Intercept

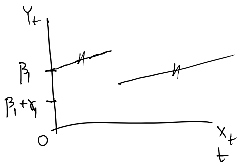
$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

$Y_t = \beta_1 + \gamma_1 D_t + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

$D_t = 0 \Rightarrow Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

$D_t = 1 \quad Y_t = (\beta_1 + \gamma_1) + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

$\gamma_1 < 0$



(A)

Intercept & Slope Dummy

$D_t = 0$ / 1

$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

$Y_t = \beta_1 + \gamma_1 D_t + \beta_2 X_{2t} + \gamma_2 D_t X_{2t} + \beta_3 X_{3t} + \gamma_3 D_t X_{3t} + u_t$

$D_t = 0 \Rightarrow Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

$D_t = 1 \quad Y_t = (\beta_1 + \gamma_1) + (\beta_2 + \gamma_2) X_{2t} + (\beta_3 + \gamma_3) X_{3t} + u_t$

(B)

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① $\quad y_{1t} = \beta_{10} + \beta_{11} x_{1t} + \beta_{12} x_{2t} + u_{1t}$

② $\quad y_{2t} = \beta_{20} + \beta_{22} x_{2t} + \beta_{23} x_{3t} + u_{2t}$

① $t=1, t=2$

$$\begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1t} \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{1t} & x_{2t} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{10} \\ \beta_{11} \\ \beta_{12} \end{bmatrix} + \begin{bmatrix} u_{11} \\ u_{12} \\ \vdots \\ u_{1t} \end{bmatrix}$$

② $t=1$

$$\begin{bmatrix} y_{21} \\ y_{22} \\ \vdots \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & x_{21} & x_{31} \\ 1 & x_{22} & x_{32} \\ \vdots & \vdots & \vdots \\ 1 & x_{2t} & x_{3t} \end{bmatrix} \begin{bmatrix} \beta_{20} \\ \beta_{22} \\ \beta_{23} \end{bmatrix} \quad \begin{bmatrix} u_{21} \\ u_{22} \\ \vdots \\ u_{2t} \end{bmatrix}$$

$= V$

$$Y = X \beta + \varepsilon$$

$$\hat{\beta} = (X'X)^{-1} X'Y \rightarrow E(\varepsilon\varepsilon') = \text{var-cov}(\varepsilon) = \sigma^2 I$$

$$\hat{\beta}_{SUR} = \hat{\beta}_{FGLS} = (X' \hat{V}^{-1} X)^{-1} X' \hat{V}^{-1} Y$$

$$\boxed{Y = X\beta + \varepsilon}$$

$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$= (X'X)^{-1} X'(X\beta + \varepsilon)$$

$$= \cancel{(X'X)^{-1}} \cancel{X'X} \beta + (X'X)^{-1} X'\varepsilon$$

$$\hat{\beta} = \beta + (X'X)^{-1} X'\varepsilon$$

<u>Unbias</u>

$E(\hat{\beta}) = \beta$

$$E(\hat{\beta}) = E\left(\beta + (X'X)^{-1} X'\varepsilon\right)$$

$$= \beta + E\left[(X'X)^{-1} X'\varepsilon\right]$$

$$= \beta + (X'X)^{-1} E\left[X'\varepsilon\right]$$