The Greek Letters

Chapter 17

Outline

- Naked and covered positions
- Stop-loss strategy
- Delta hedging
  - European options
  - Forward & futures
- Other Greek letters
- Reality of hedging

Example

- A bank has sold for \$300,000 a European call option on 100,000 shares of a nondividend paying stock
- S_0 = 49, K = 50, r = 5%, σ = 20%, T = 20 weeks, μ = 13%
- The Black-Scholes value of the option is \$240,000
- How does the bank hedge its risk?

Naked & Covered Positions

- Naked position (take no action) leaves the bank in a very risky position
- Covered call position involves buying 100,000 shares today and hold it until the options expires
- If the stock drops to $40 (from $49) the option position expires unexercised
- The bank loses $900,000 on the stock position when it received $300,000 for the option premiums
- Both strategies leave the bank exposed to significant risk

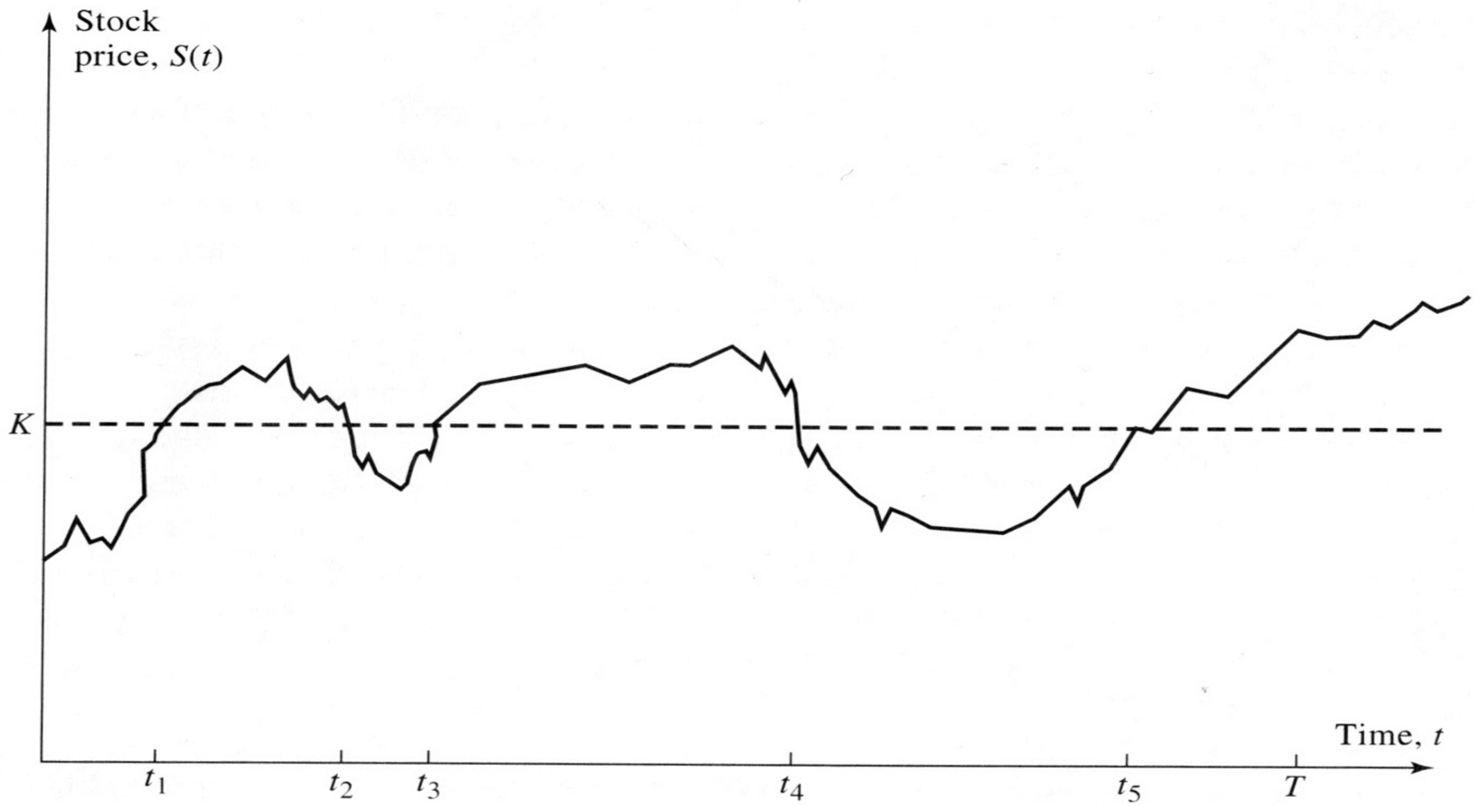
Stop-Loss Strategy

This involves:

- Buying 100,000 shares as soon as price reaches the strike price (in this example $50)
- Selling 100,000 shares as soon as price falls below the strike price
- Intuition: the probability of an option ending in the money increases as the stock price increases above the strike price and vice versa

Stop-Loss Strategy

Figure 15.1 A stop-loss strategy



Stop-Loss Strategy

- This deceptively simple hedging strategy does not work well

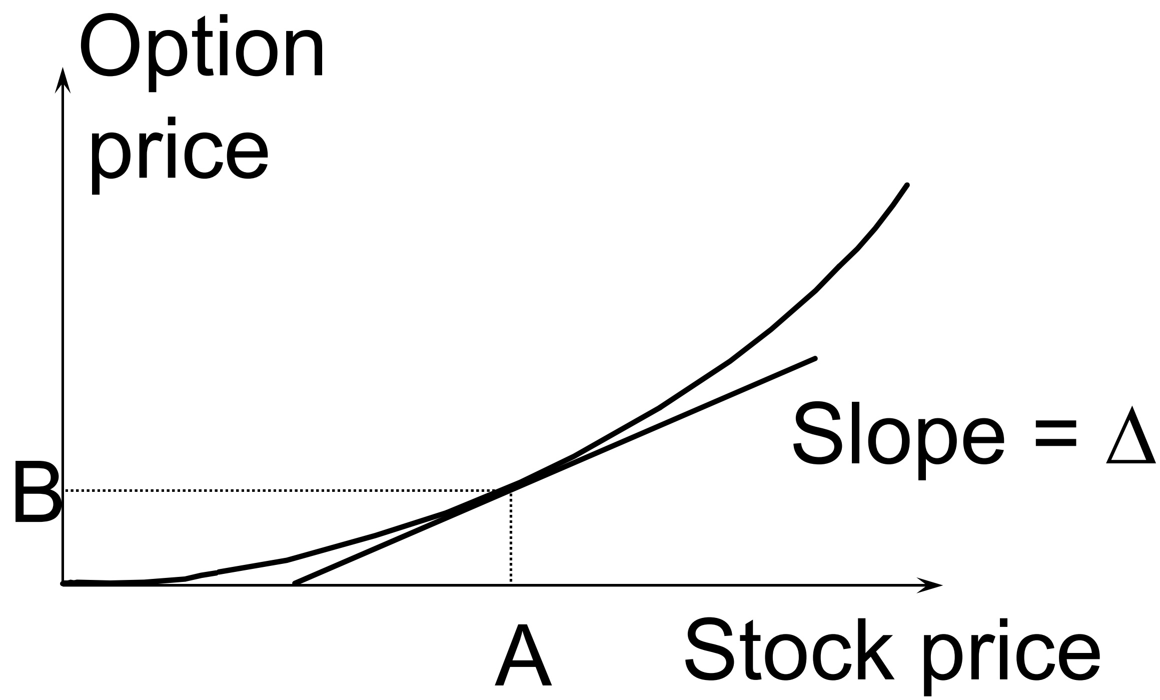
1. Because, stock price follows a random walk, one cannot predict whether the stock price is on the way up or down when it is at exactly at the strike price

Stop-Loss Strategy

- Alternatively we can buy the stock when the stock is K+a and sell when the stock is K-a where a is a small number
- But what should a be?
  - If a is too large we increase the risk of losing money from writing an option
  - If a is too small we will buy and sell very often and incur very large transaction costs

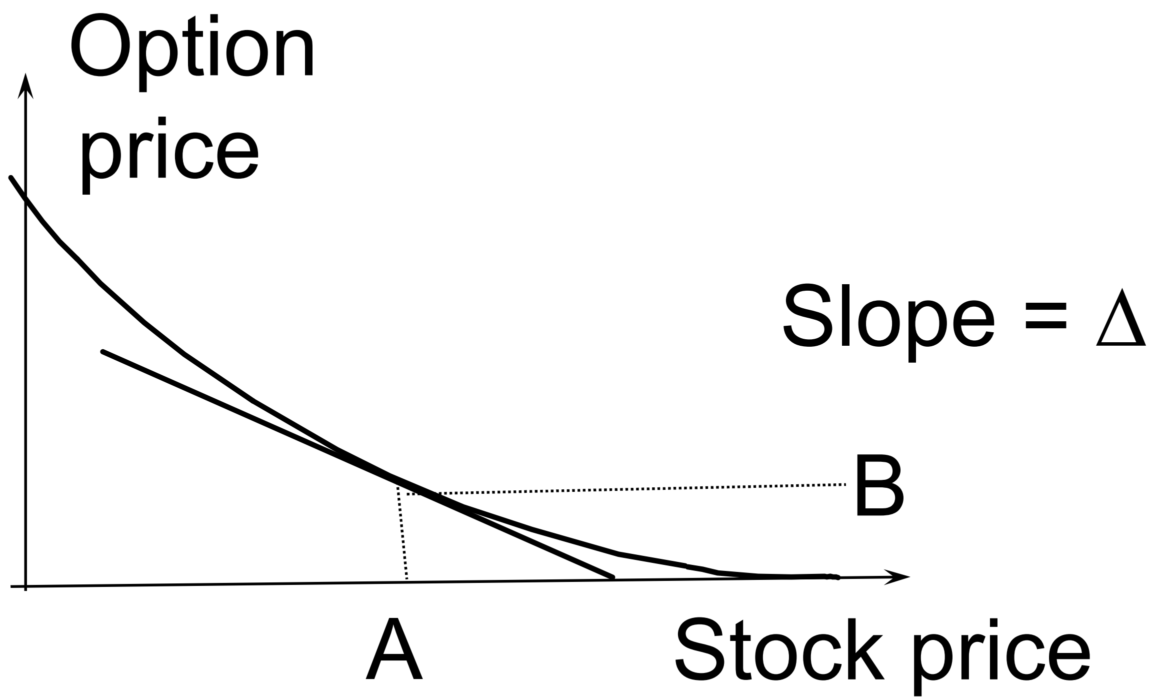
Delta of a call option is (0 to 1)

- Delta (Δ) is the rate of change of the option price with respect to the underlying



Delta of a put option is (-1 to 0)

- Delta (Δ) is the rate of change of the option price with respect to the underlying



Delta

- Delta is the derivative of the call or put option with respect to the underlying asset price. Consider the B-S model

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$\text{where} \quad d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

Delta

- The delta of a European call option on a non-dividend paying stock $N(d_1)$
- The delta of a European put option on a non-dividend paying stock $N(d_1)$-1
- The delta of a European call on a stock paying dividends at rate q is $N(d_1)e^{-qT}$
- The delta of a European put is $e^{-qT}[N(d_1)-1]$
- The delta of a European call and put on foreign currency are $N(d_1)e^{-r_fT}, e^{-r_fT}[N(d_1)-1]$

Delta Hedging

- Delta hedging involves matching the delta of the portfolio and the hedged portfolio.
- Example: The stock price is $100 and the option price is $10.
- Consider a trader who sells 20 call options each for buying 100 stocks.
- The trader position can be hedged by buying 0.6*2000=1200shares.
- The gain/loss on the option position would tend to be offset by the loss/gain on the stock positions.

Delta Hedging

- If the stock price goes up by $1 producing a gain of 1200 on the shares
- The option price will go up by 0.6*1=0.6 producing a loss of 6*2000=1200.
- If the stock price goes down by $1 then the losses of 1200 from the share price drop cancels the increase in short call option position of 0.6*2000=1200

Delta Hedging (example)

- A US bank has sold six-month put options on 1M UK pounds with a strike price of 1.6 and wishes to make its portfolio delta neutral. Suppose that the current exchange rate is 1.62, the risk-free interest rate in the UK is 13% per annum, the risk free interest rate in the US is 10% per annum

Delta Hedging (example)

- Volatility of sterling is 15%. In this case, S=1.62, K=1.6, r=0.1, rf=0.13, sigma=0.15, T=0.5. The delta of a put option on a currency is $e^{-r_f T}[N(d_1) - 1]$
- We can show that d1 = 0.0287 and N(d1) = 0.5115, and the delta of the put option is -0.458.
- What is the delta of the bank's total position?
- How does the bank hedge?

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- We can show that d1 = 0.0287 and N(d1) = 0.5115, and the delta of the put option is -0.458. The delta of the bank's total position is +485000. To make the position neutral we must add a short position in UK pound of 485000.
- This short position has a delta of -485000

Delta Hedging

	HEDGING strategy
LONG CALL	short the underlying
SHORT CALL	long the underlying asset
LONG PUT	long the underlying asset
SHORT PUT	short the underlying asset

Delta of other assets

- The value of the forward contract is
- For $1 change in the underlying security, the value of the forward contract changes by 1, everything else equal.
- The delta of the forward contract is 1.
- Because the futures contract is marked to market every day, the delta of the futures contract is e^{rT} for non-dividend-paying stock and equal e^{r-qT} for dividend paying stock, where q is the dividend yield

Using Futures for Delta Hedging

- The delta of a futures contract is $e^{(r-q)T}$ times the delta of a spot contract
- The position required in futures for delta hedging is therefore $e^{-(r-q)T}$ times the position required in the corresponding spot contract

Example

- The option in the previous example where hedging using the currency requires a short position of 458000 pounds.
- $e^{(r-q)T}$ futures = 458000
- Hedging using nine-month futures requires a short futures of $e^{-(0.1-0.13)9/12}$ 458000=468442.
- Each futures contract is for the delivery of 62500 pounds, we need to short 468442/62500 contracts

The dynamic aspects of hedging

- As time change, underlying asset change, delta change
- The delta hedge position changes
- The decision to rebalance a hedged portfolio depends on the trading cost vs how much risk (incomplete hedge) we are willing to take.

The dynamic aspects of hedging

Table 15.2 Simulation of delta hedging. Option closes in the money and cost of hedging is $263,300

Week	*Stock price*	*Delta*	*Shares purchased*	*Cost of shares purchased (\$000)*	*Cumulative cost including interest (\$000)*	*Interest cost (\$000)*
0	49.00	0.522	52,200	2,557.8	2,557.8	2.5
1	48.12	0.458	(6,400)	(308.0)	2,252.3	2.2
2	47.37	0.400	(5,800)	(274.7	1,979.8	1.9
3	50.25	0.596	19,600	984.9	2,966.6	2.9
4	51.75	0.693	9,700	502.0	3,471.5	3.3
5	53.12	0.774	8,100	430.3	3,905.1	3.8
6	53.00	0.771	(300)	(15.9)	3,893.0	3.7
7	51.87	0.706	(6,500)	(337.2)	3,559.5	3.4
8	51.38	0.674	(3,200)	(164.4)	3,398.5	3.3
9	53.00	0.787	11,300	598.9	4,000.7	3.8
10	49.88	0.550	(23,700)	(1,182.2)	2,822.3	2.7
11	48.50	0.413	(13,700)	(664.4)	2,160.6	2.1
12	49.88	0.542	12,900	643.5	2,806.2	2.7
13	50.37	0.591	4,900	246.8	3,055.7	2.9
14	52.13	0.768	17,700	922.7	3,981.3	3.8
15	51.88	0.759	(900)	(46.7)	3,938.4	3.8
16	52.87	0.865	10,600	560.4	4,502.6	4.3
17	54.87	0.978	11,300	620.0	5,126.9	4.9
18	54.62	0.990	1,200	65.5	5,197.3	5.0
19	55.87	1.000	1,000	55.9	5,258.2	5.1
20	57.25	1.000	0	0.0	5,263.3	

Delta of a portfolio

- Delta of a portfolio with n options equals the weighted average of the delta of the components of the portfolio where the weights are value of each component

$$\Delta = \sum_{i=1}^{n} w_i \Delta_i$$

Delta of a portfolio

- Suppose a financial institution in the United states has the following three positions in options on the Australian dollar:

1. Long position in 100000 call options with strike price 0.55, expiration date in 3 months. The delta of each option is .533
2. Short position in 200000 call options with strike price 0.56 and an expiration date in 5 moths. The delta of each option is 0.468

Delta of a portfolio

3. A short position in 50000 put options with strike price of 0.56 and expiration date in two months. The delta of each option is -0.508

The delta of the whole portfolio is

100K*0.533 -200K*0.468-50K*(-.508) =-14,900

Concept check question

If we want to achieve a delta neutral position using a six-months futures contract, how many would we need? The US interest rate=0.05, the Australian interest rate=0.08

G) 14900

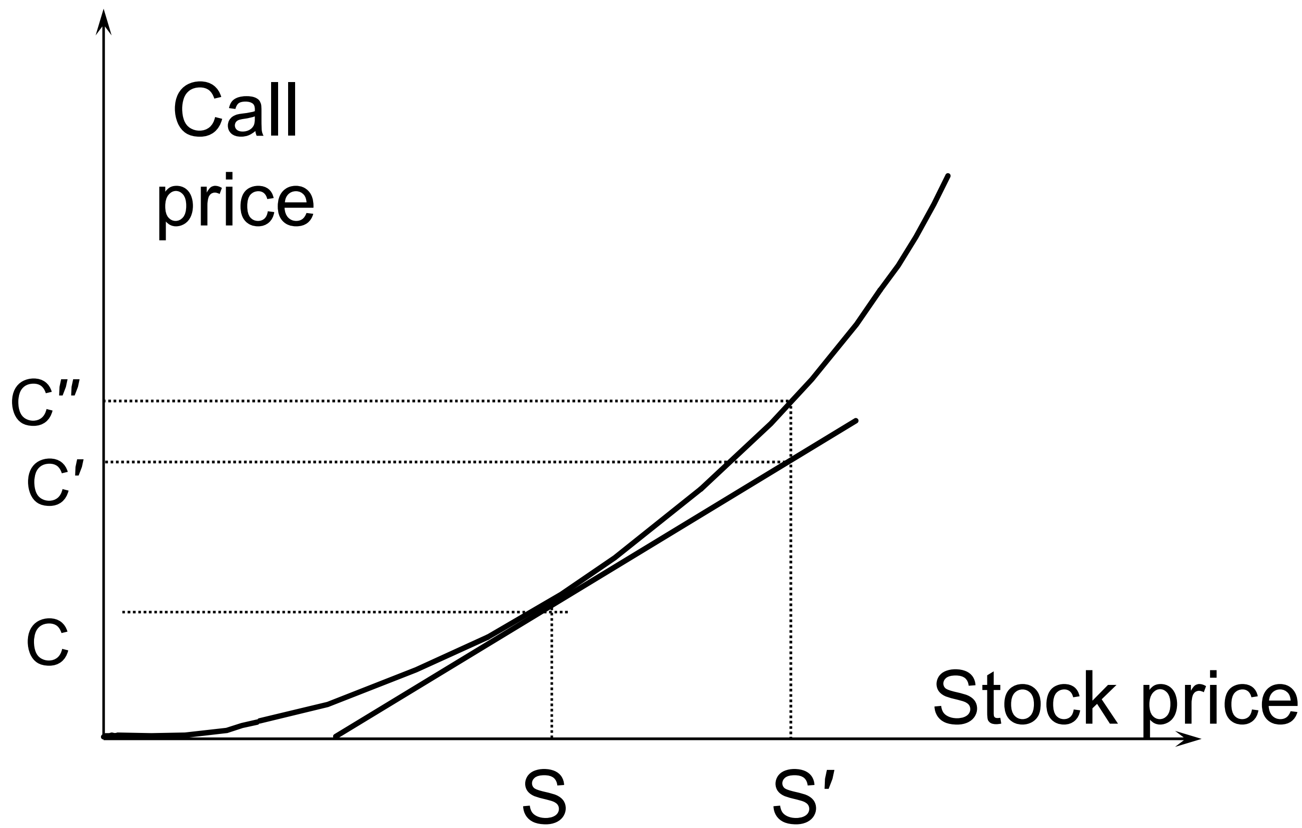
Y) 15125

R) 15000

Gamma

- Gamma (Γ) is the rate of change of delta (Δ) with respect to the price of the underlying asset
- See Figure 15.9 for the variation of Γ with respect to the stock price for a call or put option

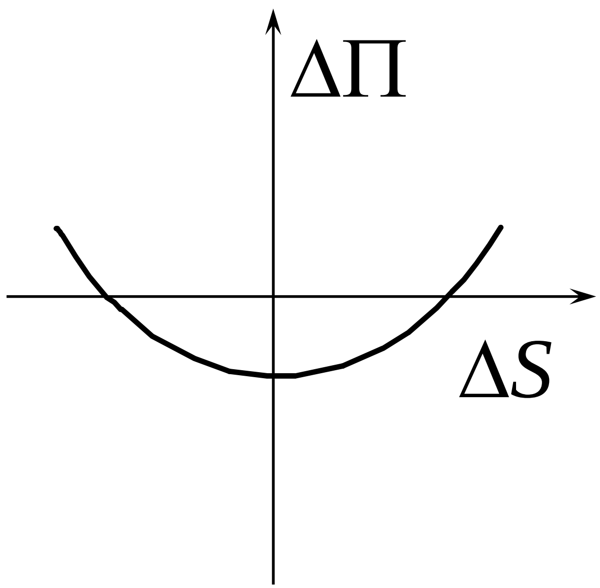
Gamma Addresses Delta Hedging Errors Caused By Curvature



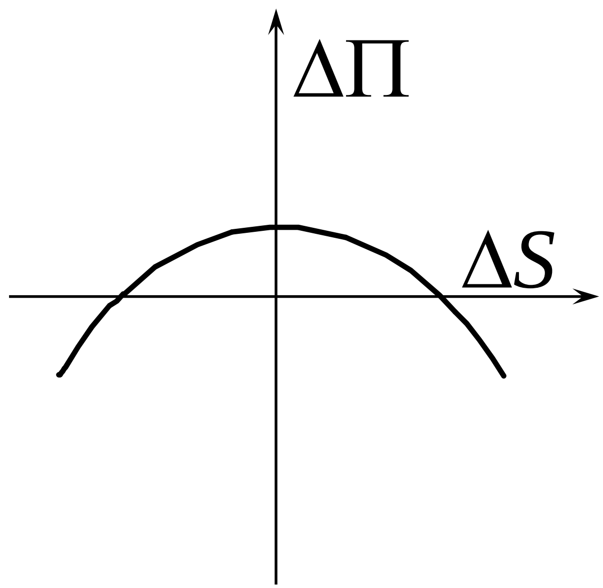
Interpretation of Gamma

- For a delta neutral portfolio,

$$\Delta\Pi \approx \Theta\,\Delta t + ½\Gamma\Delta S^2$$



Positive Gamma



Negative Gamma

Theta

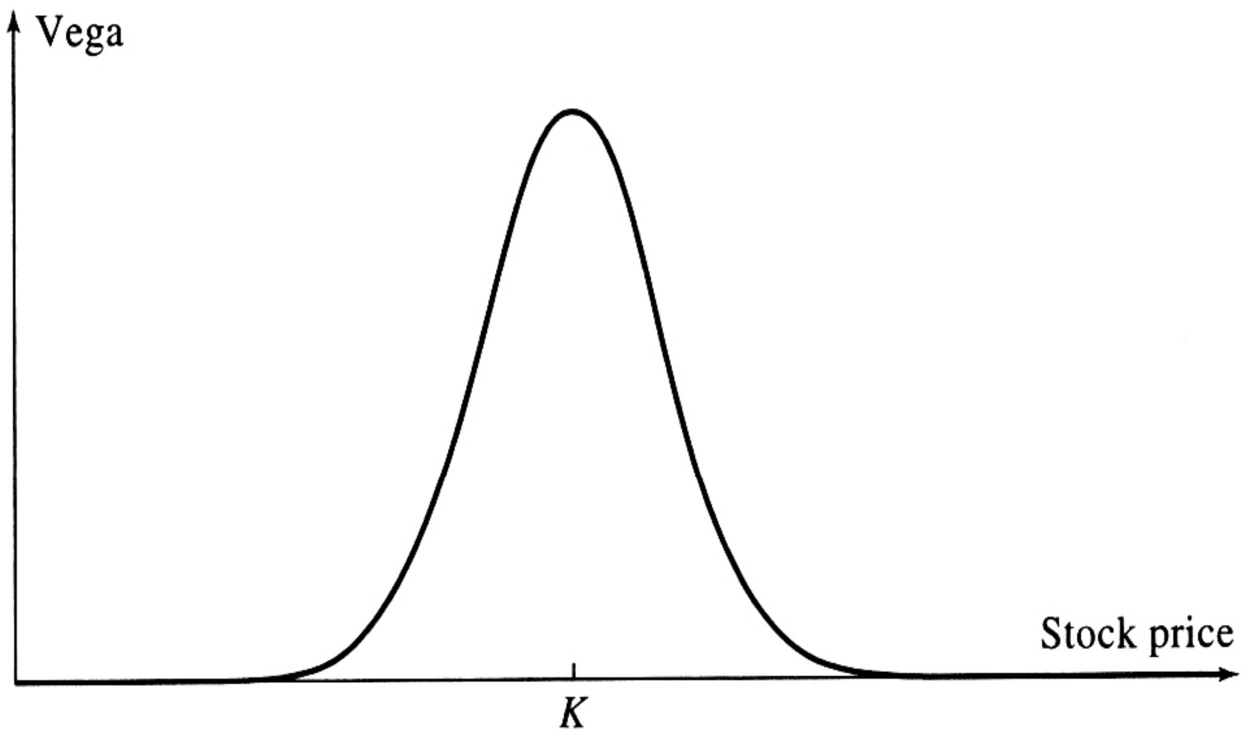
- Theta (Θ) of a derivative (or portfolio) is the rate of change of the value with respect to the passage of time
- The theta of a call or put is usually negative. This means that, if time passes with the price of the underlying asset and its volatility remaining the same, the value of a long option declines
- See Figure 17.5 for the variation of Θ with respect to the stock price for a European call

Vega

- Vega (ν) is the rate of change of the value of a derivatives portfolio with respect to volatility
- The price of an option increases as volatility increase, therefore vega is always positive

Vega

Figure 15.11 Variation of vega with stock price for an option



Rho

- Rho is the rate of change of the value of a derivative with respect to the interest rate

- For currency options there are 2 rhos

Managing Delta, Gamma, & Vega

- Delta, Δ, can be changed by taking a position in the underlying asset
- To adjust gamma, Γ, and vega, ν, it is necessary to take a position in an option or other derivative

Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega
- As portfolio becomes larger hedging becomes less expensive

Scenario Analysis

A scenario analysis involves testing the effect on the value of a portfolio of different assumptions concerning asset prices and their volatilities

Hedging vs Creation of an Option Synthetically

- When we are hedging we take positions that offset Δ, Γ, ν, etc.
- When we create an option synthetically we take positions that match Δ, Γ, & ν

Portfolio Insurance

- In October of 1987 many portfolio managers attempted to create a put option on a portfolio synthetically
- This involves initially selling enough of the portfolio (or of index futures) to match the Δ of the put option

Portfolio Insurance continued

- As the value of the portfolio increases, the Δ of the put becomes less negative and some of the original portfolio is repurchased
- As the value of the portfolio decreases, the Δ of the put becomes more negative and more of the portfolio must be sold
- The strategy did not work well on October 19, 1987...

Summary

- Naked and covered positions
- Stop-loss strategy
- Delta hedging
  - European options
  - Forward & futures
- The other Greek letters
- Reality of hedging