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Hw.

1. find cournot equilibrium when there are 3 firm in the mkt

$P = a - bQ$, $Q = q_1 + q_2 + q_3$ $\quad C_1 = C_2 = C_3$

what is equilibrium price : P^*

what 're firm profit : $\Pi_1 = \Pi_2 = \Pi_3 = ?$

2. if there're n firm, $q_i^* = f(n)$, $P = f(n)$, $\Pi = f(n) \longrightarrow$ un f(n)

3. from question 2, what happened if $n \to \infty$,

what happened if $n \to 1$

(1) $\Pi_1 = TR_1 - TC_1$

$= PQ - C$

$= (a - b(q_1 + q_2 + q_3))q_1 - C_1$

$= (a - bq_1 - bq_2 - bq_3)q_1$

FOC : $\frac{\partial \Pi_1}{\partial q_1} = 0$

$a - 2bq_1 - bq_2 - bq_3 = 0$

$q_1 = \frac{a - bq_2 - bq_3}{2b}$

substitut q_3

$2bq_1 = a - bq_2 - b\left(\frac{a - bq_2}{3b}\right)$

$6bq_1 = 3a - 3bq_2 - a + bq_2$

$q_1 = \frac{a - bq_2}{3b}$

substitute q_2

$q_1 = \frac{a - b\left(\frac{a}{4b}\right)}{3b}$

$= \frac{a}{4b}$

$\Pi_2 = TR_2 - TC_2$

$= (a - bq_1 - bq_2 - bq_3)q_2$

FOC : $\frac{\partial \Pi_2}{\partial q_2} = 0$

$a - bq_1 - 2bq_2 - bq_3 = 0$

$q_2 = \frac{a - bq_1 - bq_3}{2b}$

substitute q_1 to q_2

$2bq_2 = a - \cancel{b}\left(\frac{a - bq_2 - bq_3}{2\cancel{b}}\right) - bq_3$

$4bq_2 = 2a - a + bq_2 + bq_3 - 2bq_3$

$q_2 = \frac{a - bq_3}{3b}$

substitute q_1

$3bq_2 = a - b\left(\frac{a}{4b}\right)$

$12bq_2 = 4a - a$

$q_2 = \frac{a}{4b}$

$\Pi_3 = TR_3 - TC_3$

$= (a - bq_1 - bq_2 - bq_3)q_3$

FOC : $\frac{\partial \Pi_3}{\partial q_3} = 0$

$a - bq_1 - bq_2 - 2bq_3 = 0$

$q_3 = \frac{a - bq_1 - bq_2}{2b}$

substitute q_1

$2bq_3 = a - b\left(\frac{a - bq_2 - bq_3}{2b}\right) - bq_2$

$4bq_3 = 2a - a + bq_2 + bq_3 - 2bq_2$

$q_3 = \frac{a - bq_2}{3b}$

substitute q_2

$3bq_3 = a - b\left(\frac{a - bq_3}{3b}\right)$

$9bq_3 = 3a - a + bq_3$

$8bq_3 = 2a$

$q_3 = \frac{a}{4b}$

find equilibrium price

$P = a - bQ$

$= a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$

$= a - \frac{3a}{4}$

$= \frac{a}{4}$ or $0.25a$ #

find firms' profit

$\Pi_1 = P \cdot Q - C_1$

$= 0.25a\left(\frac{a}{4b}\right) - C_1$

$= \frac{a^2}{16b} - C_1$ #

$\Pi_2 = P \cdot Q - C_2$

$= \frac{a^2}{16b} - C_2$ #

$\Pi_3 = P \cdot Q - C_3$

$= \frac{a^2}{16b} - C_3$ #

(2) $P = a - b(q_1 + q_2 + q_3 + \dots + q_n)$

$= a - bq_1 - bq_2 - \dots - bq_n$

$\Pi_n = (a - bq_1 - bq_2 - \dots - bq_n)q_n - C_n$

FOC : $\frac{\partial \Pi_n}{\partial q_n} = 0$

$a - 2bq_1 - 2bq_2 - \dots - bq_n = 0$

$q_n = \frac{a - 2bq_1 - 2bq_2 - \dots - 2bq_{n-1}}{2b}$

$= \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 + \dots + q_{n-1})$

assume that $q_1 + q_2 + \dots + q_n = Q$

$\therefore q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 + \dots + q_n)$

$0.5q_1 = \frac{a}{2b} - 0.5Q$

$q_1 = \frac{a}{b} - Q$

So, $q_n = \frac{a}{b} - Q$

Since, $q_1 + q_2 + \dots + q_n = Q$

$= n\left(\frac{a}{b} - Q\right)$

$= \frac{na}{b} - nQ$

$Q = \frac{na}{(n+1)b}$

Substitute Q to Q_1

$Q_1 = \frac{a}{(n+1)b}$

$Q_i = \frac{a}{(n+1)b}$

find equilibrium price

$P = a - b(Q)$

$= a - b\left(\frac{na}{(n+1)b}\right)$

$= \frac{a(n+1) - na}{n+1}$

$= \frac{a}{n+1}$

$\Pi_i = P \cdot Q_i - C_i$

$= \frac{a}{n+1}\left(\frac{a}{(n+1)b}\right) - C_i$

$= \frac{a^2}{(n+1)^2 b} - C_i$

(3) if $n \to \infty$, there will be infinity quantity that is produced. Since it becomes completely competitive market price will get lower and lower, eventually profit will be losser

if $n = 1$, it will be monopoly market where firm is able to set price as high as they want and can decided how many quantity will be produced. However, the quantity produced is less than competitive market because $Q = \frac{a}{2b} < Q = \frac{na}{(n-1)b}$ and profit will be higher $\left(\Pi = \frac{a^2}{4b} - C > \Pi_i = \frac{a^2}{(n+1)^2 b} - C_i\right)$