Production and Costs in the Long run

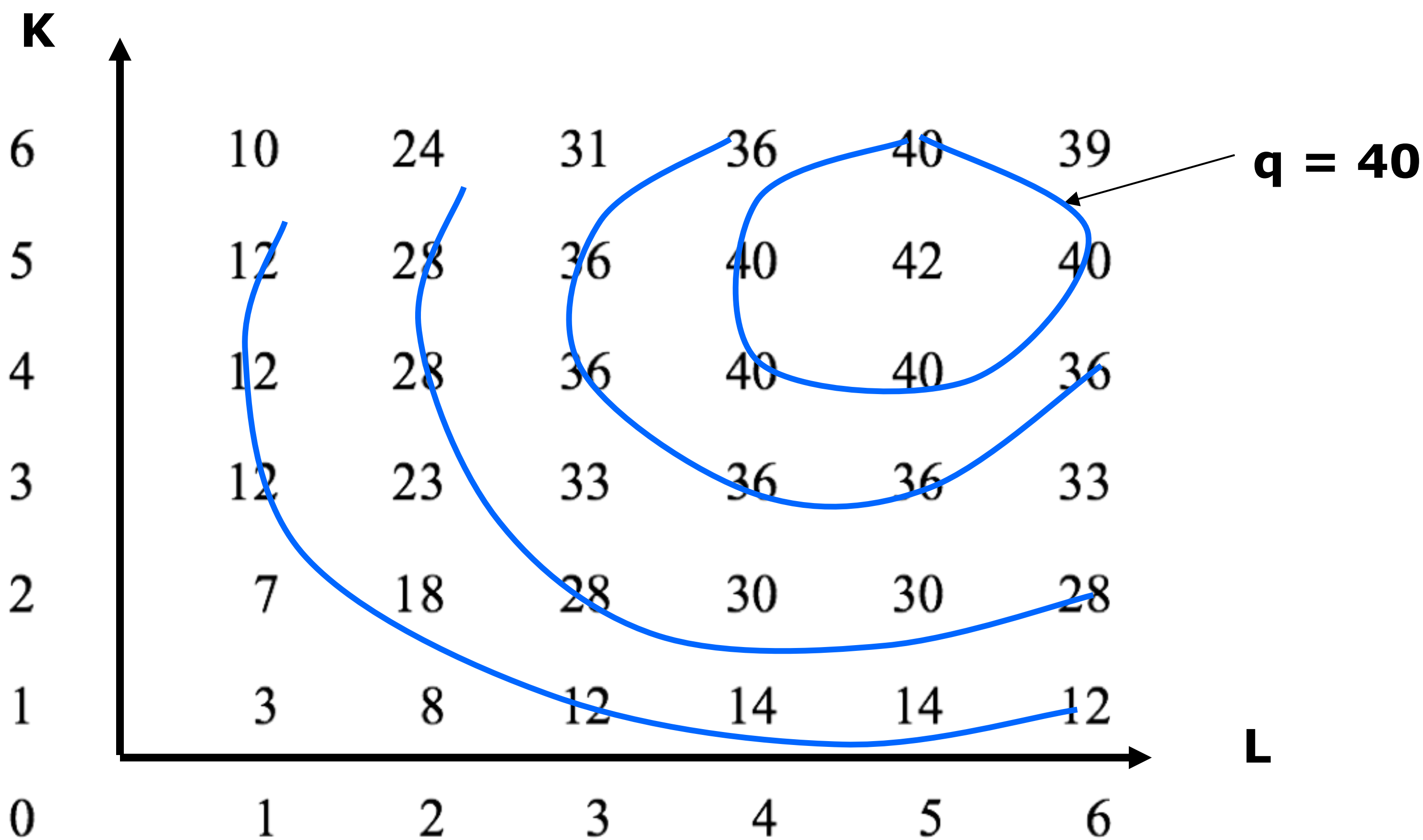
Topics to be discussed

- Isoquants
- Production with Two Variable Inputs
- Returns to Scale
- Cost in the Long Run
- Production with Two Outputs--Economies of Scope
- Learning Curve

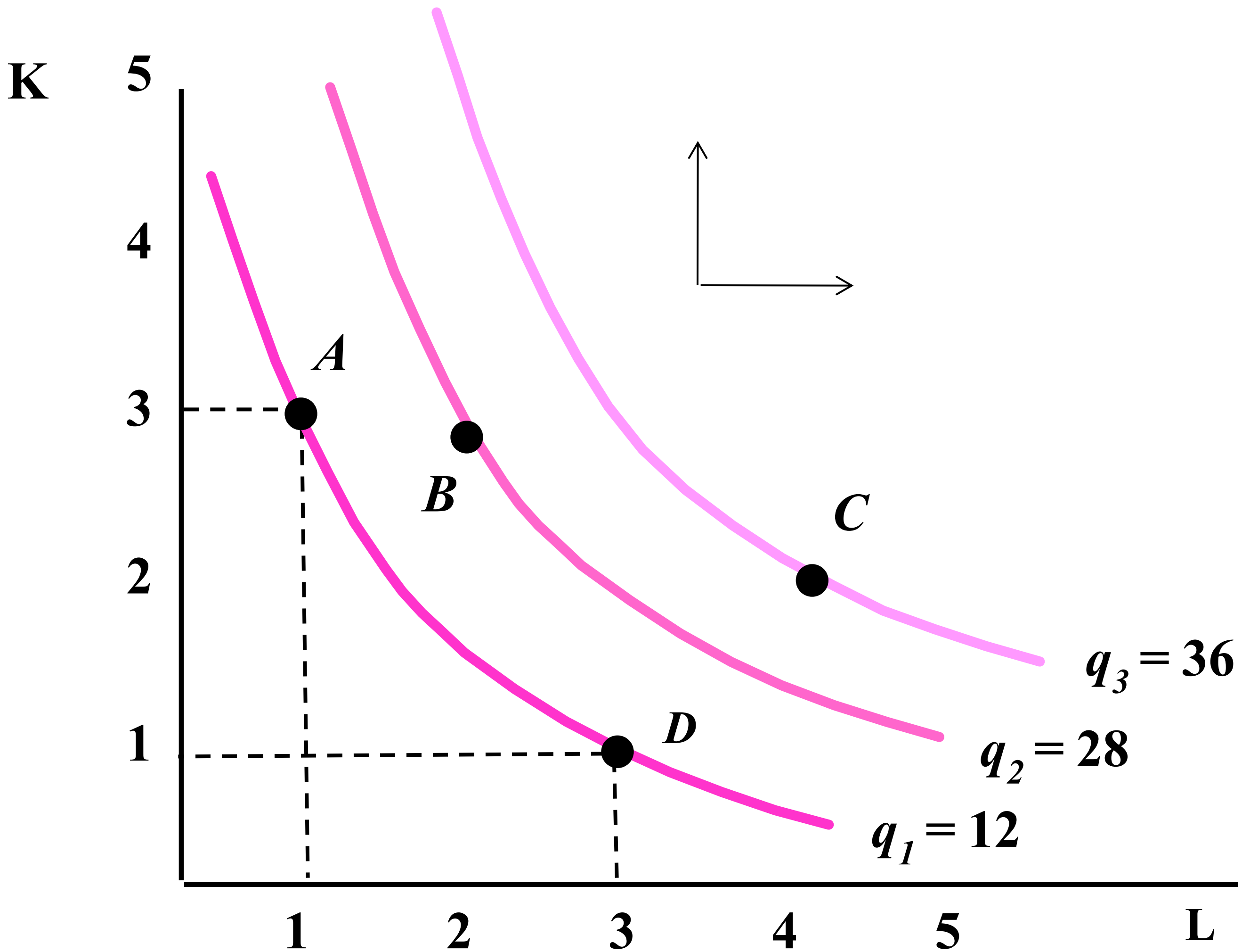
Production: Two Variable Inputs

- Firm can produce output by combining different amounts of labour and capital
- In the long-run, capital and labour are both variable.
- We can look at the output we can achieve with different combinations of capital and labour

Production: Two Variable Inputs



Isoquant Map



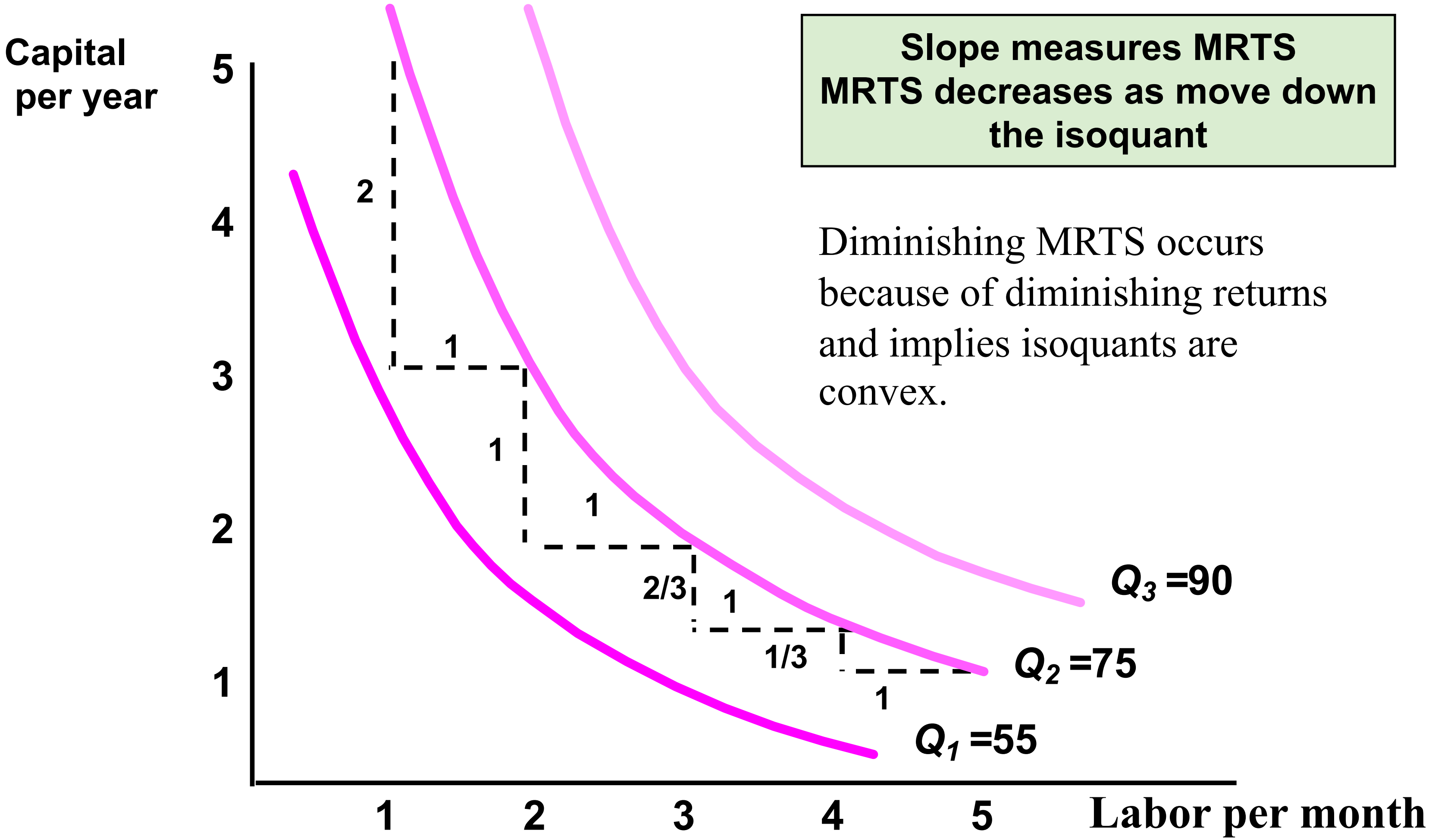
Derivation of an Isoquant

- From a production function $Q = K^{0.5}L^{0.5}$
- A a fixed Q, $K = Q^2/L$
- Suppose Q = 100, the function for this isoquant is: $K = 10000/L$
- Any combination of K & L according to the above function will always give Q = 100 and the slope is declining as L increases

Production: Two Variable Inputs

- Substituting Among Inputs
  - Slope of the isoquant shows how one input can be substituted for the other and keep the level of output the same.
  - Negative slope is the marginal rate of technical substitution (MRTS) = dK/dL
    - Amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant.

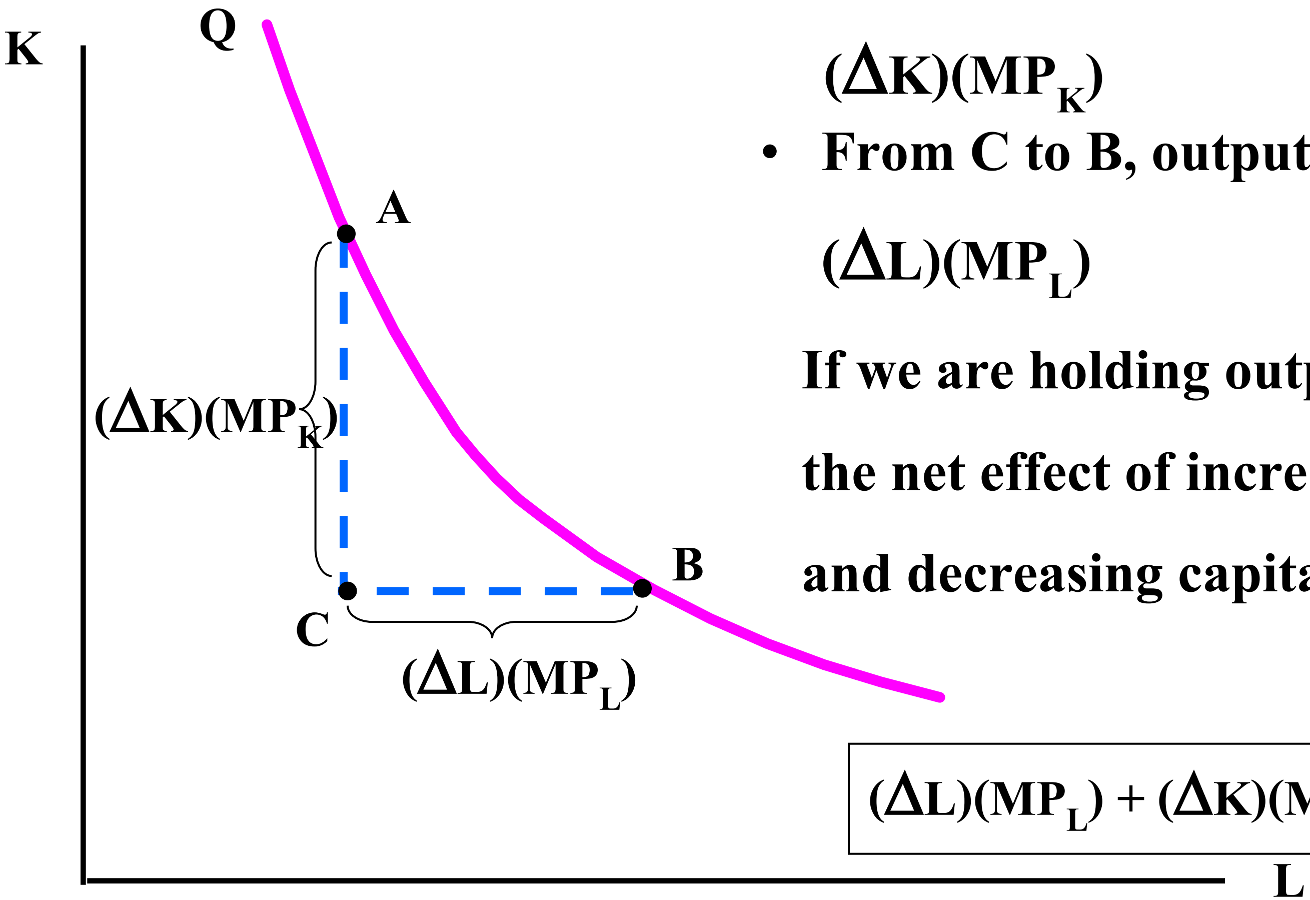
Marginal Rate of Technical Substitution



Slope measures MRTS
MRTS decreases as move down the isoquant

Diminishing MRTS occurs because of diminishing returns and implies isoquants are convex.

MRTS and Marginal Products



- **From A to C, output decreases by $(\Delta K)(MP_K)$**
- **From C to B, output increases by $(\Delta L)(MP_L)$**

If we are holding output constant, the net effect of increasing labor and decreasing capital must be zero

$$(\Delta L)(MP_L) + (\Delta K)(MP_K) = 0$$

MRTS and Marginal Products

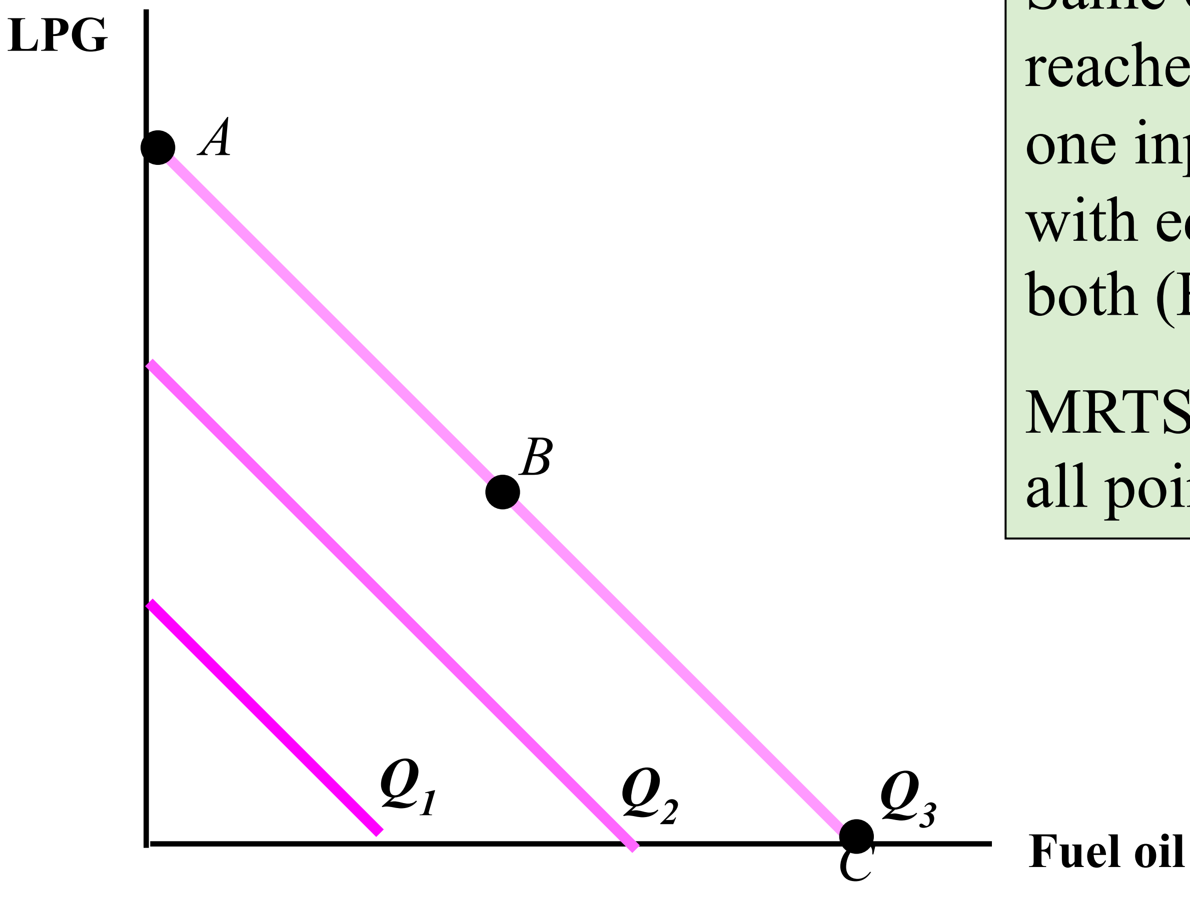
- Rearranging equation, we can see the relationship between MRTS and MPs

$$(MP_L)(\Delta L) + (MP_K)(\Delta K) = 0$$

$$(MP_L)(\Delta L) = -(MP_K)(\Delta K)$$

$$\frac{(MP_L)}{(MP_K)} = -\frac{\Delta K}{\Delta L} = MRTS$$

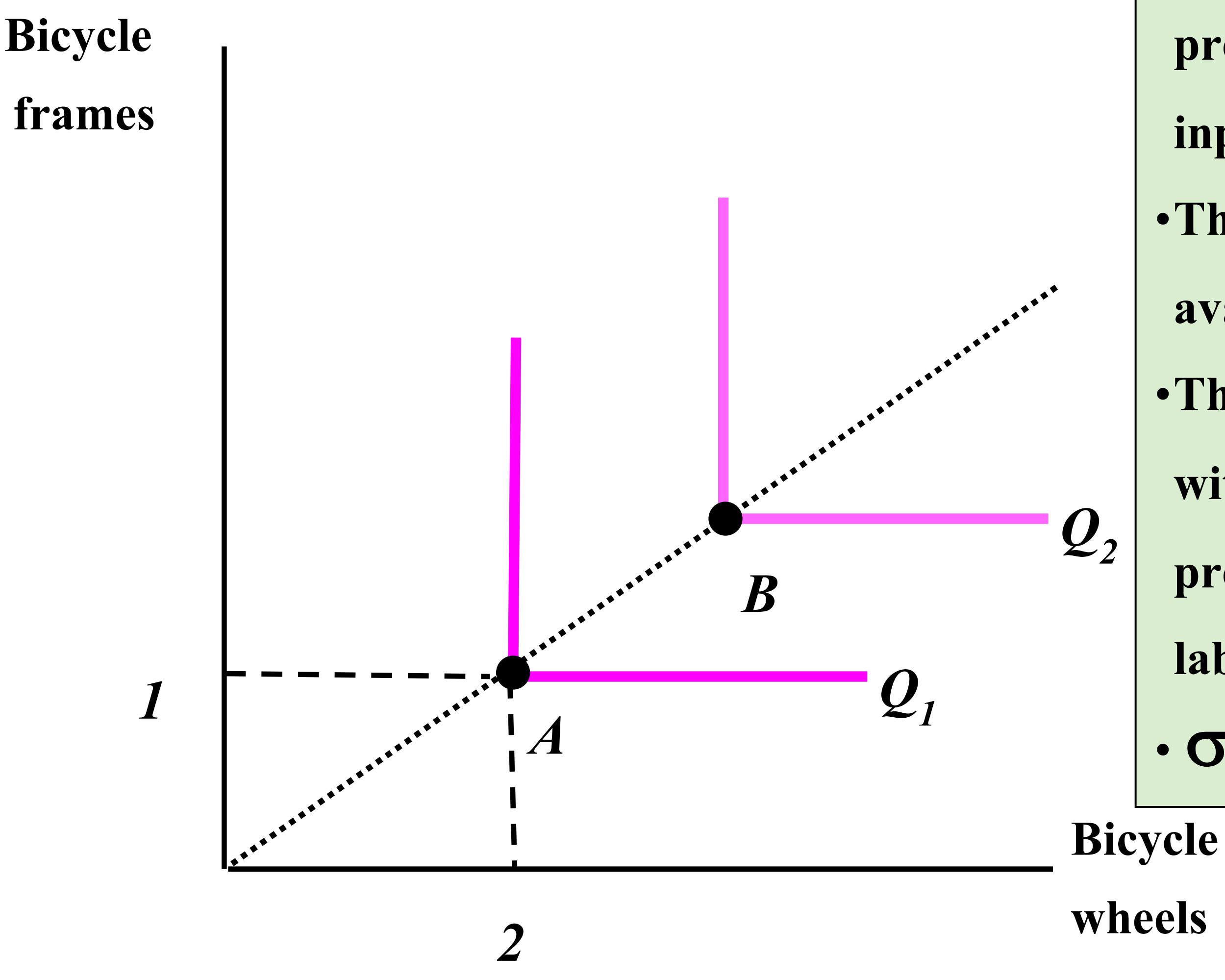
Perfect Substitutes



Same output can be reached with mostly one input (A or C) or with equal amount of both (B)

MRTS is constant at all points on isoquant

Fixed-Proportions Production Function



- **Same output can only be produced with one set of inputs.**
- **There is no substitution available between inputs.**
- **The output can be made with only a specific proportion of capital and labor**
- **$\sigma = 0$**

Returns to Scale

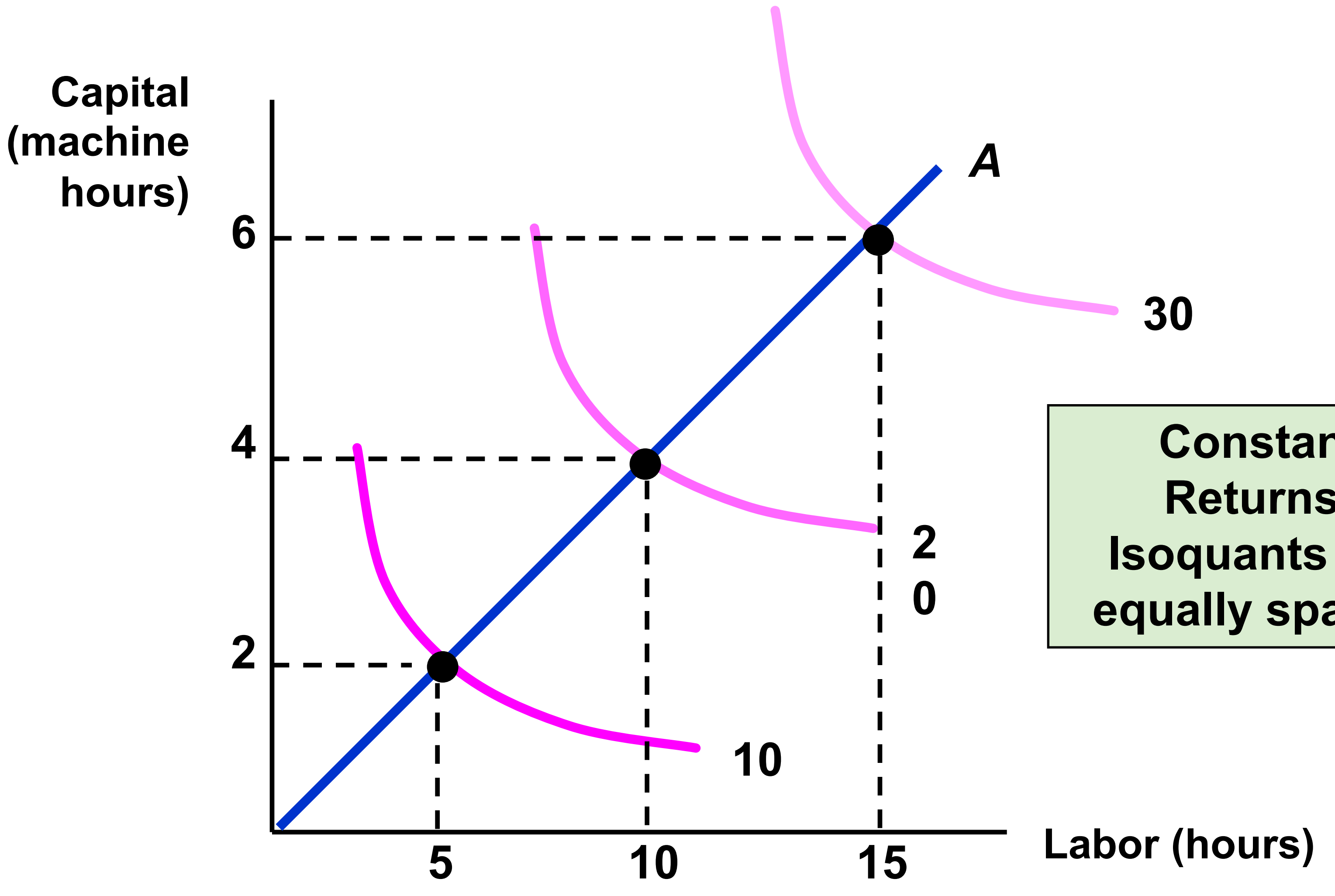
- How does a firm decide, in the long run, the best way to increase output
  - Can change the scale of production by increasing all inputs in proportion

- Rate at which output increases as inputs are increased proportionately
  - Constant returns to scale
  - Increasing returns to scale
  - Decreasing returns to scale

Constant Returns to Scale

- Output doubles when all inputs are doubled
  - Size does not affect productivity
  - May have a large number of producers
  - Isoquants are equidistant apart

Returns to Scale

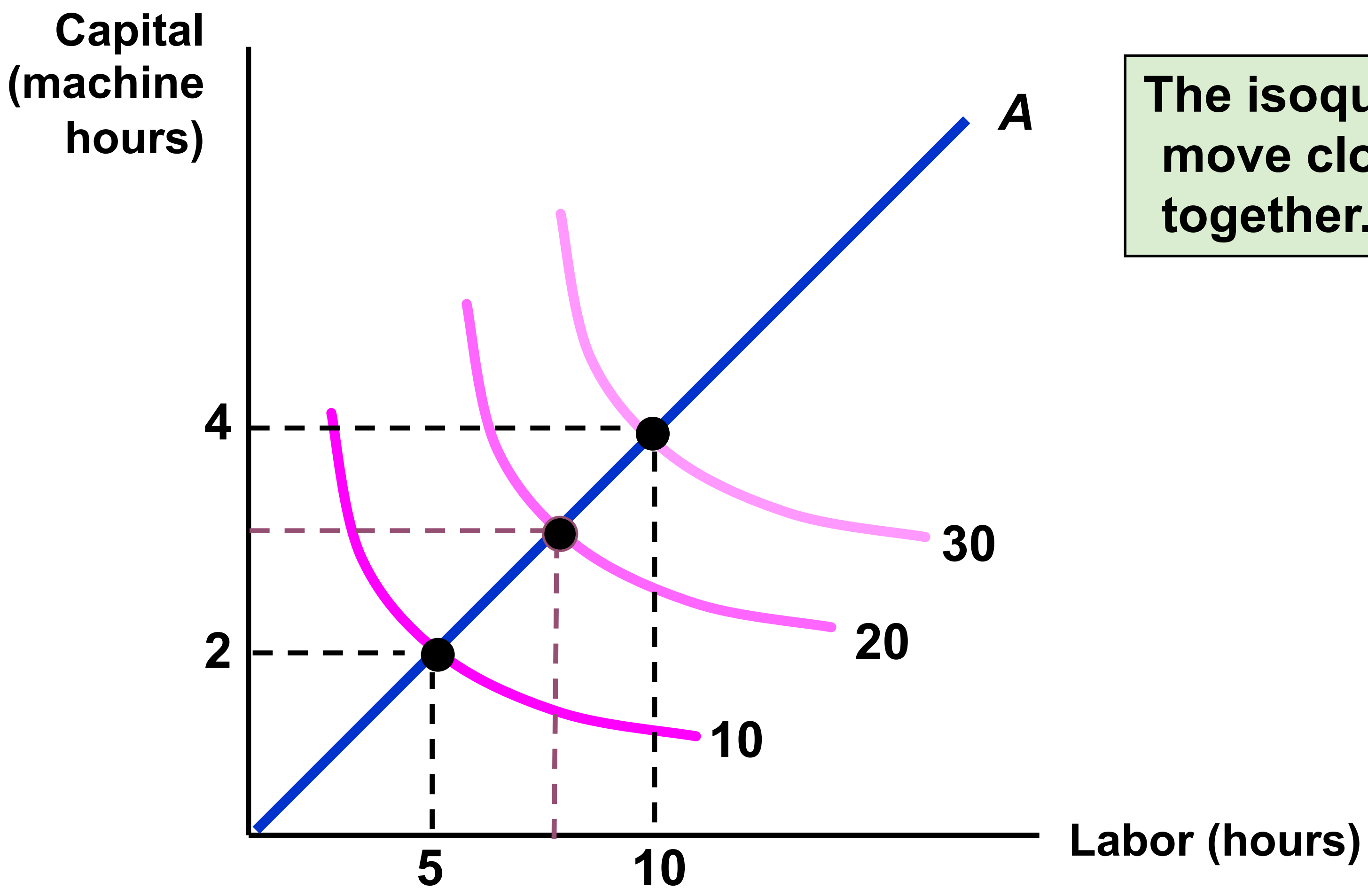


Constant Returns: Isoquants are equally spaced

Increasing Returns to Scale

- output more than doubles when all inputs are doubled
  - Larger output associated with lower cost (cars)
  - One firm is more efficient than many (utilities)
  - The isoquants get closer together
- Reasons
  - Specialisation and division of labor
  - Technical increasing returns to scale
- It leads to a declining cost per unit

Increasing Returns to Scale



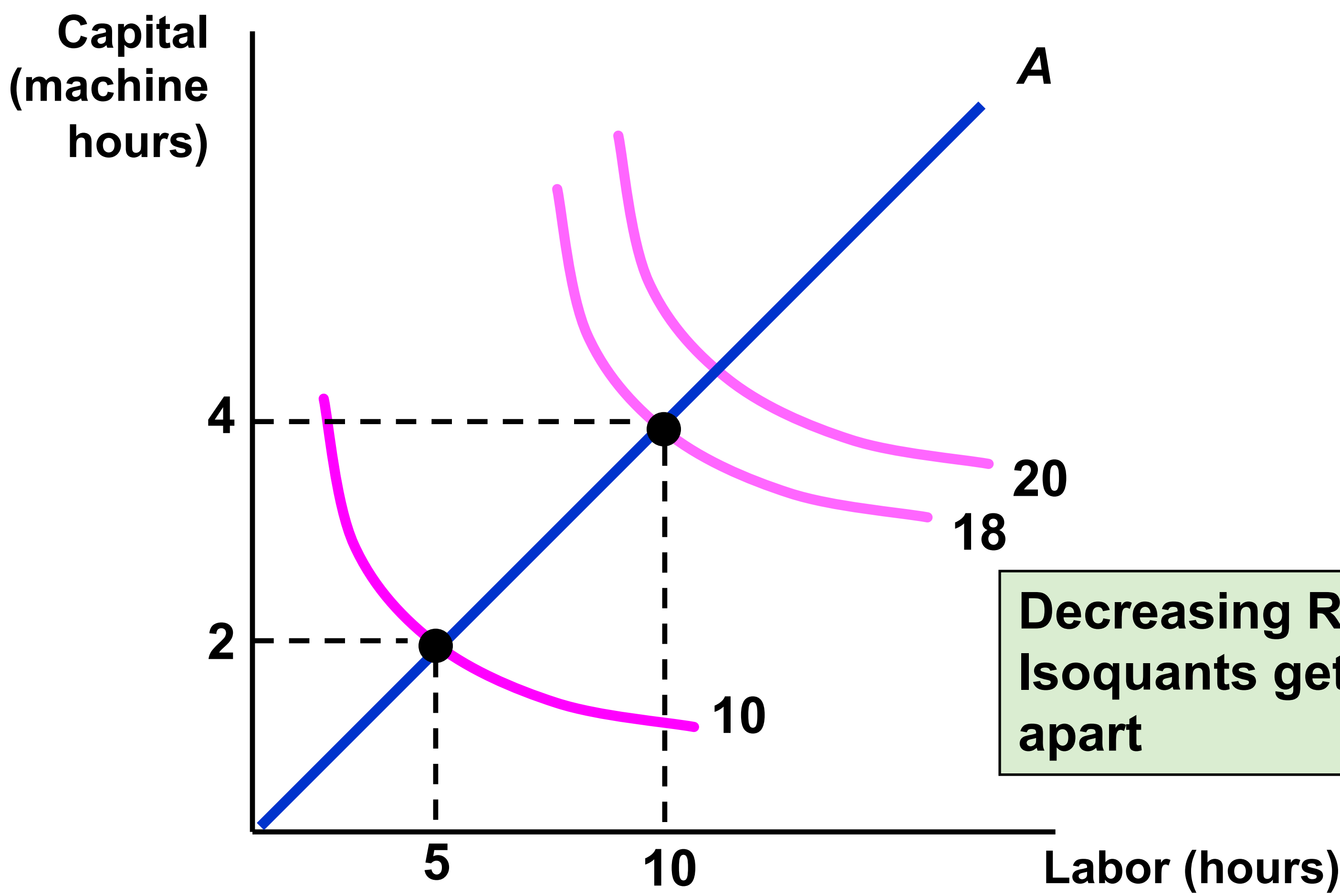
The isoquants move closer together.

Decreasing Returns to Scale

- output less than doubles when all inputs are doubled
  - Decreasing efficiency with large size
  - Reduction of entrepreneurial abilities
  - Isoquants become farther apart

- Reasons
  - Coordination problems, limitation of manager

Returns to Scale



Decreasing Returns: Isoquants get further apart

Cost in the Long Run

- In the long run a firm can change all of its inputs
- Assumptions
  - Two Inputs: Labour (L) & capital (K)
  - Price of input: wage rate (w), rental rate (r)
- The Isocost Line
  - A line showing all combinations of L & K that can be purchased for the same cost
  - Total cost of production is sum of firm's labor cost, wL and its capital cost rK

$$TC = wL + rK$$

Cost in the Long Run

- Rewriting TC as an equation for a straight line:
  - K = TC/r - (w/r)L
  - Slope of the isocost: $\Delta K / \Delta L = -\left(w / r \right)$

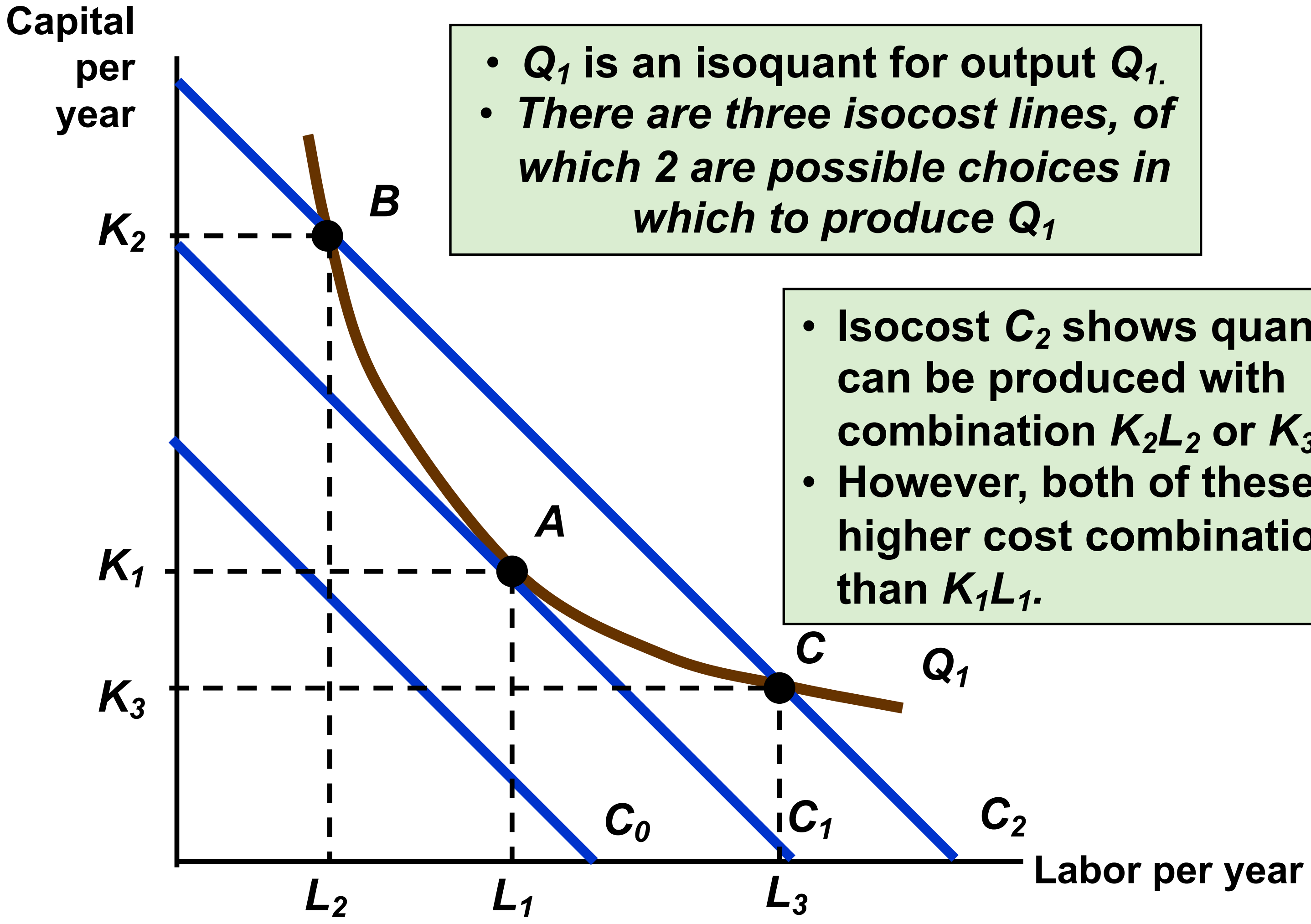
- - w/r is the ratio of the wage rate to rental cost of capital.

- This shows the rate at which capital can be substituted for labor with no change in cost.

Least Cost Combination of Inputs

- We will address how to minimise cost for a given level of output by combining isocosts with isoquants
- We choose the output we wish to produce and then determine how to do that at minimum cost
  - Isoquant is the quantity we wish to produce
  - Isocost is the combination of K and L that gives a set cost

Least Cost Combination of Inputs



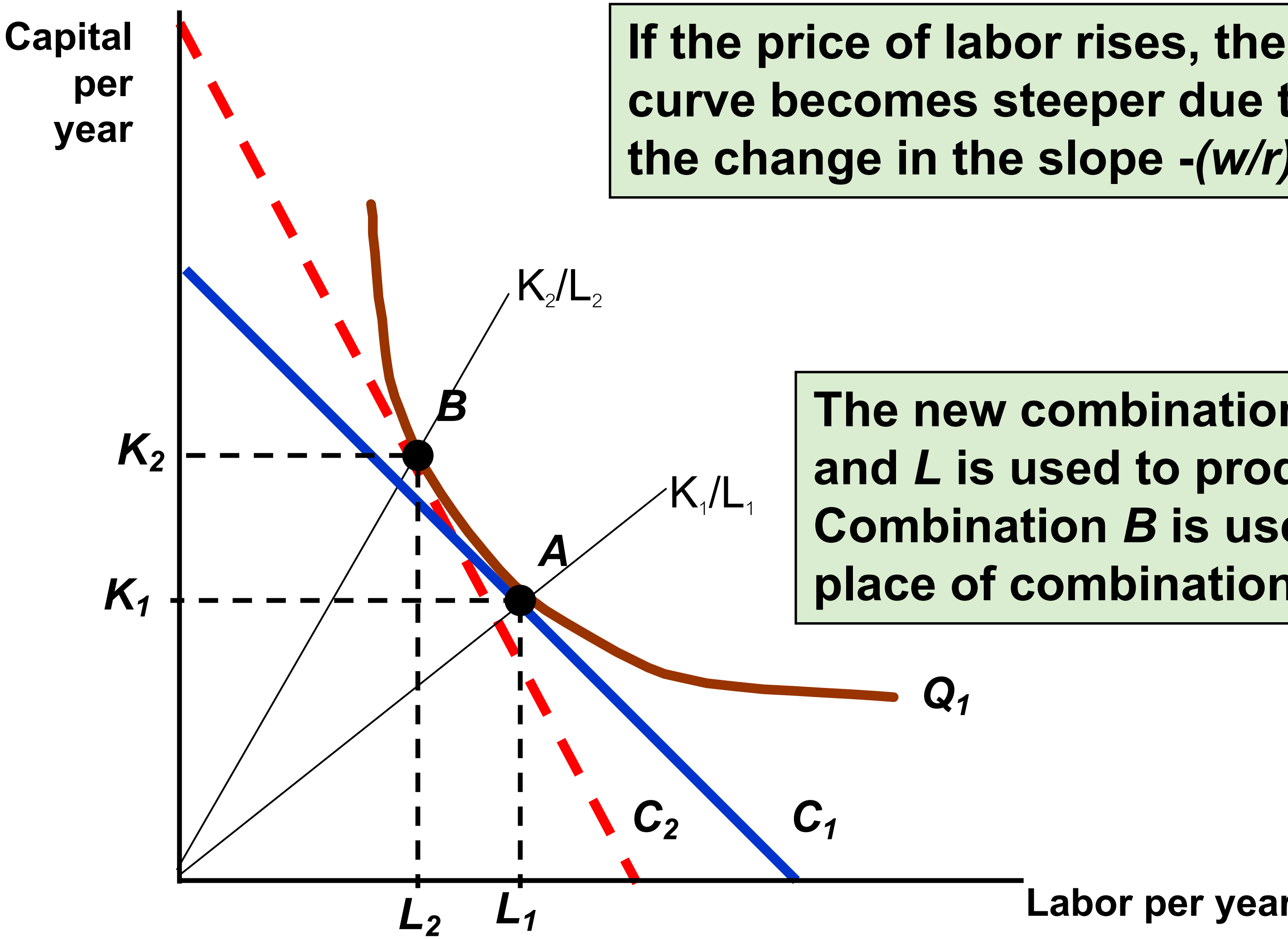
- Q_1 is an isoquant for output Q_1.
- *There are three isocost lines, of which 2 are possible choices in which to produce Q_1*

- Isocost C_2 shows quantity Q_1 can be produced with combination K_2L_2 or K_3L_3.
- However, both of these are higher cost combinations than K_1L_1.

Input Substitution When an Input Price Changes

- If the price of labor changes, then the slope of the isocost line change, w/r
- It now takes a new quantity of labor and capital to produce the output
- If price of labor increases relative to price of capital, and capital is substituted for labor

Input Substitution When an Input Price Changes



If the price of labor rises, the isocost curve becomes steeper due to the change in the slope -*(w/r)*.

The new combination of *K* and *L* is used to produce Q_1. Combination *B* is used in place of combination *A*.

Elasticity of substitution

%Δ in capital-labour ratio divided by the %Δ in the slope of the isoquant

$$\sigma = \frac{\%\Delta\left(K/L\right)}{\%\Delta\, MRTS} = \frac{d\left(K/L\right)}{d\left(MRTS\right)}\,\frac{MRTS}{K/L} = \frac{\partial \ln\left(K/L\right)}{\partial \ln\left(MRTS\right)}$$

$\sigma = 0$ means fixed proportion

$\sigma = \infty$ means perfect substitutes

Example

From Cobb-Douglas production function: $Q = K^{\alpha}L^{\beta}$,

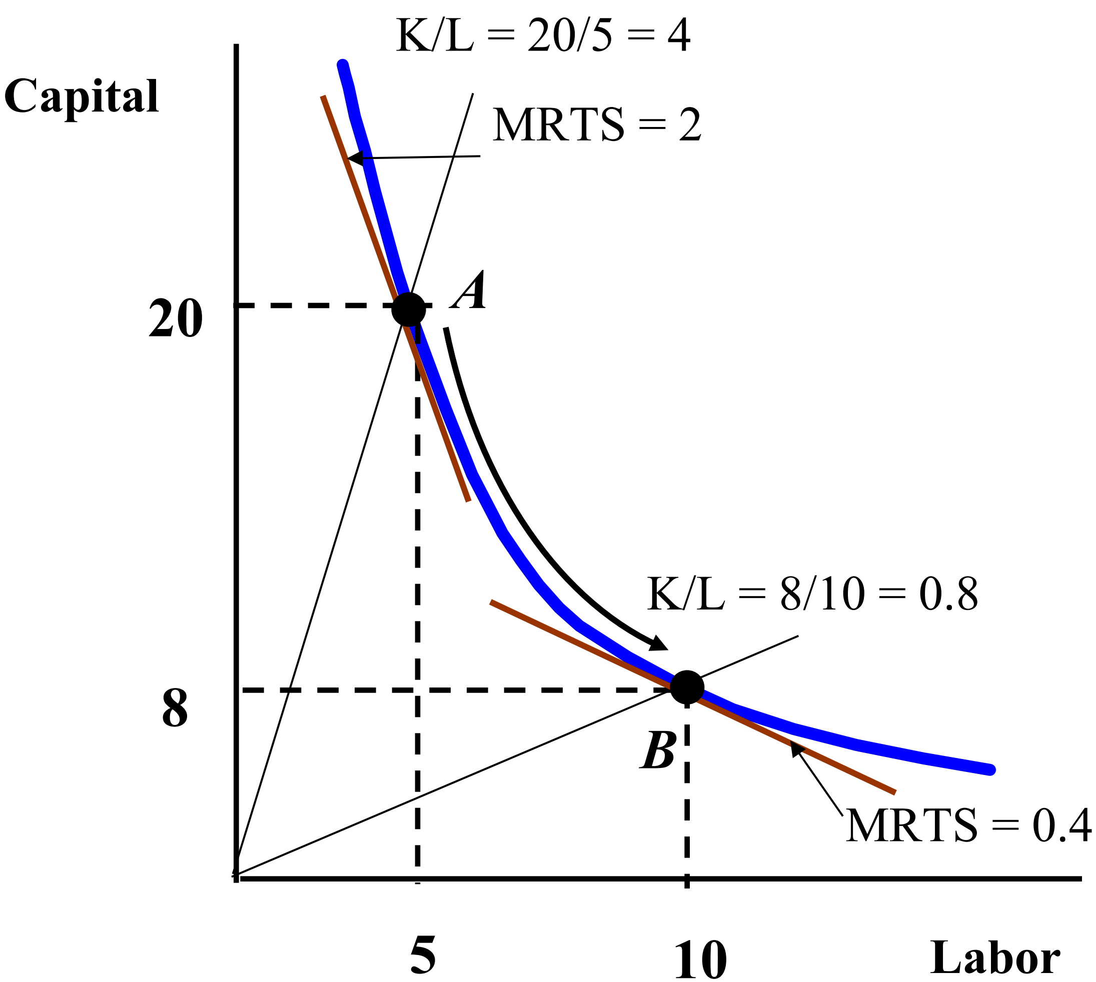
$$MRTS = \frac{MP_L}{MP_K} = \frac{\beta K^{\alpha} L^{\beta-1}}{\alpha K^{\alpha-1} L^{\beta}} = \frac{\beta K}{\alpha L}$$

$$\frac{K}{L} = \frac{\alpha}{\beta} MRTS$$

$$\ln \frac{K}{L} = \ln \frac{\alpha}{\beta} + \ln MRTS$$

$$\sigma = 1$$

Elasticity of substitution

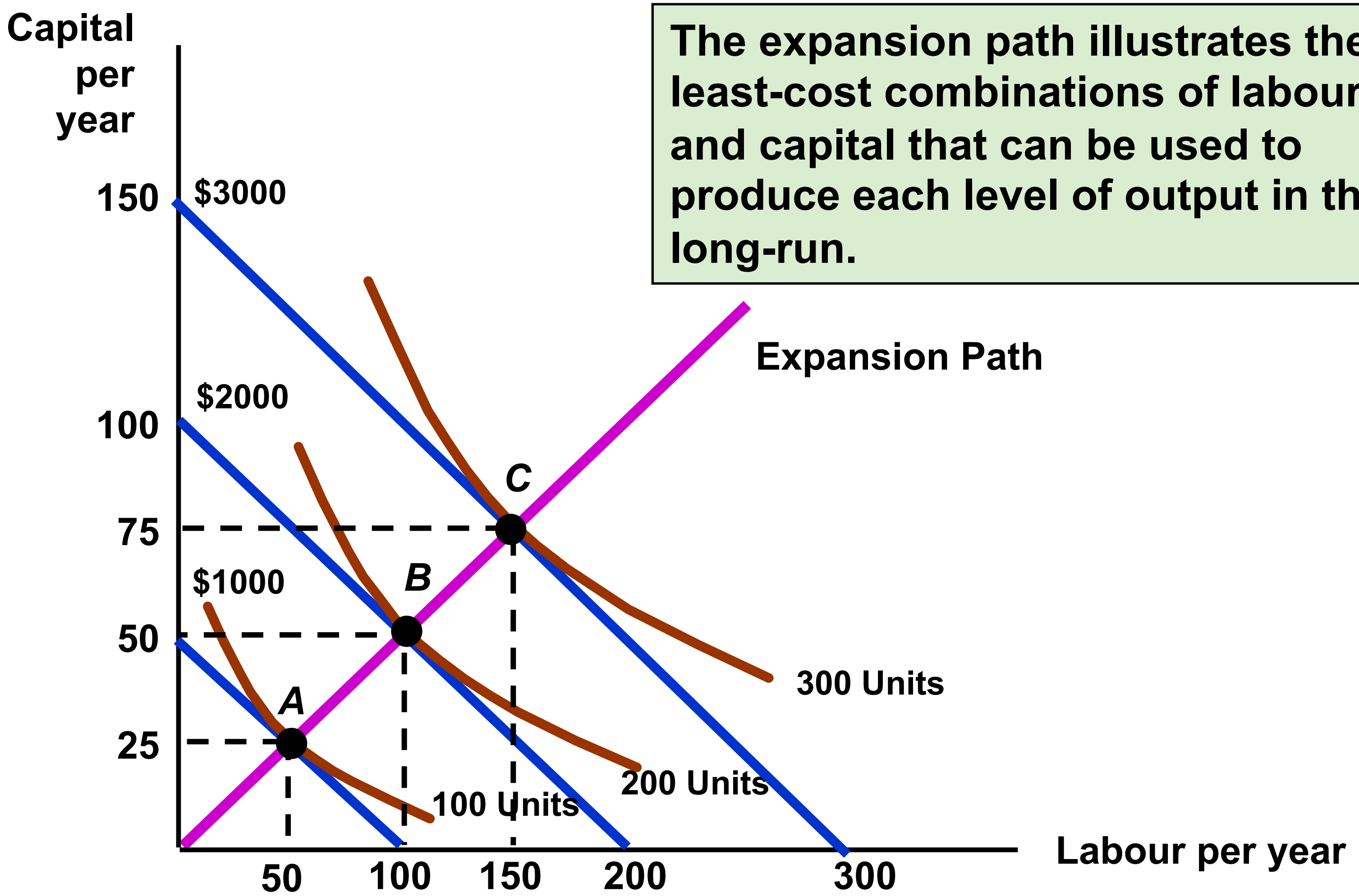


- **Point *A* is more capital-intensive, and *B* is more labor-intensive.**
- **Moving from A to B, K/L decreases by 80% and MRTS also decreases by 80% --> σ = 1**

Cost in the Long Run

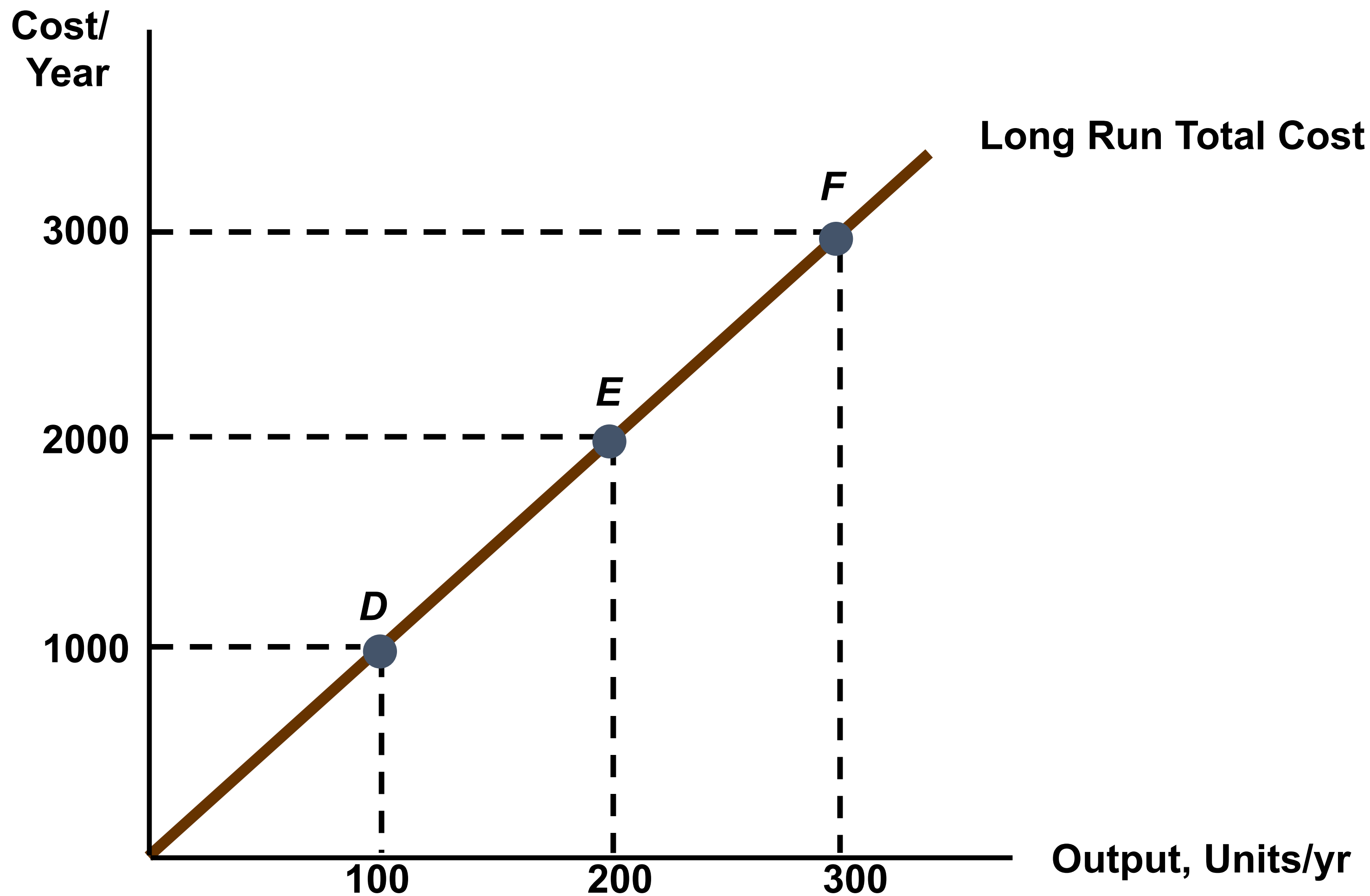
- Cost minimisation with Varying Output Levels
  - For each level of output, there is an isocost curve showing minimum cost for that output level
  - A firm's expansion path shows the minimum cost combinations of labor and capital at each level of output.

A Firm's Expansion Path



The expansion path illustrates the least-cost combinations of labour and capital that can be used to produce each level of output in the long-run.

A Firm's Long-Run Total Cost Curve: CRTS case



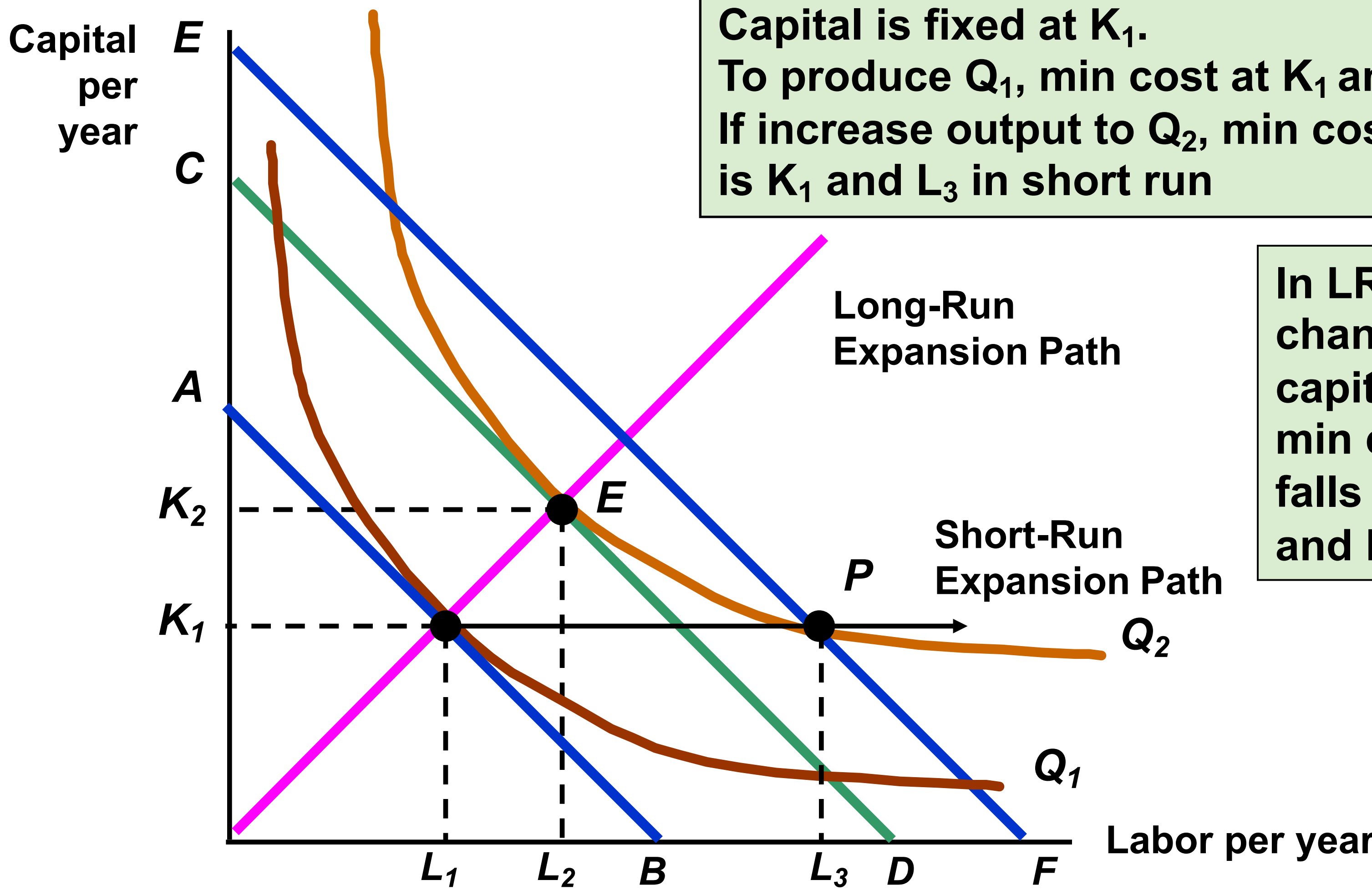
Expansion Path & Long-run Costs

- Firms expansion path has same information as long-run total cost curve
- To move from expansion path to LR cost curve
  - Find tangency with isoquant and isocost
  - Determine min cost of producing the output level selected
  - Graph output-cost combination

Long-Run Versus Short-Run Cost Curves

- In the short run some costs are fixed
- In the long run firm can change anything including plant size
  - Can produce at a lower average cost in long run than in short run
  - Capital and labor are both flexible
- We can show this by holding capital fixed in the short run and flexible in long run

The Inflexibility of Short-Run Production



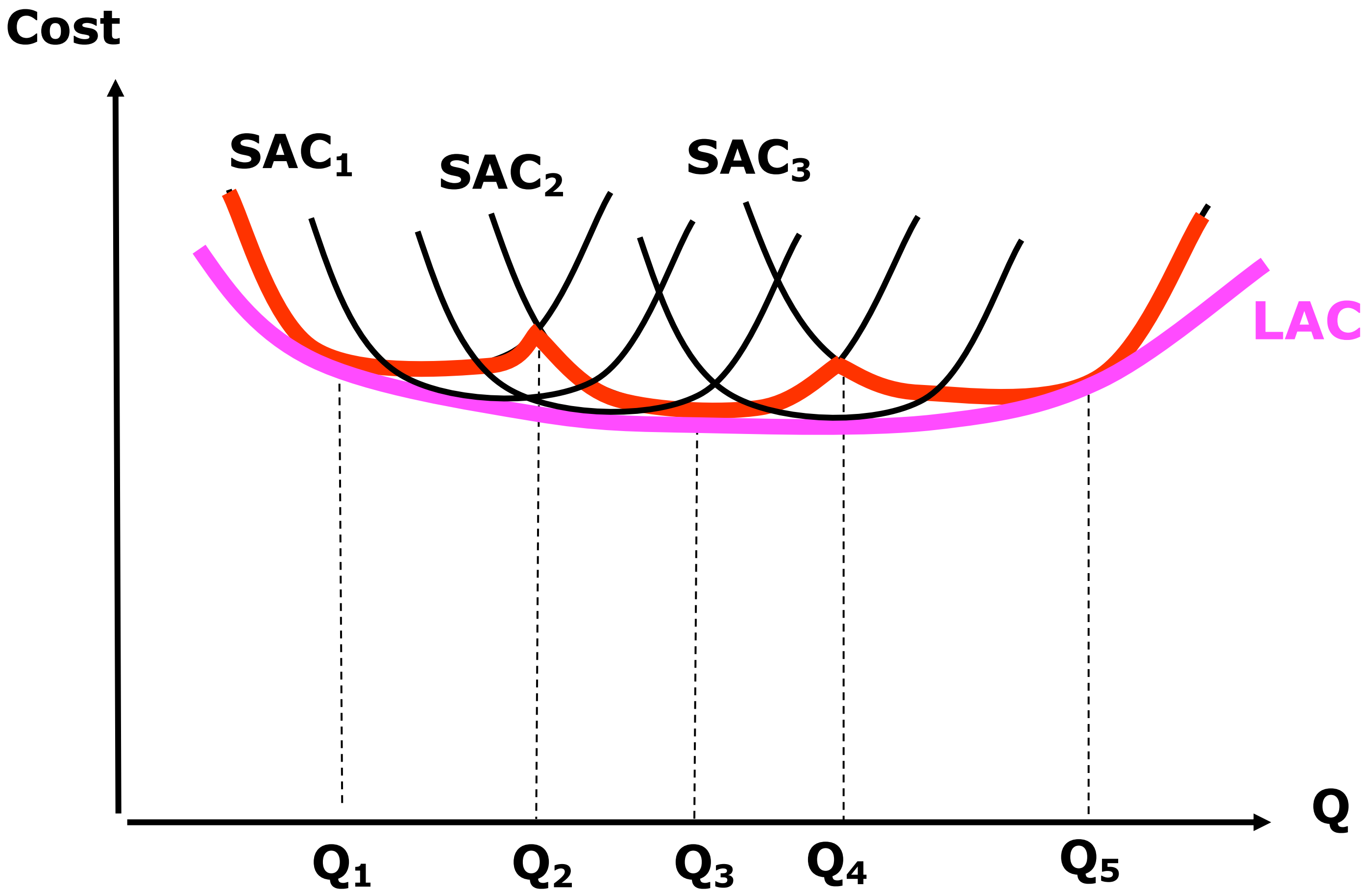
Capital is fixed at K_1.
To produce Q_1, min cost at K_1 and L_1
If increase output to Q_2, min cost is K_1 and L_3 in short run

In LR, can change capital and min costs falls to K_2 and L_2

Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
  - Most important determinant of the shape of the LR AC and MC curves is relationship between scale of the firm's operation and inputs required to min cost
- Constant Returns to Scale
  - If input is doubled, output will double
  - AC cost is constant at all levels of output.

Derivation of LAC from SAC



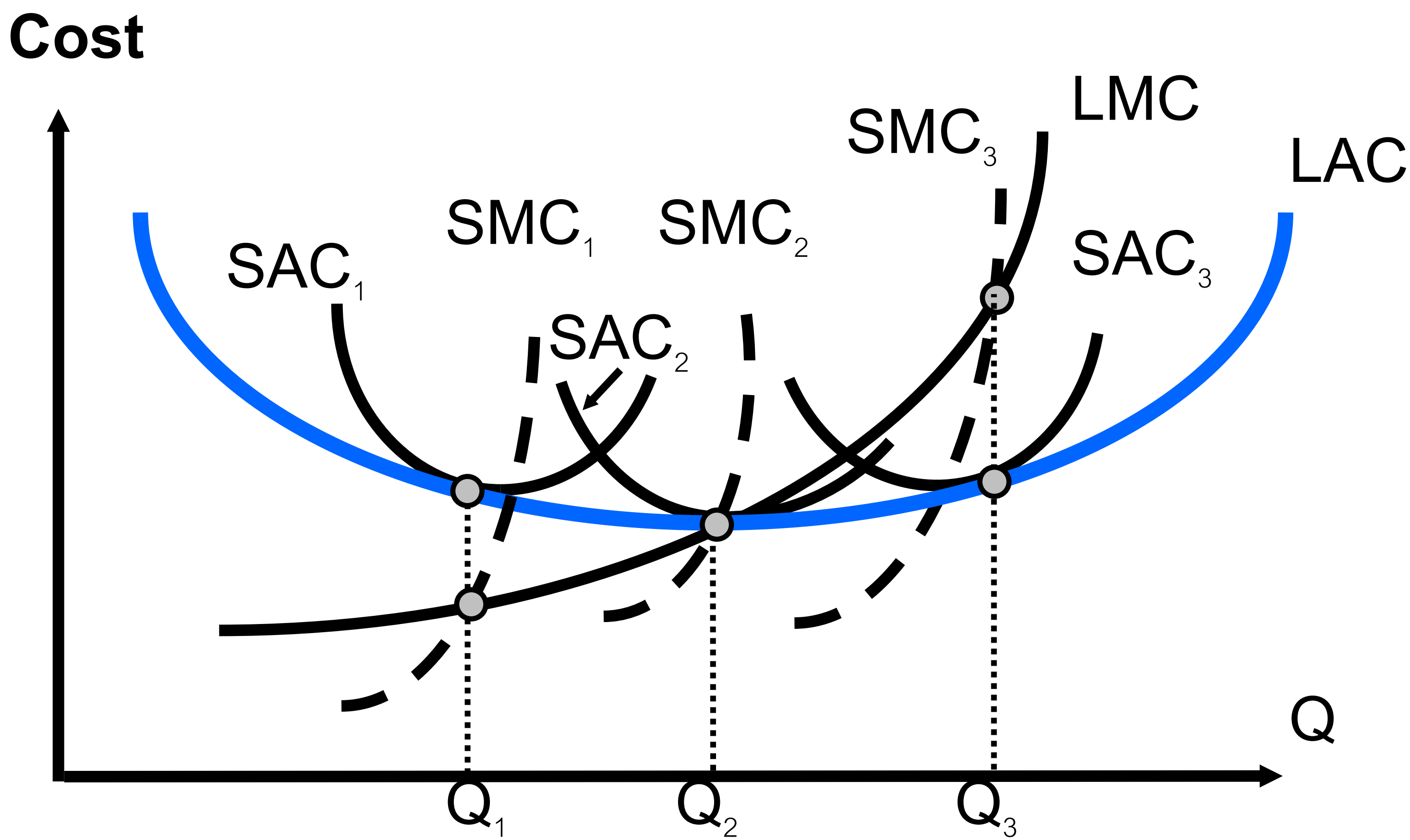
Long-Run Versus Short-Run Cost Curves

- Increasing Returns to Scale
  - If input is doubled, output will more than double
  - AC decreases at all levels of output.
- Decreasing Returns to Scale
  - If input is doubled, output will less than double
  - AC increases at all levels of output

Long-Run Versus Short-Run Cost Curves

- In the long-run:
  - Firms experience increasing and decreasing returns to scale and therefore long-run average cost is "U" shaped.
  - Source of U-shape is due to returns to scale instead of diminishing marginal returns like the short run curve
  - Long-run marginal cost curve measures the change in long-run total costs as output is increased by 1 unit

Short and Long-run Average Cost Curves



Long-Run Versus Short-Run Cost Curves

- Long-run marginal cost leads long-run average cost:
  - If LMC < LAC, LAC will fall
  - If LMC > LAC, LAC will rise
  - Therefore, LMC = LAC at the minimum of LAC
- In special case where LAC if constant, LAC and LMC are equal

Long Run Costs

- **Economies of Scale**: a situation when LAC declines with a larger output due to
  - increasing returns to scale
  - On a larger scale, workers can better specialize
  - Firm can use more efficient machine
  - Lumpiness in investment
  - Scale can provide flexibility – managers can organise production more effectively
  - Firm may be able to get inputs at lower cost if can get quantity discounts. Lower prices might lead to different input mix

Long Run Costs

- **Diseconomies of scale**: a situation when LAC increases with a larger output due to
  - Factory space and machinery may make it more difficult for workers to do their job efficiently
  - Managing a larger firm may become more complex and inefficient as the number of tasks increase
  - Bulk discounts can no longer be utilized. Limited availability of inputs may cause price to rise

Production with Two Outputs: Economies of Scope

- Many firms produce more than one product and those product are closely linked
- Examples:
  - Chicken farm--poultry and eggs
  - Automobile company--cars and trucks
  - University--Teaching and research
  - Nation Group
  - Choke Chai Farm

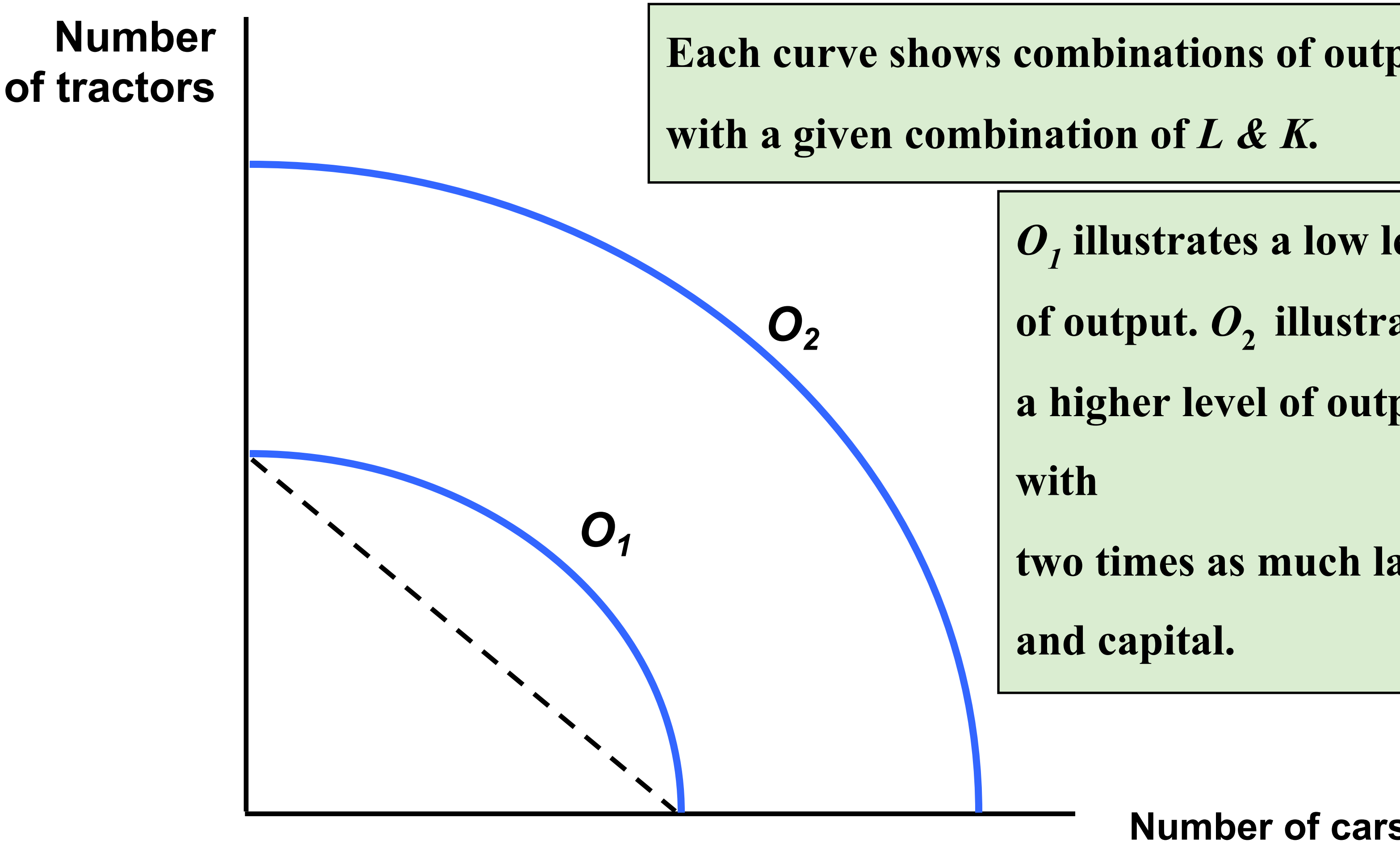
Advantages

- Both use capital and labor.
- The firms share management resources.
- Both use the same labor skills and type of machinery.

Production with Two Outputs: Economies of Scope

- Firms must choose how much of each to produce.
- The alternative quantities can be illustrated using product transformation curves
  - Curves showing the various combinations of two different outputs (products) that can be produced with a given set of inputs

Product Transformation Curve



Each curve shows combinations of output with a given combination of *L & K*.

O_1 illustrates a low level of output. O_2 illustrates a higher level of output with two times as much labor and capital.

Product Transformation Curve

- Product transformation curves are negatively slope
  - To get more of one output, must give up some of the other output
- Constant returns exist in this example
  - Second curve lies twice as far from origin as the first curve
- Curve is concave
  - Joint production has its advantages

Production with Two Outputs: Economies of Scope

- There is no direct relationship between economies of scope and economies of scale.
  - May experience economies of scope and diseconomies of scale
  - May have economies of scale and not have economies of scope

Dynamic Changes in Costs: The Learning Curve

- Firms may lower their costs not only due to economies of scope, but also due to managers and workers become more experienced at their jobs
- As management and labor gain experience with production, the firm's marginal and average costs may fall.

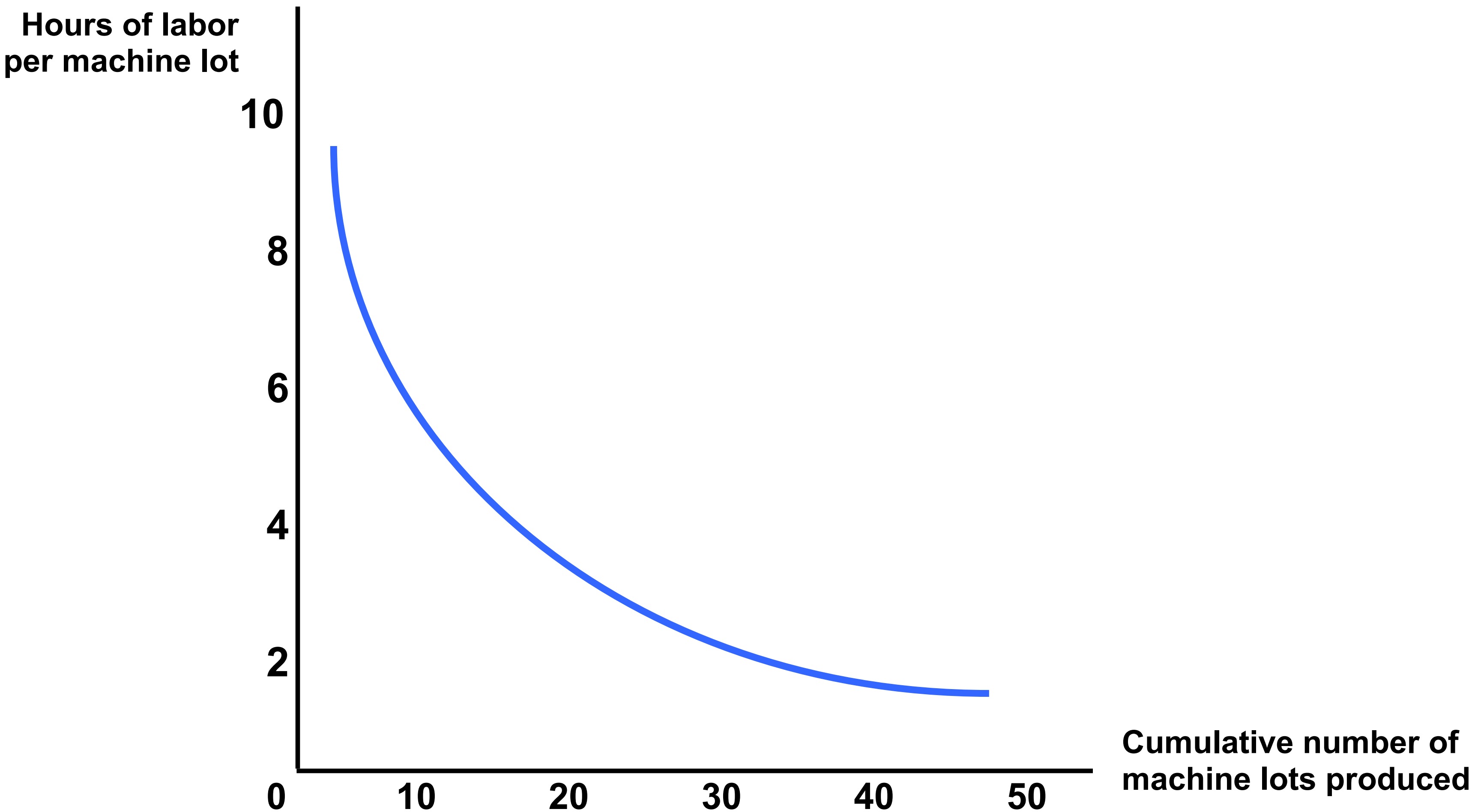
Reasons

- Speed of work increases with experience
- Managers learn to schedule production processes more efficiently
- More flexibility is allowed with experience. May include more specialised tools and plant organisation
- Suppliers become more efficient passing savings to company

Dynamic Changes in Costs: The Learning Curve

- The learning curve measures the impact of worker's experience on the costs of production.
- It describes the relationship between a firm's cumulative output and amount of inputs needed to produce a unit of output.

The Learning Curve



Dynamic Changes in Costs: The Learning Curve

- New firms may experience a learning curve, not economies of scale.
  - Should increase production of many lots regardless of individual lot size
- Older firms have relatively small gains from learning.
  - Should produce its machines in very large lots to take advantage of lower costs associated with size

Economies of Scale Versus Learning