Student ID.................................

EE431 Economics of Financial Markets and Institutions
Homework 1: Debt Market and Structure of Interest Rate (1)

1. Describe the time value of money concept and how it affects your investment programs.

   Answer. "The time value of money allows people to invest money over a period of time. At the end of the investment period, people receive not only what they have invested, but also the earnings (dividends and interest) the investment has earned. For most people, accumulation is built on the principle of investing small sums of money over a long period of time. Thus, the time value of money can really help people accumulate wealth."

2. The relationship between a bond's coupon rate, the yield required by the market and the bond's price relative to par value is as follows:

   - Coupon rate $<$ yield required by market $\rightarrow$ price_...$<$...par value ($>$, $<$, $\leq$, $\geq$ or $=$), the bond is sold at ..discount...
   - Coupon rate $=$ yield required by market $\rightarrow$ price ...$=$... par value ($>$, $<$, $\leq$, $\geq$ or $=$), the bond is sold at ..par...
   - Coupon rate $>$ yield required by market $\rightarrow$ price ...$>$... par value ($>$, $<$, $\leq$, $\geq$ or $=$), the bond is sold at ..premium...

3. When interest rates fall, the prices of outstanding bonds ....rise.. (rise or fall).

4. The market price of longer maturity bonds fluctuates .....more............ (more or less) compared with shorter maturity bonds as interest rates change.

5. Consider two bonds, A and B. Both bonds presently are selling at their par value of \$1,000. Each pays interest of \$120 annually. Bond A will mature in 5 years while bond B will mature in 6 years. If the yields to maturity on the two bonds change from 12% to 10%, both bonds will ....increase......... (increase or decrease) in value, but bond B will increase ....more...... (more or less) than bond A

6. What is yield to maturity (YTM)?

   Answer. Yield to maturity is the rate that equates the price of the bond with the discounted cashflows.

   Mathematically, Bond Price $=\sum_t \frac{\text{Cash flow received at time t}}{(1+\text{discount rate})^t} = \sum_t \frac{CF_t}{(1+k)^t}$.

7. Consider bond XYZ with a modified duration of 6.56 . Suppose that the market value of this bond is \$3 million. The approximate dollar price change for a 1 **percent** change in yield to maturity is .....196,800 dollars.....

   Reason : .....Modified duration is the percentage change in price from a 1% change in yield. The modified duration is 6.56. We can say that a 1% change in yield to maturity leads to a $1\% \times 6.56 = 6.56\%$ change in price in the opposite direction. If the yield to maturity increases by 1%, the percentage decrease in the price of bond XYZ is approximately 6.56%. The current value of the bond is \$3 million. Hence, as the yield to maturity increases by 1%, the value of the bond will decrease approximately by $6.56\% \times 3 = 0.1968$ million dollars $=$ 196,800 dollars....

8. Ron Logan, CFA, is a bond manager. He purchased \$50 million in 6.0% coupon Southwest Manufacturing bonds at par three years ago. Today, the bonds are priced to yield 6.85%. The bonds mature in nine years. The Southwest bonds are trading at a ..discount... and the yield to maturity has ..increased... since purchase.

9. Calculate the present value of 1000 Baht zero coupon bond with 10 years to maturity, if the required annual interest rate is 6%

   Table shows cashflows from the zero coupon bond.

year	Cashflows
10	1,000

$$
\begin{aligned}
\text{present value of cashflows} &= \frac{1,000}{(1+k)^{10}} \\
&= \frac{1,000}{(1+0.06)^{10}} \\
&= 1,000 \times PVIF_{6\%,10} \\
&= 1,000 \times 0.55839 \\
&= 558.39
\end{aligned}
$$

   The present value of the zero coupon bond is 558.39 Baht.

10. Repeat the calculation in question 9 for the interest rate of 7%

   Answer .

$$
\begin{aligned}
\text{present value of cashflows} &= \frac{1,000}{(1+7)^{10}} \\
&= \frac{1,000}{(1+0.07)^{10}} \\
&= 1,000 \times PVIF_{7\%,10} \\
&= 1,000 \times 0.50835 \\
&= 508.35
\end{aligned}
$$

   The present value of the zero coupon bond is 508.35 Baht.

11. Consider a 7.5% coupon bond with the par value of 1,000 Baht, selling for 879.09175 Baht. The bond will mature in 15 years. The coupon payment is made annually.

   (a) Calculate the yield to maturity.

   Answer. Yield to maturity is defined as the discount rate that equates present value of cashflows from bonds to its market price.

$$
\begin{aligned}
\text{Bond Price} &= \sum_t \frac{CF_t}{(1+k)^t}, \\
879.09175 &= \frac{75}{(1+YTM)} + \frac{75}{(1+YTM)^2} + ... + \frac{75}{(1+YTM)^{15}} + \frac{1,000}{(1+YTM)^{15}}, \\
&= 75 \times PVIFA_{YTM\%,15} + 1,000 \times PVIF_{YTM\%,15}.
\end{aligned}
$$

   The yield to maturity usually cannot be solved directly. It is determined by using trial and error or iterative method. The method consists of a sequence of iterations, involving repeated calculations.

   The bond in the question is sold at discount. Hence, the coupon rate is lower than the discount rate.

   Therefore, guess the initial value of yield to maturity as 8%. Then,

$$
\begin{aligned}
\text{Bond Price} &= 75 \times PVIFA_{8\%,15} + 1,000 \times PVIF_{8\%,15}, \\
&= (75 \times 8.55948) + (1,000 \times 0.31524), \\
&= 641.961 + 315.24, \\
&= 957.201.
\end{aligned}
$$

   957.201> 879.09175. Therefore, we increase the trial discount rate to 9% and recalculate.

$$
\begin{aligned}
\text{Bond Price} &= 75 \times PVIFA_{9\%,15} + 1{,}000 \times PVIF_{9\%,15}, \\
&= (75 \times 8.06069) + (1{,}000 \times 0.27454), \\
&= 604.555175 + 274.54, \\
&= 879.09175.
\end{aligned}
$$

Therefore, yield to maturity of this bond is equal to 9%.

(b) Later in the same year, the bond interest rate is fallen to 5%. What is the price of the 7.5% coupon bond?

Answer. Bond price is equal to its present value of cashflows. Therefore, the price of the bond can be calculated as follows.

$$
\begin{aligned}
\text{Bond Price} &= 75 \times PVIFA_{5\%,15} + 1{,}000 \times PVIF_{5\%,15}, \\
&= (75 \times 10.37966) + (1{,}000 \times 0.48102), \\
&= 778.4745 + 481.02, \\
&= 1{,}259.4945.
\end{aligned}
$$

Therefore, the price of the 7.5% coupon bond becomes 1,259.4945, when the interest rate is 5%.

Using the answer in question 11 with that in question 12, we can see that when the interest rate is 9%, the bond price is 879.09175 Baht.

When the interest rate is 5%, the bond price is 1,259.4945.

Thus, when interest rate increases, the bond price decreases.

When interest rate decreases, the bond price increases.

12. Consider a 5% coupon bond with the par value of 2,000 Baht, selling for 2,228.46372 Baht. The bond will mature in 4 years. Find the modified duration of the coupon bond and explain its meaning.

$D = \sum_t \frac{\text{Present Value of } CF_t}{\text{Bond Market Price}} \times t$, where Bond Market Price $= \sum_t \frac{CF_t}{(1+k)^t}$.

First, find the yield to maturity of the bond. Yield to maturity is defined as the discount rate that equates present value of cashflows from bonds to its market price.

$$
\begin{aligned}
\text{Bond Price} &= \sum_t \frac{CF_t}{(1+k)^t}, \\
2228.46372 &= \frac{100}{(1+YTM)} + \frac{100}{(1+YTM)^2} + \frac{100}{(1+YTM)^3} + \frac{100}{(1+YTM)^4} + \frac{2{,}000}{(1+YTM)^4}, \\
&= 100 \times PVIFA_{YTM\%,4} + 2{,}000 \times PVIF_{YTM\%,4}.
\end{aligned}
$$

The yield to maturity usually cannot be solved directly. It is determined by using trial and error or iterative method. The method consists of a sequence of iterations, involving repeated calculations.

The bond in the question is sold at premium. Hence, the coupon rate is greater than the discount rate.

Therefore, guess the initial value of yield to maturity as 4%. Then,

$$
\begin{aligned}
\text{Bond Price} &= 100 \times PVIFA_{4\%,4} + 2{,}000 \times PVIF_{4\%,4}, \\
&= (100 \times 3.62990) + (2{,}000 \times 0.85480), \\
&= 362.99 + 1709.6, \\
&= 2072.59.
\end{aligned}
$$

2,072.59$<$ 2,228.46372. Therefore, we decrease the trial discount rate to 3% and recalculate.

$$
\begin{aligned}
\text{Bond Price} &= 100 \times PVIFA_{3\%,4} + 2{,}000 \times PVIF_{4\%,4}, \\
&= (100 \times 3.71710) + (2{,}000 \times 0.88849), \\
&= 371.71 + 1{,}776.98, \\
&= 2{,}148.69.
\end{aligned}
$$

2,148.69 $<$ 2,228.46372. Therefore, we decrease the trial discount rate to 2% and recalculate.

$$\begin{aligned}
\text{Bond Price} &= 100 \times PVIFA_{2\%,4} + 2,000 \times PVIF_{2\%,4}, \\
&= (100 \times 3.80773) + (2,000 \times 0.92385), \\
&= 380.773 + 1,847.7, \\
&= 2,228.473.
\end{aligned}$$

2,228.473 ≈ 2,228.46372. (Note that the two numbers are not exactly equal. This is because the calculation is based on the present value table in which the numbers are rounded off to five decimal places. The difference between the two numbers is lower than 0.01. The two numbers are close enough. In this case, we conclude that yield to maturity is approximately equal to 2%. If we try 1% discount rate, we would get the bond price equal to 2,312.157, which is even further from 2,228.46372. Of all the numbers we can obtain from the calculation by using the present value table, 2,228.473 is the closest number to 2228.46372. If we use a financial calculator or a computer program, we would find that the yield to maturity for this bond is equal to 2%. .)

Second, find the duration of the bond.

$D = \sum_t \frac{\text{Present Value of } CF_t}{\text{Bond Market Price}} \times t$, where Bond Market Price $= \sum_t \frac{CF_t}{(1+k)^t}$.

(1) t	(2) Present Value of CF_t	(3) $\frac{\text{PV of } CF_t}{\text{Bond Market Price}}$	(4) $\frac{\text{PV of } CF_t}{\text{Bond Market Price}} \times t$ (3) × (1)
1	$100 \times PVIF_{2\%,1}$ $= 100 \times 0.98039$ $= 98.039$	$\frac{98.039}{2,228.473}$ $= 0.0439938$	0.0439938×1 $= 0.0439938$
2	$100 \times PVIF_{2\%,2}$ $= 100 \times 0.96117$ $= 96.117$	$\frac{96.117}{2,228.473}$ 0.0431313	0.0431313×2 $= 0.0862626$
3	$100 \times PVIF_{2\%,3}$ $=100 \times 0.94232$ 94.232	$\frac{94.232}{2,228.473}$ $= 0.0422855$	0.0422855×3 $= 0.1268565$
4	2,100 $\times PVIF_{2\%,4}$ $= 2,100 \times 0.92385$ $1,940.085$	$\frac{1,940.085}{2,228.473}$ $= 0.8705894$	0.8705894×4 3.4823576
Total	2,228.473 = Bond Market Price	1	3.7394705

The bond's duration is equal to 3.7394705.

Third, find the modified duration of the bond.

$$
\begin{aligned}
\text{Modified Duration} &= \frac{\text{Duration}}{1+\text{YTM}}, \\
&= \frac{3.7394705}{1+2\%}, \\
&= \frac{3.7394705,}{1.02} \\
&= 3.66615.
\end{aligned}
$$

Fourth, explain the meaning of the bond's modified duration.
If the interest rate (yield to maturity of the bond) changes by 1%, the bond price will change by about 3.6615%, in an opposite direction.
In other words, if the interest rate (yield to maturity of the bond) increases by 1%, the bond price will decrease by about 3.6615%.

*Note that A student may get slightly different numbers due to round-offs. The difference should be small enough; otherwise it is unacceptable.

13. You are buying your first house for $250,000, and are paying $50,000 as a down payment. You have arranged to finance the remaining $200,000 20-year mortgage with a 6% nominal interest rate and yearly payments. What are the equal yearly payments you must make?

    Answer. The house price is $250,000. The down payment is $50,000. The remaining amount is $200,000. We will pay $50,000 today and pay equal installments of $ PMT at 6% interest rate for 20 years. The present value of the installments is equal to $200,000. Mathematically, we can write the following equation and we can solve for $ install payment per year.

$$
\begin{aligned}
\text{Present Value of Installments} &= \$ \text{ installment per year} \times PVIFA_{6\%,20}, \\
200,000 &= PMT \times PVIFA_{6\%.20}, \\
&= PMT \times 11.46992. \\
\text{Solve for PMT.} & \\
PMT &= \frac{200,000}{11.46992}, \\
&= 17,436.9132.
\end{aligned}
$$

    Therefore, the equal yearly payment is equal to $17,436.9132.

14. Suppose an investor who has 1,000 Baht is considering buying one of 3 years bonds:

    - Bond A has 1,000 Baht face value, 12% annual coupon, 10% yield to maturity.
    - Bond B has 100 Baht face value, 10% annual coupon, 12% yield to maturity.

    Assume that the investor can buy fractions of a bond. The investor plans to hold the bond until its maturity and expects that interest rate will be constant. Which bond the investor should buy to get higher returns? Explain the reason.

    Assume that the investor can buy fractions of a bond. If the investor plans to hold the bond until its maturity, which bond the investor should buy to get more returns? Explain the reason.

Answer. Yield to maturity is the rate of return on a bond if held until maturity date. Bond A and Bond B have the same time to maturity. Bond B has a higher yield to maturity than Bond A. Since the investor plans to hold the bond until its maturiy, the investor should buy Bond B to get more returns.

Note:
It is quite difficult to answer this question numerically. I do not expect a student to answer this question numerically. Basically, the rate of return on a financial asset is defined as the discount rate that equates its present value of cashflows with its market price. When we calculate yield to maturity, we assume that the bond is held to its maturity date. Therefore, yield to maturity is the the rate of return on a bond if held until maturity date. The question assumes that investor can buy any fraction of a bond. This means that investors can always use up his or her 1,000 Baht to the bond regardless of what the price of the bond.is
Bond A's price is equal to $120 \times PVIFA_{10\%.3} + 1,000 \times PVIF_{10\%,3} = 1,049.732$. With 1,049.732 Baht, you can buy 1 of Bond A. With 1,000 Baht, you can buy $\frac{1,000}{1,049.732} \approx 0.9526$ of Bond A. You buy $\frac{1,000}{1,049.732}$ of Bond A, then you would receive $\left(\frac{1,000}{1,049.732}\right) \times 120$ at the end of the year for 3 years and $\left(\frac{1,000}{1,049.732}\right) \times 1,000$ at the end of year 3. You pay 1,000 Baht for these cashflows. The rate of return on this investment is the discount rate that solves this following equation.

$$\begin{aligned}
\text{Price (Baht invested)} &= \text{Present Value of Cashflows,} \\
1,000 &= \left\{\left[\left(\frac{1,000}{1,049.732}\right) \times 120\right] \times PVIFA_{k\%.3}\right\} \\
&\quad + \left\{\left[\left(\frac{1,000}{1,049.732}\right) \times 1,000\right] \times PVIF_{k\%,3}\right\}.
\end{aligned}$$

Rearrage,

$$\begin{aligned}
1,000 &= \left(\frac{1,000}{1,049.732}\right)\{120 \times PVIFA_{k\%.3}\} \\
&\quad + \left(\frac{1,000}{1,049.732}\right)\left\{1,000 \times PVIF_{k\%,3}\right\}, \\
\left(\frac{1,049.732}{1,000}\right) \times 1,000 &= \{120 \times PVIFA_{k\%.3}\} + \left\{1,000 \times PVIF_{k\%,3}\right\}, \\
1,049.732 &= \{120 \times PVIFA_{k\%.3}\} + \left\{1,000 \times PVIF_{k\%,3}\right\}.
\end{aligned}$$

Then, the discount rate that solves the equation is equal to the yield to maturity of the bonds. In other words, *you get the same rate of return on the bond no matter how much (how many units) you buy the bond.*
The rate of return on Bond B from 1,000 Baht investment in Bond B can be calculated in the same way.
The rate of return from investing in a bond if held to maturity is equal to the bond's yield to maturity.
Therefore, for this question, the investor just simply goes for the one with the higher yield to maturtiy.