Example 3.J: Excess burden *formula under linear model & Tax-Revenue-maxizing tax rate*

Demand: $p \quad = a - bQ \qquad ; \;\; a \geq 0, \quad b \leq 0.$

Supply : $p \quad = c + dQ \qquad ; \;\; d \geq 0.$

- Solve for quantity and prices equilibrium when the unit tax is imposed. Analyze the result

before tax : solve for P^* & Q^*

$P^* = f(a, b, c, d)$

$Q^* = f(a, b, c, d)$

after tax : assume that tax / unit = t

new supply curve : $p = c + dQ + t$

$P^*_{tax} = f(a, b, c, d, t)$

$Q^*_{tax} = f(a, b, c, d, t)$

after tax : $P_s = P_d$

$c + dQ + t = a - bQ$

$dQ + bQ = a - c - t$

$Q(d+b) = a - c - t$

$Q^* = \dfrac{a-c-t}{d+b}$

$= c + d\left(\dfrac{a-c-t}{d+b}\right) + t \qquad = a - b\left(\dfrac{a-c-t}{d+b}\right)$

$= c + \dfrac{d(a-c-t)}{d+b} + t \qquad = a - \dfrac{b(a-c-t)}{d+b}$

There is no P^* when Q^* is $\dfrac{a-c-t}{d+b}$ so the answer after tax is written above

- Derive the excess burden formula for buyers and sellers

consumers' burden

$$= (P_B - P^*) \times Q_{tax}$$

$$= \left(a - b - a - \frac{b(a-c-t)}{d+b}\right)\left(\frac{a-c-t}{d+b}\right)$$

$$= \frac{\left(-\frac{b(a-c-t)}{d+b} - b\right)(a-c-t)}{d+b}$$

$$= \frac{(-bd - b^2 - ba + bc + bt)(a-c-t)}{(d+b)(d+b)}$$

$$= \frac{(-bd - b^2 - ba + bc + bt)(a-c-t)}{(d+b)^2}$$

producers' burden

$$= (P^* - P_S) \times Q_{tax}$$

$$= \left(c + \frac{d(a-c-t)}{d+b} + t - c + d\right)\left(\frac{a-c-t}{d+b}\right)$$

$$= \frac{\left(t + \frac{d(a-c-t)}{d+b} + d\right)(a-c-t)}{d+b}$$

$$= \frac{(td + d^2 + da + dc + db)(a-c-t)}{(d+b)(d+b)}$$

$$= \frac{(td + d^2 + da - dc + db)(a-c-t)}{(d+b)^2}$$

- Calculate the tax rate that maximizes the tax revenue of government.

1. Find tax revenue

$= t \times Q_{tax}$

$= t \times \frac{a-c-t}{d+b}$

$= \frac{t(a-c-t)}{a+b}$

2. $\frac{\partial \text{ tax revenue}}{\partial t} = 0$

$\frac{\partial \frac{t(a-c-t)}{d+b}}{\partial t} = 0$

$t^* = \frac{a-c}{d+b}$