6304640391 Kanchal

$n = 46$	$\sum_{i=1}^{n} X_i = 3{,}959.80$	$\sum_{i=1}^{n} Y_i = 3{,}180.80$
$\bar{X} = 86.0826$	$\bar{Y} = 69.1478$	
$\sum_{i=1}^{n}(X_i)^2 = 364{,}023.30$	$\sum_{i=1}^{n} X_i Y_i = 319{,}943.18$	
$\sum_{i=1}^{n}(X_i-\bar{X})^2 = 23{,}153.3861$	$\sum_{i=1}^{n}(Y_i-\bar{Y})^2 = 94{,}525.1748$	
$\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y}) = 46{,}131.6183$	$\sum_{i=1}^{n}\hat{u}_i^2 = 2{,}610.9211$	

Cheat sheet

› $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$

› $\hat{\beta}_2 = \frac{\sum(X_i-\bar{X})(Y_i-\bar{Y})}{\sum(X_i-\bar{X})^2} = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i(Y_i-\bar{Y})}{\sum x_i(X_i-\bar{X})}$

where $x_i = (X_i-\bar{X}), y_i = (Y_i-\bar{X}), x_i^2 = (X_i-\bar{X})^2$

› $r^2 = \frac{ESS}{TSS} = \frac{\sum(\hat{Y}_i-\bar{Y})^2}{\sum(Y_i-\bar{Y})^2}$ or

› $r^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum \hat{u}_i^2}{\sum(Y_i-\bar{Y})^2}$ $\sum(Y_i-\hat{Y}_i)^2$

answer the following questions. Show your work.

a) (4 points) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0,\sigma^2)$, find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) (2 points) Find R^2 and explain its meaning.

c) (1 points) If $X_i = 60$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) (3 points) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

1a) $\beta_2 = \frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sum_{i=1}^{n}(X_i-\bar{X})^2} = \frac{46{,}131.6183}{23{,}153.3861} = 1.9924$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} = 69.1478 - 1.9924(86.0826) = -102.3632$

1b) $r^2 = \frac{ESS}{TSS} = \frac{\sum(\hat{Y}_i-\bar{Y})^2}{\sum(Y_i-\bar{Y})^2}$

or

$r^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum\hat{u}_i^2}{\sum(Y_i-\bar{Y})^2} = 1 - \frac{2610.9211}{94525.1748} = 0.9724$

the values of $r^2 = 0.9724$ suggest that X_i explain about 97.24% of the variation in Y_i

1c) $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_i = -102.3632 + 1.9924(60)$

$\hat{Y}_i = 17.1808$

when $X_i = 60$, $\hat{Y}_i$ is 17.1808

intercept: -102.3632
slope: 1.9924

1d) $Var(u_i) = \sigma^2$
estimated from

$\hat{\sigma}^2 = \frac{\sum\hat{u}_i^2}{n-k} = \frac{\sum(Y_i-\hat{Y})^2}{n-k} = \frac{2610.9211}{46-2} = 59.3391$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$

$\hat{\beta}_2 = \frac{\sum(X_i-\bar{X})(Y_i-\bar{Y})}{\sum(X_i-\bar{X})^2} = \frac{\sum x_i y_i}{\sum x_i^2}$, $\frac{\Sigma x_i (Y_i - \bar{y})}{\Sigma x_i (X_i - \bar{x})}$

where $x_i = (X_i - \bar{X}), y_i = (Y_i - \bar{X}), x_i^2 = (X_i - \bar{X})^2$

C

$n = 46$	$\sum_{i=1}^{n} X_i = 3{,}959.80$	$\sum_{i=1}^{n} Y_i = 3{,}180.80$
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$\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y}) = 46{,}131.6183$		$\sum_{i=1}^{n}\hat{u}_i^2 = 2{,}610.9211$

1d. continued)

$$Var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n\Sigma x_i^2}\sigma^2 = \frac{\Sigma X_i^2}{n\Sigma(X_i-\bar{X})^2}$$

$$= \frac{364023.3}{(46)(23153.3861)}(59.3391) = 0.3418$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{\sigma^2}{\Sigma(X_i-\bar{X})^2} = \frac{59.3391}{23153.3861} = 2.5629\times10^{-3}$$

e) (2.5 points) What are the 95-percent confident intervals for β_2? Interpret the meaning.

f) (2.5 points) Test the hypothesis whether coefficients (both β_1 and β_2) are different from zero at 0.05 level of significance.

1e) $95\% = 1-\alpha$ $\quad \alpha = 0.05)$ $\quad n-k = 44$

$\hat{\beta}_2 \pm t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{\beta}_2}$

U.B: $1.9924 + (2.021 \cdot \sqrt{2.5629\times10^{-3}}) = 2.0947$

L.B: $1.9924 - (2.021 \cdot \sqrt{2.5629\times10^{-3}}) = 1.8901$

$$P[1.8901 - \le \hat{\beta}_2 \le 2.0947] = 0.95$$

We expect that the value of $\hat{\beta}_2$ will fall in that interval 95 out of 100 times.

1f) $\hat{\beta}_1 \pm t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{\beta}_2}$

UB: $-102.3632 + (2.021 \cdot \sqrt{0.3418})$

LB: $-102.3632 - (2.021 \cdot \sqrt{0.3418})$

UB $= -101.1816$

LB $= -103.5448$

$$P[-103.5448 \le \hat{\beta}_1 \le -101.1816] = 0.95$$

1f cont.) for $\hat{\beta}_2$

#1 hypothesis

$H_0 : \beta_2 = 0$ Null

$H_a : \beta_2 \neq 0$ Alternative

#2 $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{1.9924 - 0}{\sqrt{2.5629 \times 10^{-3}}} = 39.3560$

#3 LB: $t_{\frac{\alpha}{2}} = -2.021$

UB: $t_{\frac{\alpha}{2}} = 2.021$

#4 t cal lies beyond boundary, reject null hypothesis, at significance level of 95%.
We are sure that β_2 is not zero 95 out of 100 times when we sample

for $\hat{\beta}_1$

#1 $H_0 : \beta_1 = 0$ null

$H_a : \beta_2 \neq 0$ alternative

#2 $t_{cal} = \dfrac{\hat{\beta}_1 - \beta_1}{\sigma_{\beta_1}} = \dfrac{-102.3632 - 0}{\sqrt{0.3418}} = -175.0886$

#3 critical interval LB $= -2.021$

UB $= 2.021$

#4 t cal lies beyond critical interval so we reject null hypothesis, and we are sure that β_1 is not zero 95 out of 100 times when we sample.

2a) No, an SRF is the line that is drawn using the process of estimation of many points if there is only one point then there will be infinite line possible for SRF.

2b) No, the causation of X and Y can not be prove with regression function we can only test how reliable the data is, or find its correlation, but we can not say for sure that X causes Y. For example if the data tested for both X and Y are income and students test scores, the regression function could have a positive relation, but it doesn't tell us that X causes Y.

2c) We can say that when we sample the value of β_2 will not be zero in the percent confidence interval that we conduct. The distribution of β_2 is very far apart with the normal distribution.

2D) The interval estimators allowed for error so prediction will be more accurate. (confidence interval) The confidence interval will decrease as more samples is being recorded so the study will be more precise

3a) 7.6581 Baht

3b) 0.3180 baht per 1 hour of work

3c) The coef. $\frac{0.318017}{24} = 0.6134$