

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$\begin{aligned} \text{a. } \sum_{i=1}^5 (a + bx_i) &= \sum_{i=1}^5 a + b \sum_{i=1}^5 x_i = 5 \cdot a + b \sum_{i=1}^5 x_i \\ &= 5a + bx_1 + bx_2 + bx_3 + bx_4 + bx_5 \end{aligned}$$

$$\begin{aligned} \text{b. } \sum_{y=0}^5 f(x+y) &= f[(x+0) + (x+1) + (x+2) + (x+3) + (x+4)] \\ &= f(5x+10) \end{aligned}$$

$$\begin{aligned} \text{c. } \sum_{i=1}^{10} i^2 &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 \\ &= 385 \end{aligned}$$

$$\begin{aligned} \text{d. } \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) &= \sum_{x=1}^2 [(2x+2) + (2x+3) + (2x+4)] \\ &= \sum_{x=1}^2 (6x+9) = (6(1)+9) + (6(2)+9) = 36 \end{aligned}$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b 0.0625	b 0.125	2.25b 0.28125	2b 0.25	1.5b 0.1875	0.5b 0.0625	0.25b 0.03125

** when b is constant number

a. Find the value of b

$$\begin{aligned} 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b &= 1 \\ b &= 0.125 \end{aligned}$$

b. Find the answer for $P(X \leq 2)$

$$\begin{aligned} &= (P=-2) + (P=-1) + (P=0) + (P=1) + (P=2) \\ &= 0.0625 + 0.125 + 0.28125 + 0.25 + 0.1875 \\ &= 0.90625 \end{aligned}$$

c. Find the answer for $P(-2 \leq X \leq 3)$

$$= 0.96875$$

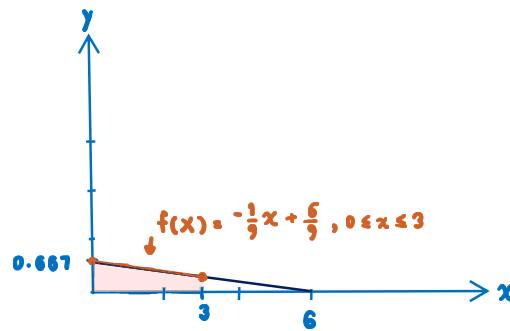
d. Find the answer for $P(X \geq 1)$

$$= 0.53125$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} \int_1^3 -\frac{1}{9}x + \frac{6}{9} &= -\frac{1}{9}x^2 + \frac{6}{9}x \Big|_1^3 \\ &= \left[-\frac{1}{9}(3)^2 + \frac{6}{9}(3) \right] - \left[-\frac{1}{9}(1)^2 + \frac{6}{9}(1) \right] \\ &= \frac{4}{9} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} \int_2^3 -\frac{1}{9}x + \frac{6}{9} &= -\frac{1}{9}x^2 + \frac{6}{9}x \Big|_2^3 \\ &= \left[-\frac{1}{9}(3)^2 + \frac{6}{9}(3) \right] - \left[-\frac{1}{9}(2)^2 + \frac{6}{9}(2) \right] \\ &= \frac{7}{9} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(x) &= \int_{-\infty}^{\infty} x \cdot f(x) dx = \int_0^3 x \cdot \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x dx = -\frac{1}{18}x^3 + \frac{6}{9}x^2 \Big|_0^3 \\ &= \left[-\frac{1}{18}(3)^3 + \frac{6}{9}(3)^2 \right] = \frac{9}{2} \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

Throwing a dice

	x	1	2	3	4	5	6	
	y							
Coin Toss	1	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/2$
	0	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/2$
		$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	1

- b. Find the marginal probability distribution function (PDF) of X

The marginal prob of X $P(X=x)$ is represented in green

- c. Find the marginal probability distribution function (PDF) of Y

The marginal prob of Y $P(Y=y)$ is represented in yellow

- d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

x	$P(X=x Y=1)$
1	$\frac{1/12}{1/2} = \frac{1}{6}$
2	$1/6$
3	$1/6$
4	$1/6$
5	$1/6$
6	$1/6$

- e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum x_i P(X=x_i|Y=1) = \frac{\sum x_i P(X=x_i, Y=1)}{P(Y=1)} = \frac{1}{0.5} \sum x_i P(X=x_i, Y=1)$$

$$= \frac{1}{0.5} \left[(1 \cdot \frac{1}{12}) + (2 \cdot \frac{1}{12}) + (3 \cdot \frac{1}{12}) + (4 \cdot \frac{1}{12}) + (5 \cdot \frac{1}{12}) + (6 \cdot \frac{1}{12}) \right] = \frac{7}{2}$$

- f. Find the variance of X given Y is equal to 1

$$V(X|Y=1) = \sum (X - E(X|Y=1))^2 \cdot P(X|Y=1)$$

$$= \left[\left(1 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(2 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(3 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(4 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(5 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(6 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right]$$

$$= \left(\frac{25}{4} \cdot \frac{1}{6} \right) + \left(\frac{9}{4} \cdot \frac{1}{6} \right) + \left(\frac{1}{4} \cdot \frac{1}{6} \right) + \left(\frac{1}{4} \cdot \frac{1}{6} \right) + \left(\frac{9}{4} \cdot \frac{1}{6} \right) + \left(\frac{25}{4} \cdot \frac{1}{6} \right)$$

$$= \frac{120}{24} = \frac{10}{3}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$$\bar{X} \text{ is estimator used to estimate mean value. } \bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{1}{N} \sum_{i=1}^3 X_i\right)$$

$$= \frac{1}{N} E(X_1 + X_2 + X_3)$$

$$= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3)]$$

$$= \frac{1}{3} [\mu_x + \mu_x + \mu_x]$$

$$= \frac{1}{3} \cdot 3\mu_x$$

$$= \mu_x$$

$$\text{Var}(\bar{X}) = \text{var}\left(\frac{1}{N} \sum_{i=1}^3 X_i\right)$$

$$= \frac{1}{N^2} \text{Var}(X_1 + X_2 + X_3)$$

$$= \frac{1}{N^2} [V(X_1) + V(X_2) + V(X_3) + 2\text{Cov}(X_1, X_2) + 2\text{Cov}(X_1, X_3) + 2\text{Cov}(X_2, X_3)]$$

$$= \frac{1}{3^2} \left[\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2 \right]$$

$$= \frac{\sigma_x^2}{4}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . \bar{X} is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left(\frac{1}{N} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{N} E(X_1 + X_2 + X_3 + X_4)$$

$$= \frac{1}{4} \cdot 4\mu_x$$

$$= \mu_x$$

$$\text{var}(\bar{X}) = \text{var}\left(\frac{1}{N} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{N^2} \text{Var}(X_1 + X_2 + X_3 + X_4)$$

$$= \frac{1}{16} \cdot 4\sigma_x^2$$

$$= 0.25\sigma_x^2$$

$$\tilde{x} = \frac{1}{4} (0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that \tilde{X} is an unbiased estimator of μ

$$\begin{aligned} E(\tilde{x}) &= E\left(\frac{1}{4} \sum_{i=1}^4 x_i\right) \\ &= \frac{1}{4} E(0.5x_1 + x_2 + 0.5x_3 + 2x_4) \\ &= \frac{1}{4} [0.5E(x_1) + E(x_2) + 0.5E(x_3) + 2E(x_4)] \\ &= \frac{1}{4} (0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x) \\ &= \frac{1}{4} \cdot 4\mu_x = \mu_x \end{aligned}$$

thus \tilde{x} is unbiased estimator of μ

$$\begin{aligned} \text{Var}(\tilde{x}) &= \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 x_i\right) \\ &= \frac{1}{4^2} \text{Var}(0.5x_1 + x_2 + 0.5x_3 + 2x_4) \\ &= \frac{1}{4^2} [0.25 \text{Var}(x_1) + \text{Var}(x_2) + 0.25 \text{Var}(x_3) + 4 \text{Var}(x_4)] \\ &= \frac{1}{4^2} [0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2] \\ &= \frac{5.5\sigma_x^2}{16} \\ &= 0.34\sigma_x^2 \end{aligned}$$

- c. Between \bar{X} and \tilde{X} , which one is the better estimator for μ ? Why?

$$\text{Var}(\bar{x}) = 0.25\sigma_x^2$$

$$\text{Var}(\tilde{x}) = 0.34\sigma_x^2$$

$\text{var}(\tilde{x}) > \text{var}(\bar{x})$ so, \bar{x} is a better estimator because \bar{x} has smaller variance indicates that data point are close to the mean