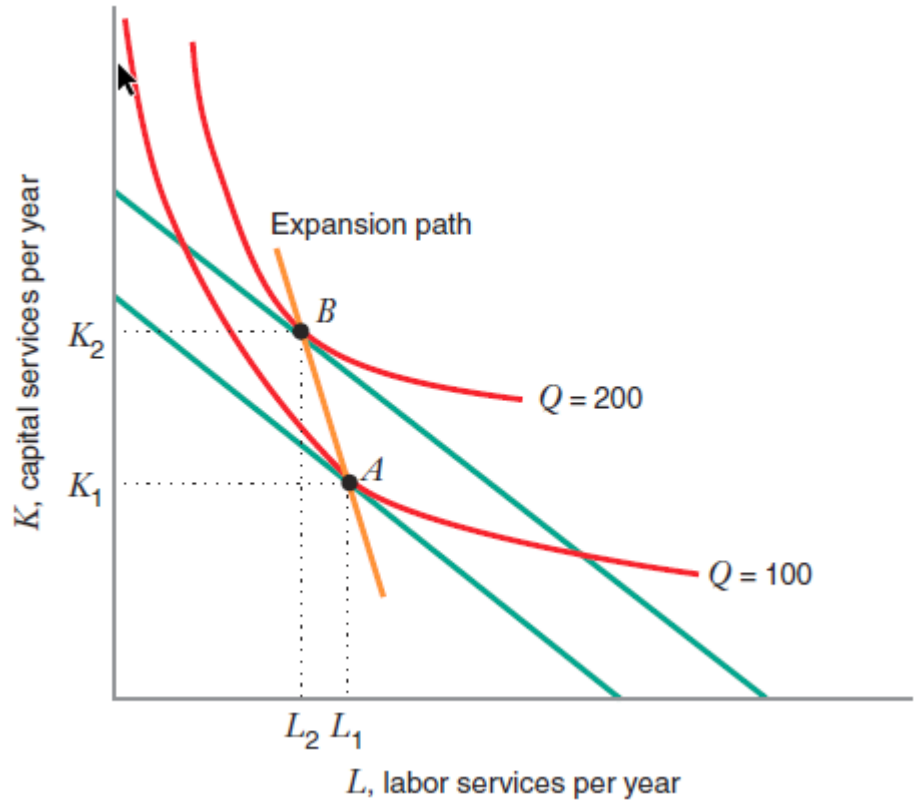
Normal Input

- The price of capital and the price of labor are held constant
- As output increases, the cost minimization path moves from point A to B to C when inputs are normal
- The expansion path is upward sloping



Inferior Input

- The price of capital and the price of labor are held constant
- As output increases, the cost minimization path moves from point A to B to C when labor is an inferior input
- The expansion path is downward sloping

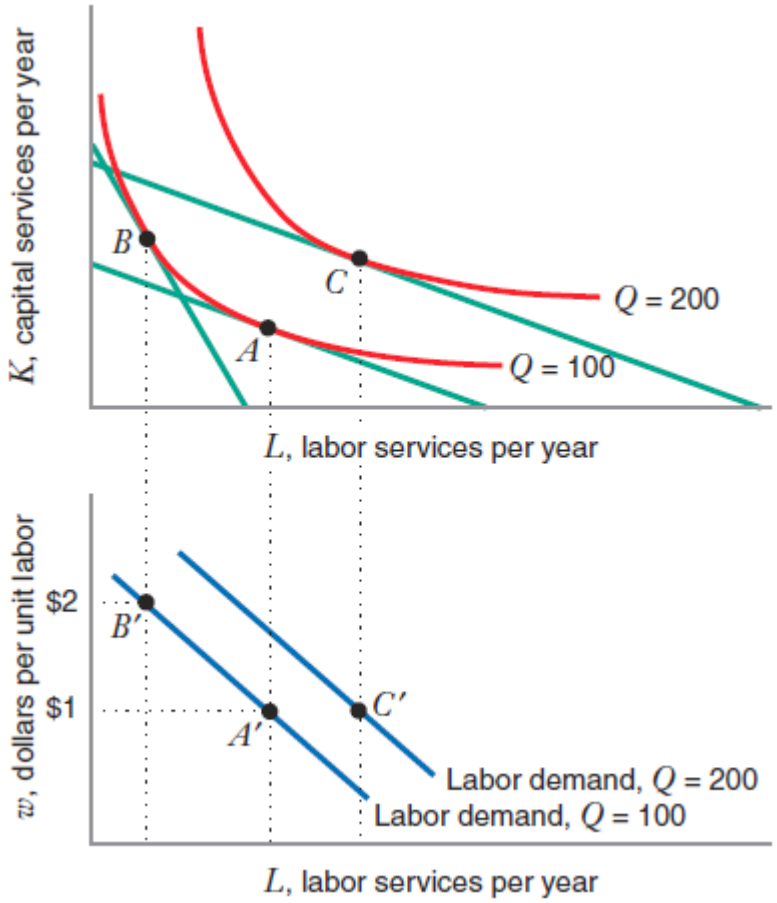


Input Demand

- A function that shows how the firm's cost-minimizing quantity of input varies with the price of that input
- Labor demand curve: Shows how the firm's cost-minimizing quantity of labor varies with the price of labor
- Capital demand curve: Shows how the firm's cost-minimizing quantity of capital varies with the price of capital

Comparative Statics Analysis and the Labor Demand Curve

- For a fixed quantity, as price of labor increases from $1 to $2, firm moves along its labor demand curve from A to B
- Increase in output shifts the demand curve



Price Elasticity of Demand for Inputs

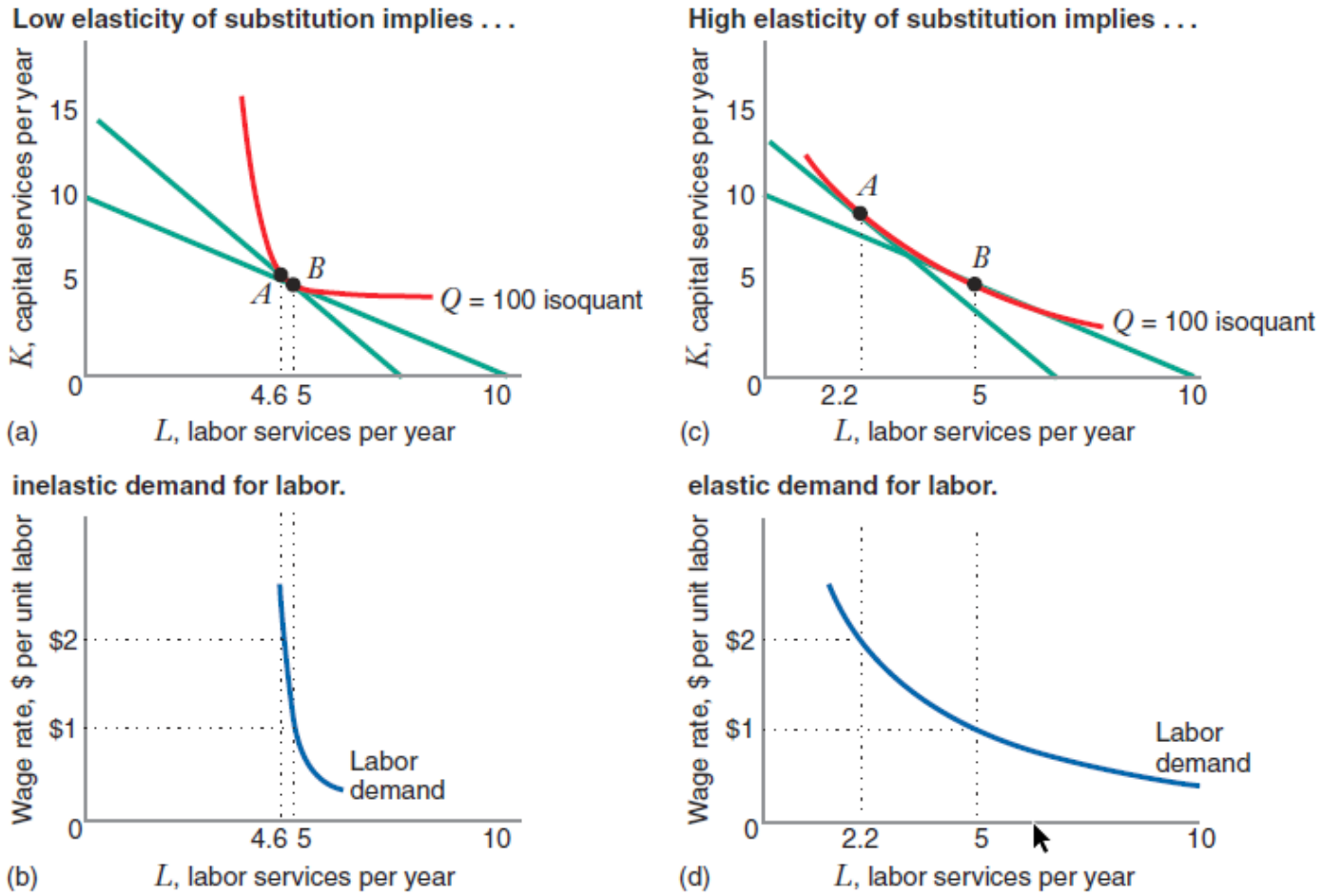
- Percentage change in the cost-minimizing quantity of labor with respect to a 1% change in the price of labor

$$-\varepsilon_{L,w} = \frac{\Delta L}{\Delta w}\frac{w}{L}$$

- Percentage change in the cost-minimizing quantity of capital with respect to a 1% change in the price of capital

$$-\varepsilon_{K,r} = \frac{\Delta K}{\Delta r}\frac{r}{K}$$

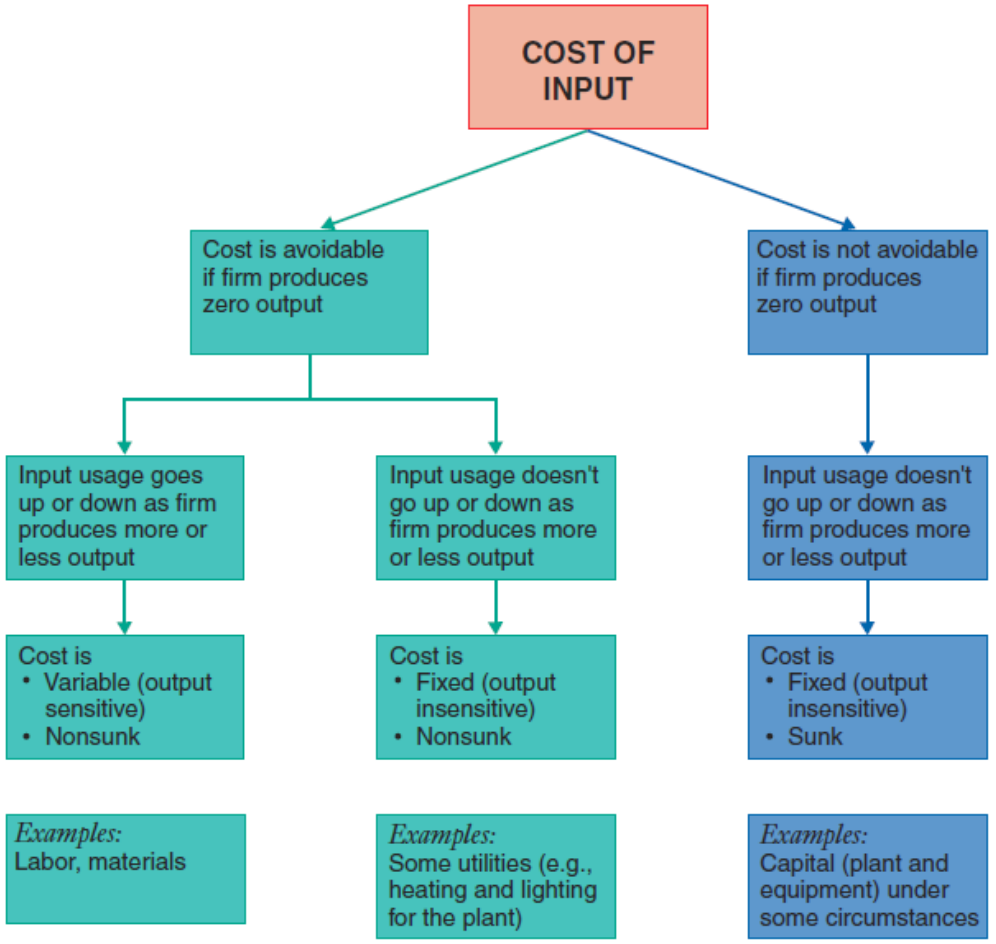
Price Elasticity of Demand for Labor



Short-Run Cost Minimization

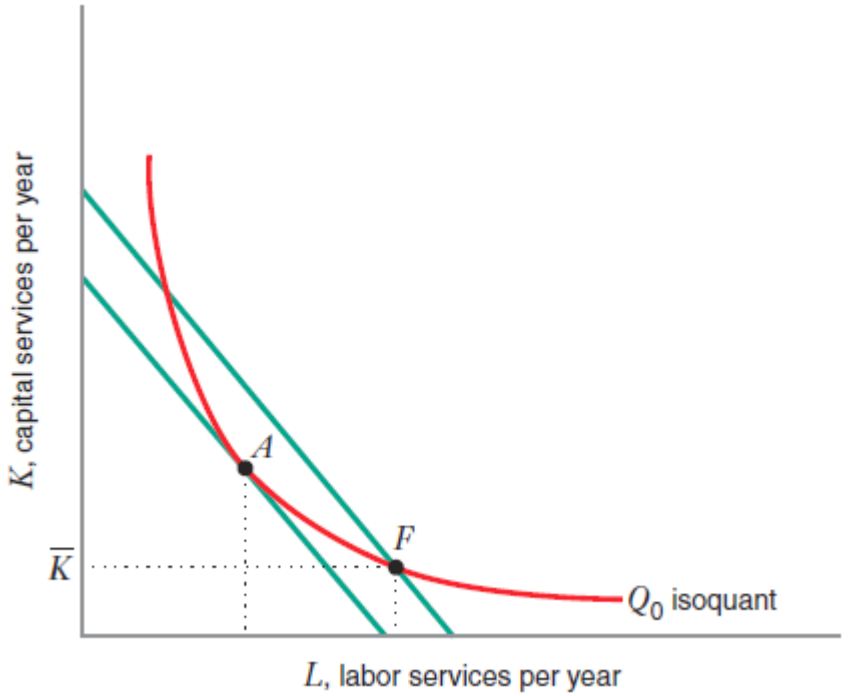
- Total variable costs – the sum of total expenditures on variable inputs, such as labor and materials, at the short-run cost-minimizing input combination
- Total fixed costs – the cost of fixed inputs; it does not vary with output

Classifying Costs in the Short Run



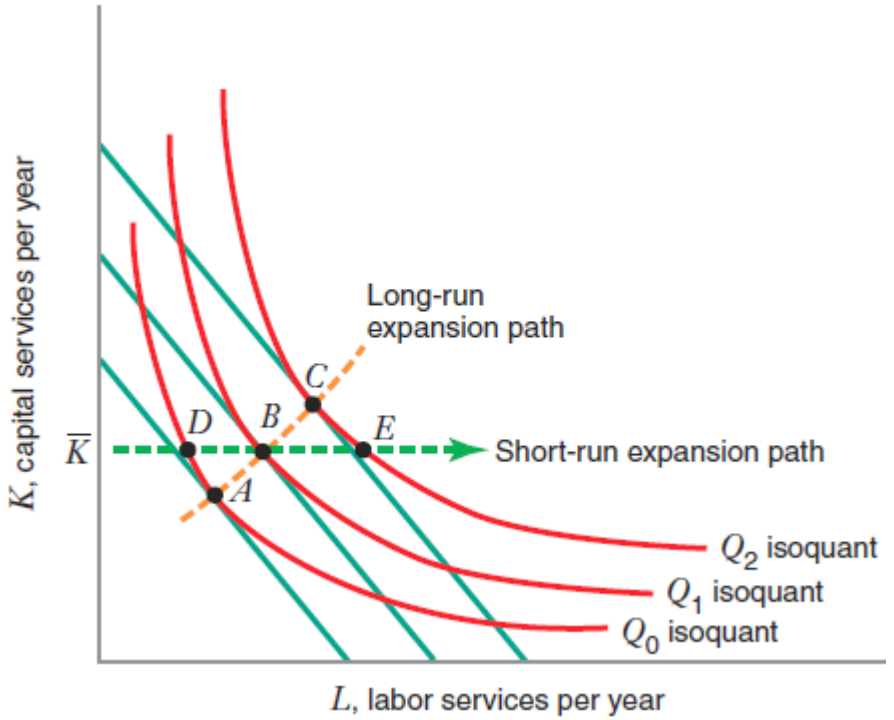
Short-Run Cost Minimization with One Fixed Input

- When the firm's capital is fixed at $\overline{K}$
- Short run combination is point F
- If the firm were free to adjust all of its inputs, the cost-minimizing combination is at point A



Short-Run Input Demand versus Long-Run Input Demand

- Long run-all variables are variable and the expansion path is from A – B – C
- Short run-some variables are fixed (capital)-the expansion path is from D –E –F



More than one Variable Input in the Short Run

- Short run: One input is fixed, capital
  - Firm can vary the other input, labor
  - So demand for labor will be independent of price
- Short run demand for labor will also depend on quantity produced
- As quantity increased, labor used increases holding capital fixed

Short-Run Cost Minimization with Two Variable Inputs

- More than one variable input – analysis similar to long-run cost minimization
- Three inputs
  1. Labor (L)
  2. Capital ($\bar{K}$)
  3. Raw materials (M)
- $MRTS_{L,M} = \frac{MP_L}{MP_M} = \frac{w}{m}$