

Chapter 10

Integration and Its Application

Rules of Integration

1. Integral of a constant (k)

$$\int k dx = kx + c$$

2. Integral of 1

$$\int 1 dx = x + c$$

3. The Power Rule

$$\int x^n dx = \frac{1}{n+1} (x^{n+1}) + c$$

4. Integral of x^{-1}

$$\begin{aligned} \int x^{-1} dx &= \int \frac{1}{x} dx \\ &= \ln x + c \end{aligned}$$

where $x > 0$

If $x < 0$,

$$\int x^{-1} dx = \ln |x| + c, x \neq 0$$

5. The Exponential Rule

$$\int a^{kx} dx = \frac{a^{kx}}{k \ln a} + c$$

6. Integral of Natural Exponential Function

$$\int e^{kx} dx = \frac{e^{kx}}{k} + c$$

7. Integral of a multiple

$$\int k \cdot f(x) dx = k \int f(x) dx$$

8. Integral of a sum or a difference

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

9. Integral of the negative of a function

$$\int -f(x) dx = -\int f(x) dx$$

10. The Substitution Rule

$$\begin{aligned} \int f(x) dx &= \int \left(u \frac{du}{dx}\right) dx \\ &= \int u du \\ &= F(u) + c \end{aligned}$$

11. Integration by parts

$$\int f(x) g'(x) dx = f(x) g(x) - \int g(x) f'(x) dx$$

From:

$$\frac{d}{dx} [f(x) g(x)] = f(x) g'(x) + g(x) f'(x)$$

$$f(x) g(x) = \int f(x) g'(x) dx + \int g(x) f'(x) dx$$

Some Properties of Definite Integrals

$$1. \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$2. \int_a^{b=a} f(x)dx = F(b=a) - F(a)$$
$$= F(a) - F(a)$$
$$= 0$$

$$3. \int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx ; a \leq b \leq c$$

$$4. \int_a^b f(x)dx \pm \int_a^b g(x)dx = \int_a^b [f(x) \pm g(x)]dx$$

$$5. \int_a^b k \cdot f(x)dx = k \int_a^b f(x)dx$$