

Chapter 11 Consumption: Utility, Indifference Curve and Budget Line

Consumer's Problem The consumer wants to maximize satisfaction (utility) by deciding what and how much to consume under a limitation of income

2 Types of Utility

1. Cardinal Utility—the consumer assigns a numerical value to denote his level of satisfaction. For example, a noodle for lunch gives the consumer a utility of 3 utils (units of utility).

This numerical value of 3 is entirely up to the consumer. He can assign value of 30, 300, or 17. However, if a pizza gives him twice the satisfaction he gets from noodle, then he will assign a number double that given to the noodle.

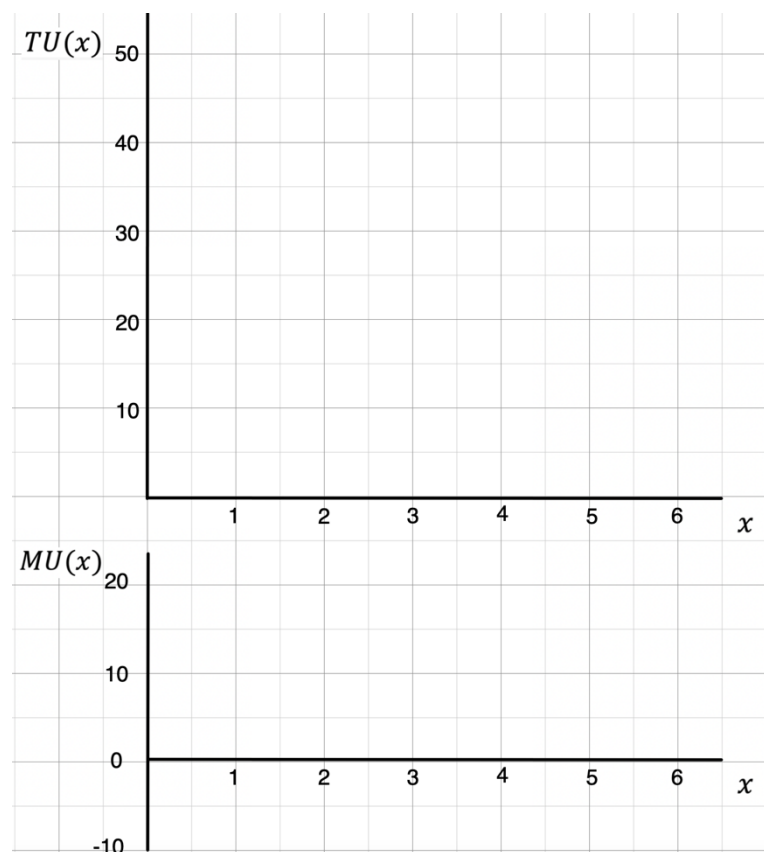
A total utility of consuming an increasing quantity of a product always increases at the beginning. Beyond a quantity, the total utility will decrease.

$TU(x)$ = Total Utility from consuming x units

$MU(x)$ = Marginal Utility from consuming x units
= the change of TU per 1 unit change of x

If the consumer consumes 1, 2, 3, ... units of x , we may have an example of $TU(x)$ and $MU(x)$ as given in the following table

x	$TU(x)$	$MU(x)$
0	0	
1	10	10
2	24	14
3	36	12
4	44	8
5	46	2
6	42	-4



If the consumer is allowed to consume any decimal of x ,

$$\begin{aligned} MU(x) &= \text{marginal utility from consuming } x \text{ units} \\ &= \text{rate of change of } TU(x) \text{ per unit of } x \\ &= \frac{d}{dx} TU(x) \end{aligned}$$

After certain amount of consumption x , the Marginal Utility of x always is decreasing. This is one explanation why the Demand curve has a negative slope. To induce the consumer to buy the another unit, the price has to be lowered.

Given the value of $MU(x)$ is known, if we allow a change in $x = \Delta x$, then we can approximate the change in $TU(x)$ as follows:

$$\Delta TU(x) \approx MU(x)\Delta x$$

For example, when $x_0 = 10$, we have $MU(x) = 12$. If we increase x by $\Delta x = 0.1$, then TU will change approximately by

$$\begin{aligned} \Delta TU(x) &\approx MU(x)\Delta x \\ &= 12(0.1) = 1.2. \end{aligned}$$

- The eventual decline of Marginal Utility can explains the Law of Demand.

2. Ordinal Utility Assume the consumer has 2 products X and Y to consume. A **bundle** is a quantity of X and a quantity of Y to be consumed by a consumer. A bundle (x_0, y_0) is represented by a point in the XY plane.

Ordinal utility approach assumes that the consumer is ‘rational’ according to the following assumptions:

Assumptions:

1) Given two bundles $A = (x_0, y_0)$ and $B = (x_1, y_1)$, the consumer can say one of these:

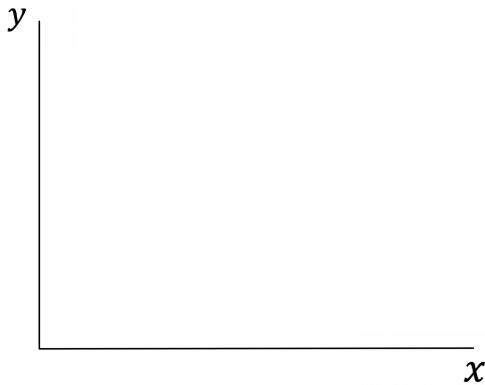
- 1) $A \succ B$ (A is preferred to B)
- 2) $B \succ A$ (B is preferred to A)
- 3) $A \sim B$ (A and B are indifferent)

Equivalently, we can say:

- 1) $A \succcurlyeq B$ (A is not less preferred to B)
- 2) $B \succcurlyeq A$ (B is not less preferred to A)

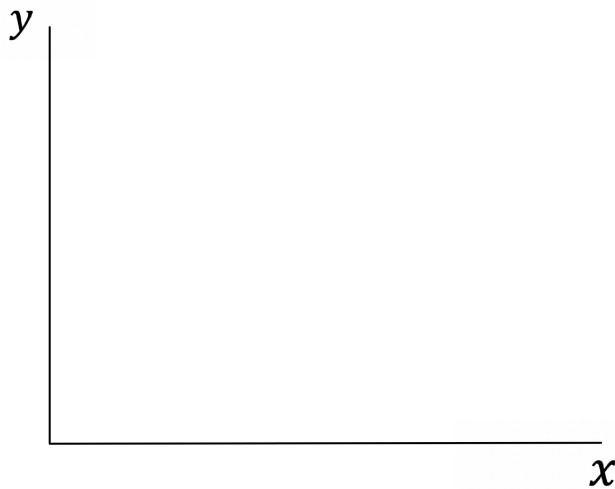
3) $A \sim B$ (A and B are indifferent)

2. More is always better



3. **Transitivity:** If $A > B$, and $B > C$, then $A > C$.

4. Given a bundle A, the consumer can tell all other bundles that are equally preferred. This creates an Indifference Curve (IC) passing through point A.

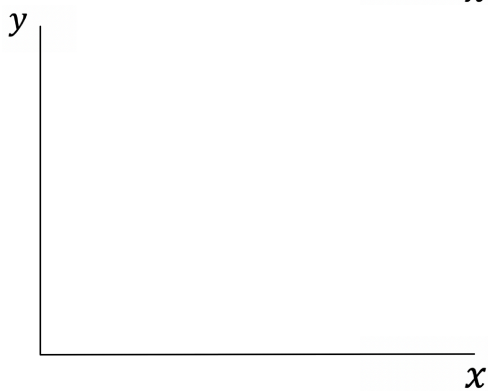
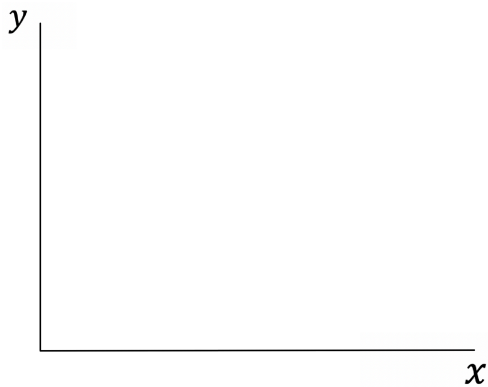


Indifference Curve (IC) is a curve whose every point gives the same satisfaction (utility) to the consumer.

- An IC separates the graph into 3 parts
 - a) All the bundles **on** the IC
 - b) All the bundles **above** the IC
 - c) All the bundles **below** the IC

Properties of Indifference Curves

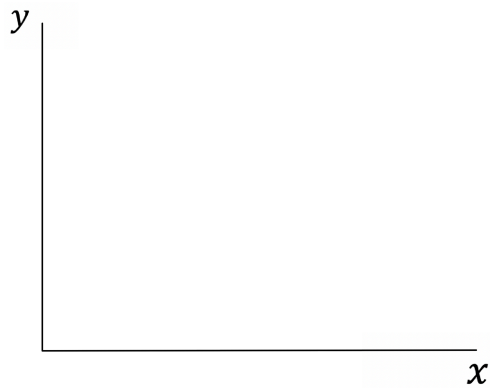
1. For any bundle $A = (x_0, y_0)$, there is exactly one IC passing through it. That is, no two IC's can intersect nor be tangent to each other.



2. There are infinite number of IC's, each never intersect nor is tangent to another

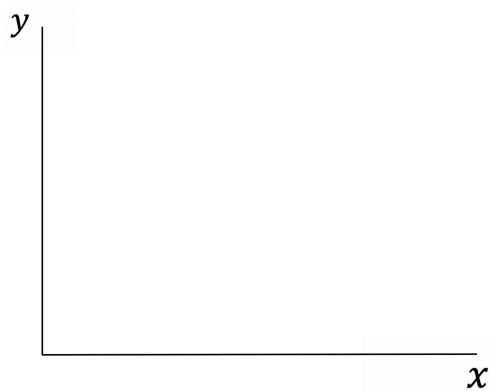


3. Higher IC means higher satisfaction.



4. Each IC always has negative slope

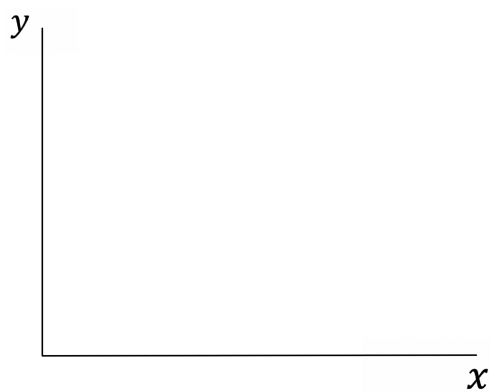
- What if there is a part of an IC with positive slope?



- Can an IC have zero slope?

- If we move from $A = (x_1, y_1)$ to $B = (x_2, y_2)$ on a same IC, there is

- an increase in $x = x_2 - x_1 = \Delta x > 0$
- a decrease in $y = y_2 - y_1 = \Delta y < 0$



The ratio $\frac{\Delta y}{\Delta x}$ is the exchange rate the between x and y with no change in the satisfaction level.

Δx causes an increase in $TU \approx MU_x(x_1, y_1)\Delta x$

Δy causes a decrease in $TU \approx MU_y(x_1, y_1)\Delta y$

With total change being zero, we have

$$MU_x(x_1, y_1)\Delta x + MU_y(x_1, y_1)\Delta y \approx 0$$

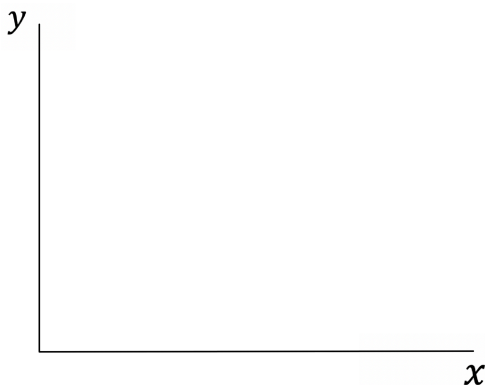
$$MU_y(x_1, y_1)\Delta y \approx -MU_x(x_1, y_1)\Delta x$$

$$\frac{\Delta y}{\Delta x} \approx -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}$$

When we move from A to B that is very close together. So close that $\Delta x \rightarrow 0$, and we have the slope of IC at A as

$$\frac{dy}{dx} = -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}$$

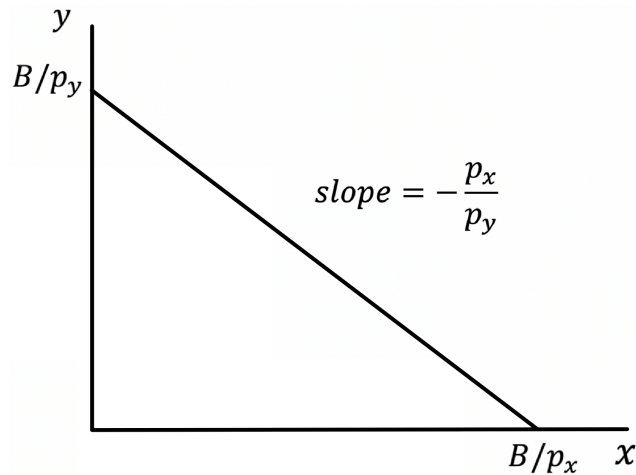
- At point A the slope can be found by drawing a line tangent to A . The slope of IC at A is equal to the slope of the tangent line.
- Slope at point $A = (x_1, y_1)$ is called the **Marginal Rate of Substitution (MRS)**. It is the **instantaneous exchange rate** between x and y such that **in the mind of the consumer** his satisfaction (utility) is unchanged.
- $MRS = -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}$
- This leads to the 5th property of IC
- **5. Diminishing MRS** On a given IC, as the consumption of x increases, the value of $|MRS|$ decrease.



Consumption Problem: maximizing satisfaction under a budget constraint

Budget Line A consumer is assumed to have a fixed income or budget B that he can buy x and y at fixed prices p_x and p_y , respectively. The equation for budget line is given by,

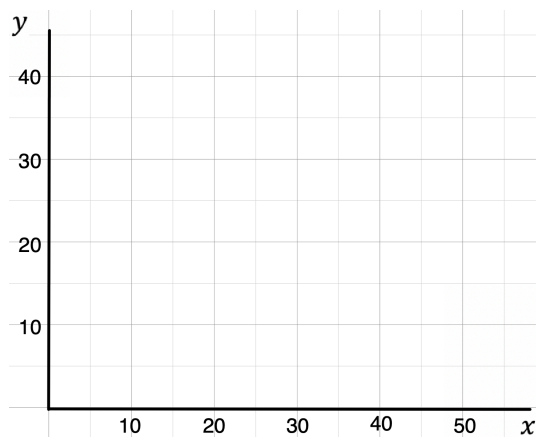
$$p_x x + p_y y = B$$



$\text{slope} = -\frac{p_x}{p_y} = \text{relative price } x \text{ interms of } y$
 $= \text{exchange rate between } x \text{ and } y \text{ in the market}$

- Every point on the budget line is a bundle the consumer can afford.

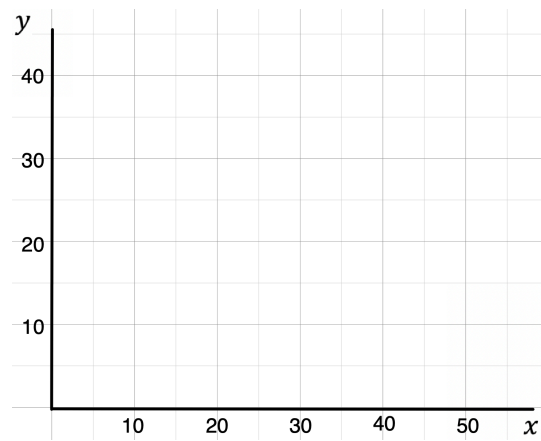
Example: $p_x = 3, p_y = 4, B = 120$



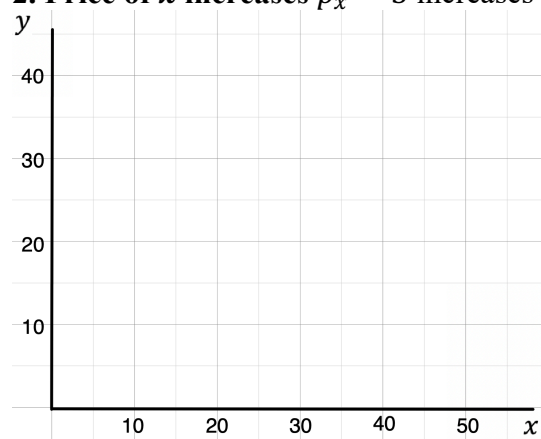
$$\text{slope} = -\frac{p_x}{p_y} =$$

Changes of Budget Line

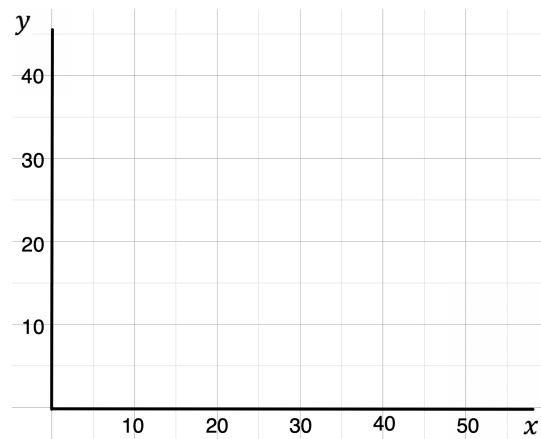
1. Income increases: Income increases from, $B = 120$ to $B' = 150$.



2. Price of x increases $p_x = 3$ increases to $p_x = 4$



Price of y decreases $p_y = 4$ increases to $p_x = 3$



- In Economics, only one thing is allowed to change at a time. That is, either income or price of a product changes.
- If we have inflation so that the prices p_x and p_y increases at the same percentage of 10%, what is the effect on the budget line?
- If we have 10% inflation and at the same time income also increase by 10%, how will the budget line change?