0.1) Given that $Z = \frac{x^3-y^3}{x^2y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

$$\frac{\partial z}{\partial x} = \frac{\left[x^2y^2\left(\frac{\partial x^3-y^3}{\partial x}\right)\right] - \left[(x^3-y^3)\left(\frac{\partial x^2y^2}{\partial x}\right)\right]}{(x^2y^2)^2}$$

$$= \frac{\left[x^2y^2(3x^2)\right] - \left[(x^3-y^3)(2xy^2)\right]}{x^4y^4}$$

$$= \frac{(3x^4y^2) - (2x^4y^2 - 2xy^5)}{x^4y^4}$$

$$= \frac{x^4y^2 + 2xy^5}{x^4y^4}$$

$$= \frac{\cancel{x}\cancel{y^2}(x^3+2y^3)}{\cancel{x^4}\cancel{y^4}\, x^3 y^2}$$

$$= \frac{x^3+2y^3}{x^3y^2}$$

$$\frac{\partial z}{\partial y} = \frac{\left[x^2y^2\left(\frac{\partial x^3-y^3}{\partial y}\right)\right] - \left[(x^3-y^3)\left(\frac{\partial x^2y^2}{\partial y}\right)\right]}{(x^2y^2)^2}$$

$$= \frac{\left[x^2y^2(-3y^2)\right] - \left[(x^3-y^3)(x^2 2y)\right]}{x^4y^4}$$

$$= \frac{-3y^4x^2 - \left[y^5 2y - x^2 2y^4\right]}{x^4y^4}$$

$$= \frac{-y^4x^2 - 2x^5y}{x^4y^4}$$

$$= \frac{\cancel{x^2}\cancel{y}(-y^3-2x^3)}{x^2\cancel{x^4}\cancel{y^4}\,y^3}$$

$$= \frac{-y^3-2x^3}{x^2y^3}$$

$$\cancel{x}\left(\frac{x^3+2y^3}{\cancel{x^3}y^2 \; x^2}\right) + \cancel{y}\left(\frac{-2y^3-2x^3}{x^2\cancel{y^3}\;y^2}\right) = \frac{-x^3+y^3}{x^2y^2}$$

$$\frac{x^3+2y^3}{x^2y^2} + \frac{-1y^3-2x^3}{x^2y^2} = \frac{-x^3+y^3}{x^2y^2}$$

$$\frac{-x^3+y^3}{x^2y^2} = \frac{-x^3+y^3}{x^2y^2}$$

0.2) Given that $Z = \frac{x-y}{x+y}$ use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$$

$$= \frac{(x+y)\,\partial(x-y) - (x-y)\,\partial(x+y)}{(x+y)^2} \times dx + \frac{(x+y)\,\partial(x-y) - (x-y)\,\partial(x+y)}{(x+y)^2} \times dy$$

$$= \frac{(x+y)(1) - (x-y)(1)}{x^2+2xy+y^2} \times dx + \frac{(x+y)(-1) - (x-y)(1)}{} dy$$

$$= \frac{x+y-x+y}{(x+y)^2} dx \qquad \frac{\cancel{} - x \cancel{+y}}{(x+y)^2} dy$$

$$= \frac{2y}{(x+y)^2} dx - \frac{2x}{(x+y)^2} dy$$

$$= \frac{2y}{(x+y)^2}(2) - \frac{2x}{(x+y)^2}(-2)$$

$$= \frac{4}{4} + \frac{4}{4}$$

$$= 2$$

0.3) If z = $2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= \left[\frac{\partial(2x^2y+3xy+y^2)}{\partial x} \times \frac{\partial(r^2+2rs)}{\partial s}\right] + \left[\frac{\partial(2x^2y+3xy+y^2)}{\partial y} \times \frac{\partial(2r-4s)}{\partial s}\right]$$

$$= \left[(4xy+3y) \times (2r)\right] + \left[(2x^2+3x+2y) \times (-4)\right]$$

$\boxed{\bullet\ r=1,\ s=0\ \bullet}$

$$= (8xy+6y) + (-8x^2-12x-8y)$$
$$= -8x^2-12x+8xy-2y$$
$$= -8(1)^2-12(1)+8(1)(2)-2(2)$$
$$= -8-12+16-4$$
$$= -8$$

$x = r^2+2rs$
$= 1^2+2(1)(0)$
$= 1$

$y = 2r-4s$
$= 2(1)-4(0)$
$= 2$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial r}$$

$$= \left[\frac{\partial(2x^2y+3xy+y^2)}{\partial x} \times \frac{\partial(r^2+2rs)}{\partial r}\right] + \left[\frac{\partial(2x^2y+3xy+y^2)}{\partial y} \times \frac{\partial(2r-4s)}{\partial r}\right]$$

$$= \left[(4xy+3y) \times (2r+2s)\right] + \left[(2x^2+3x+2y) \times (2)\right]$$

$\boxed{r=1,\ s=0}$

$$= (8xy+6y) + (4x^2+6x+4y)$$
$$= 4x^2+6x+8xy+10y$$

$x = r^2+2rs$
$= 1^2+2(1)(0)$
$= 1$

$y = 2r-4s$
$= 2(1)-4(0)$
$= 2$

$$= 4(1)^2+6(1)+8(1)(2)+10(2)$$
$$= 4+6+16+20$$
$$= 46$$

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate $\partial z/\partial y$ when x = 1, y = 2, z = -1.

$$2x^2 + 3y^2 + 2z^2 - 16 = 0$$

$$f(x, y, z) = 2x^2 + 3y^2 + 2z^2 - 16$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}$$

$$= -\frac{\frac{\partial(2x^2 + 3y^2 + 2z^2 - 16)}{\partial y}}{\frac{\partial(2x^2 + 3y^2 + 2z^2 - 16)}{\partial z}}$$

$$\frac{\partial z}{\partial y} = -\frac{6y}{4z}$$

$$= \frac{-6(2)}{4(-1)}$$

$$= \frac{-12}{-4}$$

$$= 3$$

Question 0: (required for all)

0.1) Given that $Z = \frac{x^3 - y^3}{x^2 y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

0.2) Given that $Z = \frac{x-y}{x+y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

0.3) If $z = 2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

0.5) Given that $ln\,(x + y + z) + xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

0.5) $f(x, y, z) = \ln(x+y+z) + xyz - ze^{x+y+z}$

$$\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial x} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x} = \frac{\partial 0}{\partial x} \;{}^{''}0$$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}$$

$$\frac{\partial f}{\partial x} = \left(\frac{1}{x+y+z}\right) + yz - (ze^{x+y+z})(1)$$

$$\frac{\partial f}{\partial z} = \left(\frac{1}{x+y+z}\right) + xy - (ze^{x+y+z})(1)$$

$$\left.\frac{\partial z}{\partial x}\right|_{0,1,0} = -\left[\frac{\left(\frac{1}{0+1+0}\right) + 0 - 0}{\left(\frac{1}{0+1+0}\right) + 0 - 0}\right]$$

$$= -1 \,//$$

Question 1: Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2.$$ $-50\,(P_y)^{-1/2}$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

1.1) What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative

$\frac{\partial Q_x}{\partial P_x} = -4$; $P_x \uparrow \$1 \rightarrow Q_x \downarrow 4$ units

(c1awufD)

$$\frac{\partial Q_x}{\partial P_y} = 25(P_y)^{-3/2} = \frac{25}{\sqrt{P_y^3}}$$

$P_y > 0$ $\quad P_y \uparrow \quad Q_y \downarrow \rightarrow Q_x \uparrow \frac{25}{\sqrt{P_y^3}}$ units

$P_y = 0$ $\quad P_y \uparrow \quad Q_y \downarrow \rightarrow Q_x \uparrow 25$ units

subsitute product

1.2) Is the product X considered an inferior product?

1.3) What is the level of quantity demanded if $P_x = 10, P_y = 25$ $and\ I = 10$?

1.2) $\frac{\Delta Q_X}{\Delta I} = (0.5)(2)I = I$

As income increases by \$1, quantity of good X increases by I. So, product X considered as an normal good. //

1.3) $Q_X = 100 - 4P_X - \frac{50}{\sqrt{P_y}} + 0.5I^2$

$Q_X = 100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(100)$

$= 100 - 40 - 10 + 50 = 100$ //

Question 1: Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2.$$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

1.1) What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative

1.2) Is the product X considered an inferior product?

1.3) What is the level of quantity demanded if $P_x = 10, P_y = 25$ and $I = 10$?

1.4) Calculate the own-price elasticity of demand, and evaluate the value when $P_x = 10, P_y = 25$ and $I = 10$.

1.5) Calculate the cross-price elasticity of demand when $P_x = 10, P_y = 25$ and $I = 10$.

1.6) Calculate income elasticity of demand when $P_x = 10, P_y = 25$ and $I = 10$. Is the product a necessary or luxurious product?

1.4). Calculate the own-price elasticity of demand

$$\varepsilon^P = \frac{\% \Delta Q_x}{\% \Delta P_x} = \frac{\Delta Q_x}{\Delta P_x} \times \frac{P_x}{Q_x} = (-4) \times \frac{P_x}{100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2}$$

$$= \frac{-4P_x}{100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2}$$

When $P_x = 10$, $P_y = 25$ and $I = 10$:

$$\Rightarrow \varepsilon^P = \frac{(-4) \times (10)}{100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(10)^2}$$

$$\varepsilon^P = \frac{-40}{100 - 40 - 10 + 50} = -0.4$$

1.5). Calculate the cross-price elasticity of demand

$$\varepsilon^C = \frac{\% \Delta Q_x}{\% \Delta P_y} = \frac{\Delta Q_x}{\Delta P_y} \times \frac{P_y}{Q_x} = (-50) \times \left(-\frac{1}{2}\right) \times (P_y)^{-\frac{3}{2}} \times \frac{P_y}{100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2}$$

$$= \frac{25(P_y)^{-\frac{1}{2}}}{100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2}$$

When $P_x = 10$, $P_y = 25$ and $I = 10$:

$$\Rightarrow \varepsilon^{c} = \frac{25 \times \frac{1}{\sqrt{25}}}{100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(10)^2} = 0.05$$

1.6). Calculate income elasticity of demand

$$\varepsilon^{I} = \frac{\%\Delta Q_x}{\%\Delta I} = \frac{\Delta Q_x}{\Delta I} \times \frac{I}{Q_x} = 0.5 \times 2I \times \frac{I}{100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2}$$

$$= \frac{I^2}{100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2}$$

When $P_x = 10$, $P_y = 25$ and $I = 10$:

$$\varepsilon^{I} = \frac{10^2}{100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(10)^2} = 1$$

$\Rightarrow$ The product is a necessary product.

Question 3: Suppose that the preference set of a household can be given by

$$U(x, y) = x^{1/2} + y^{1/2},$$

where x is the amount of consumption on good-x, and y is the amount of consumption on good-y. Consider the following problems.

3.1) Calculate the marginal utility of good x and good y, respectively.

3.2) Does the utility function satisfy with the law of diminishing marginal utility?

3.3) Does the marginal utility curve of good x shift up when the consumer consumes more units of good-y?

3.4) What is the level of the household utility when the consumer consumes 1 unit of good-x and 2 units of good-y?

3.5) Following (3.4), use the total differential to calculate the change in the level of utility under which the consumer increases the consumption on good x by 3 units and reduces the consumption on good y by 1 unit.

3.6) Derive the MRS and show that MRS is decreasing in x.

$U(x,y) = x^{1/2} + y^{1/2}$,

3.1) Calculate the marginal utility of good x and good y, respectively.

$MU_x = \frac{\partial U}{\partial x} = \frac{1}{2}x^{-\frac{1}{2}}$ $\quad MU_y = \frac{\partial U}{\partial y} = \frac{1}{2}y^{-\frac{1}{2}}$

3.2) Does the utility function satisfy with the law of diminishing marginal utility?

Since $MU_x = \frac{1}{2}x^{-\frac{1}{2}}$ and $MU_y = \frac{1}{2}y^{-\frac{1}{2}}$.

$\frac{dMU_x}{dx} = -\frac{1}{4}x^{-\frac{3}{2}} < 0$ $\quad \frac{dMU_y}{dy} = -\frac{1}{4}y^{-\frac{3}{2}} < 0$.

The utility function satisfies with the law of diminishing marginal utility.

3.3) Does the marginal utility curve of good x shift up when the consumer consumes more units of good-y?

$$\frac{\partial MU_x}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = 0$$

An increase in consumption in good-y does not affect MU_x

$\therefore$ Marginal utility curve of good x remains the same.

3.4) What is the level of the household utility when the consumer consumes 1 unit of good-x and 2 units of good-y?

$$U(x,y) = x^{1/2} + y^{1/2}$$

$$U(1,2) = 1^{1/2} + 2^{1/2} = 1 + \sqrt{2} \approx 1 + 1.414 \approx 2.414$$

$\therefore$ The level of the household utility is approximately 2.414.

3.5) Following (3.4), use the total differential to calculate the change in the level of utility under which the consumer increases the consumption on good x by 3 units and reduces the consumption on good y by 1 unit.

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy$$

$$= \frac{1}{2}x^{-\frac{1}{2}} \underbrace{dx}_{3} + \frac{1}{2}y^{-\frac{1}{2}} \underbrace{dy}_{-1}$$

$$dU = \frac{3}{2}x^{-\frac{1}{2}} - \frac{1}{2}y^{-\frac{1}{2}}$$

3.6). Derive the MRS and show that MRS is decreasing in x

$$MRS = -\frac{MU_x}{MU_y} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = -\frac{\left(\frac{1}{2}\right)x^{-\frac{1}{2}}}{\left(\frac{1}{2}\right)y^{-\frac{1}{2}}} = -\frac{x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$= -\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}$$

As x increases, the denominator gets bigger and MRS decreases.

As x increases, y decreases and the numerator gets smaller so MRS decreases.

Therefore, MRS is decreasing in x.