

$$1) E_0[m_{02}, f_2] = E_0[m_{02}(S_2 - F_2)] = E[m_{02} S_2] - E[m_{02} F_2]$$

$$= S_0 - D_0 - \frac{1}{R_f^2} F_{02}$$

$$= 0$$

Since we know that $F_{02} = R_f^2 (S_0 - D_0)$

$$2) V(C_{t+1}) = -S^{t+1-\alpha C_t}$$

$$V(C_{t+1}) = a \delta^{t+1-\alpha C_t} e^{\sum_{i=1}^{\infty} \frac{V(C_{t+i})}{V_0(C_{t+1}, 0)} d_i}$$

$$= E_0 \left[\sum_{i=1}^{\infty} \delta^i e^{-a(d_{t+i} - d_t)} d_{t+i} \right]$$

$$3) \frac{P_t}{d_t} = E_t \left[\sum_{j=1}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t} \right)^{\gamma-1} \left(\frac{d_{t+j}}{d_t} \right) \right]$$

$$= E_t \left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1) \ln \left(\frac{C_{t+j}}{C_t} \right) + \ln(d_{t+j}/d_t)} \right]$$

$$\frac{P_t}{d_t} = E_t \left[\sum_{j=1}^{\infty} \delta^j e^{[(\gamma-1)\mu_c + \mu_d] + \sum_{i=1}^j [(\gamma-1)G_c \eta_{t+i} + G_d \varepsilon_{t+i}]} \right]$$

$$= \sum_{j=1}^{\infty} e^{\left[\ln \delta - (1-\gamma)\mu_c + \mu_d + \frac{1}{2}[(1-\gamma)^2 G_c^2 + G_d^2] - (1-\gamma)G_c G_d \rho \right]}$$

$$= \frac{1}{1 - \delta e^{-(1-\gamma)\mu_c + \mu_d + \frac{1}{2}[(1-\gamma)^2 G_c^2 + G_d^2] - (1-\gamma)G_c G_d \rho}} - 1$$

$$S_0, P_t = d_t \frac{\delta e^{\alpha}}{1 - \delta e^{\alpha}}; \quad \alpha \equiv \mu_d - (1-\gamma)\mu_c + \frac{1}{2}[(1-\gamma)^2 G_c^2 + G_d^2] - (1-\gamma)G_c G_d \rho$$

$$4. a) E_t[b_{t+1}] = \frac{R_f}{q_t} b_t + q_t E_t[e_{t+1}] + (1-q_t) E_t[z_{t+1}] \cdot R_f b_t$$

$$b) \text{ From } E_t[b_{t+1}] = R_f b_t$$

$$\lim E_t[b_{t+1}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

b_t should > 0 , as it limits $t \rightarrow \infty$.

c) A rational bubble can't be existed, as the bond price is required to be $P_t = d_t$ at maturity and be 0 after that. Only rational price is $P_t = P_t^*$

5. $P_t = P_t^* + b_t$ w/ $b_t \neq 0$ couldn't happen,

$$P_T = P_T^* = d_T \text{ at date } T$$