HW #1

1. $y = 10 + \sqrt{x}$

Find $\frac{dy}{dx}$.

Approximate Δy when $x = 2, \Delta x = 0.1, and\ \Delta x = -0.2$

Compare the actual Δy to find the errors.

$y = 10 + \sqrt{x}$ → Domain $= [0, \infty)$ → Range $= [10, \infty)$

x	y	$dy/dx = \frac{1}{2\sqrt{x}}$
0	10	Undefined
1	11	$\frac{1}{2}$
2	$10+\sqrt{2}$	$\frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$
3	$10+\sqrt{3}$	$\frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

Find $\frac{dy}{dx}$

$y = 10 + \sqrt{x}$ → $x^{1/2}$

$\frac{dy}{dx} = 0 + \frac{1}{2}x^{1/2-1}$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ #

Approximate Δy when $x = 2$

▸ $\boxed{\Delta x = 0.1}$: $\Delta y = \left.\frac{dy}{dx}\right|_{x=2} \Delta x$

$\Delta y = \frac{\sqrt{2}}{4}(0.1)$

Approximated $\Delta y = \frac{\sqrt{2}}{40} \approx 0.0353$ #

▸ $\boxed{\Delta x = -0.2}$: $\Delta y = \left.\frac{dy}{dx}\right|_{x=2} \Delta x$

$\Delta y = \frac{\sqrt{2}}{4}(-0.2)$

Approximated $\Delta y = \frac{-\sqrt{2}}{20} \approx -0.0707$ #

Actual Δy when $x = 2$

▸ $\boxed{\Delta x = 0.1}$: $\Delta y = f(2+\Delta x) - f(2)$

$\Delta y = f(2.1) - f(2)$

$\Delta y = (\cancel{10} + \sqrt{2.1}) - (\cancel{10} + \sqrt{2})$

Actual $\Delta y = \sqrt{2.1} - \sqrt{2} \approx 0.0349$ #

▸ $\boxed{\Delta x = -0.2}$: $\Delta y = f(2+\Delta x) - f(2)$

$= f(1.8) - f(2)$

$= (\cancel{10} + \sqrt{1.8}) - (\cancel{10} + \sqrt{2})$

Actual $\Delta y = \sqrt{1.8} - \sqrt{2} \approx -0.0726$ #

Compare actual Δy to the approximated Δy

▸ $\boxed{\Delta x = 0.1}$: $\left| \text{Actual } \Delta y - \text{Approximated } \Delta y \right| = \left| 0.0349 - 0.0353 \right|$

Error $=$ 0.0004 #

▸ $\boxed{\Delta x = -0.2}$: $\left| \text{Actual } \Delta y - \text{Approximated } \Delta y \right| = \left| -0.0726 - (-0.0707) \right|$

Error $=$ 0.0019 #

Note I put the absolute to show only the difference of 2 values.

2. Find $\frac{d^2y}{dx^2}$ (second order derivative) of $y = 10 + \sqrt{x}$ and plot the graph of y and $\frac{dy}{dx}$. Is the slope of slope constant?

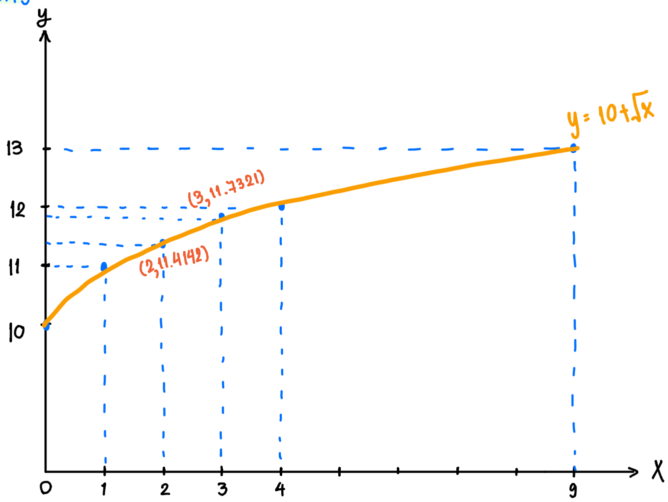
Find $\frac{d^2y}{dx^2}$

From the question 1.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{2}x^{-1/2}\right) = -\frac{1}{4}x^{-3/2} = -\frac{1}{4x^{3/2}} = -\frac{1}{4\sqrt{x^3}} \#$$

Graphs

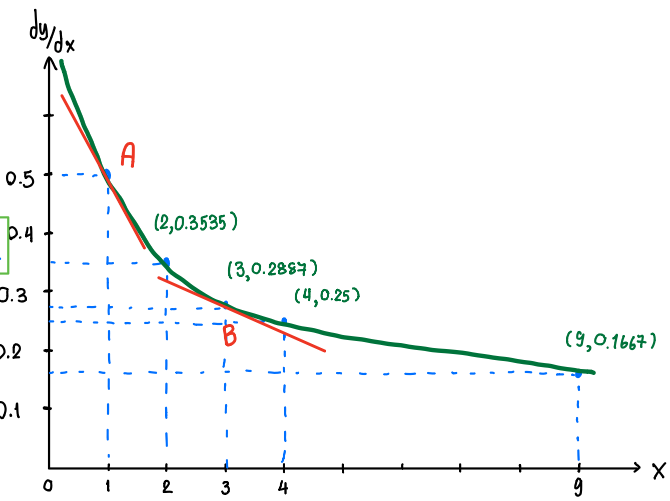
Graph I



$y = 10+\sqrt{x}$

x	y
0	10
1	11
2	11.4142
3	11.7321
4	12
⋮	
9	13

Graph II



$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

x	dy/dx
0	Undefined
1	0.5
2	0.3535
3	0.2887
4	0.25
⋮	
9	0.1667

Is the slope of slope constant ?

No, it is not constant because if you look at graph II (which represents the relationship between dy/dx and x, you can see that slope at point A is not equal to the slope at point B.

Mathematically speaking, slope of slope is the second-order derivative $\left(\frac{d^2y}{dx^2}\right)$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = \frac{-1}{4\sqrt{x^3}} = \frac{-1}{4\sqrt{(1)^3}} = -0.25$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2} = \frac{-1}{4\sqrt{x^3}} = \frac{-1}{4\sqrt{(2)^3}} = -0.0884$$

Not the same → Slope of slope is not constant

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