Name __________ **Surname** __________ **Student ID.** __________

DUE DATE : Tuesday 6^{th}, September 2016.

Assignment 1: (80 marks)

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

Student Signature: __________

1. Let X and Y have the join probability density function specified in the following table.

		X 0	1	2	
	0	0.05	0.20	0.05	$f(Y=0)=$
Y	1	0.15	0.40	0	$f(Y=1)=$
	2	0.10	0	0.05	$f(Y=2)=$
		$f(X=0)=$	$f(X=1)=$	$f(X=2)=$	

[1]. Find $E(10X^3|Y=1)$ and $var(Y|X=0)$

2. Suppose the joint distribution of X and Y is such that

$E[X] = 5 \qquad E[X^2] = 30 \qquad E[Y|X] = 3X \qquad E[XY|X] = 3X^2 \qquad Var[Y] = 81$

Find (a) Cov(X,Y) and (b) Corr(X,Y)

3. Let X and Y be random variables with the following joint distribution

$$f(x, y) = P(X = x, Y = y) = 1/8$$

If x $\in \{-1, 0, 1\}$ and y $\in \{-1, 0, 1\}$ BUT (x,y) $\neq$ (0,0)

Show that the Cov(X,Y) =0 BUT X and Y ARE NOT INDEPENDENT.

4. Show the derivation if X ,Y, and Z are three **independent random variables**, then

$$var(X + Y + Z) = var(X) + var(Y) + var(Z)$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2. X_1, X_2, X_3 **ARE NOT independent** $cov(X_1, X_2) = cov(X_1, X_3) = cov(X_2, X_3) = \frac{1}{3}\sigma^2$ and $\bar{X}$ is an estimator used to estimate mean value where $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$.
Find $E(\bar{X})$ and $var(\bar{X})$

6. The joint probability density function of X and Y is given by

$$f(1,1) = \frac{1}{8} \qquad f(1,2) = \frac{1}{4} \qquad f(2,1) = \frac{1}{8} \qquad and \qquad f(2,2) = \frac{1}{2}$$

(a) Compute the conditional probability density function of Y given X = i where i=1,2

(b) Are X and Y independent?

(c) Compute the following:

P(X+Y>2) =

P(XY $\leqslant$ 2) =

P($\frac{X}{Y}$ > 1) =

7. You arrive at bus stop at 10 o' clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

(a) What is the probability that you will have to wait longer than 10 minutes?

(b) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

8. A clock stops at random at any time during the day. Let X be the time (hours plus fractions of hours) at which the clock stops. For example, X=14.00 for 2.00 pm and X=9.52 for 9.52 am.

(a) Write down the probability density function (pdf) for X.

(b) What is the probability that the clock will stop between 8.35 am and 3.45 pm?