$\boxed{\text{Profit maximization}}$

$$P \cdot f'(L) = W$$

Don't know

If we know $f^{\underline{c}}$ form of $f(L)$

L^* in terms of $\underline{P}$ and $\underline{W}$.

↳ optimal <u>labor</u> employment.

$$L^* = L^*(P, W)$$

→ Sensitivity Analysis
Comparative Statics $\left\{ \dfrac{\partial L^*}{\partial P}, \dfrac{\partial L^*}{\partial W} \right.$

<u>What If</u> we don't know f^{z} form, but only some properties of the function.

$\Rightarrow$ L^* <u>Cannot be</u> solve for explicitly.

$\Rightarrow$ $\frac{\partial L^*}{\partial p}$; $\frac{\partial L^*}{\partial w}$ <u>not derivable ist</u> ?

No, derivable but you need to know "<u>Implicit derivative Concept.</u>"

$L^* = L^*(P, w)$ as we know.

$$\Rightarrow \quad P \cdot f'(L^*(P,w)) = w$$

$$d\left[P \cdot f'(L^*(P,w))\right] = dw$$

$$P \cdot f''(L^*)\left(dL^*(P,w)\right) + f'(L^*(P,w)) \cdot dp = dw$$

$$\Rightarrow \quad dL^* = \frac{1}{P \cdot f''(L^*)} \cdot \left[dw - f'(L^*) \cdot dp\right]$$

dL^* depend on "dw" and "dp"

If $dp = 0$;

$$dL^* = \frac{1}{p \cdot f''(L^*)} \, dw.$$

Unit of change in $\underline{\underline{w}}$. !

Marginal Effect of w on L^* !

$\therefore$

$$\frac{dL^*}{dw} = \frac{1}{p \cdot f''(L^*)} < 0 \quad ; \quad f''(\cdot) < 0$$

$\left\{ \begin{array}{l} \text{production } f^n \\ \text{is concave} \end{array} \right\}$

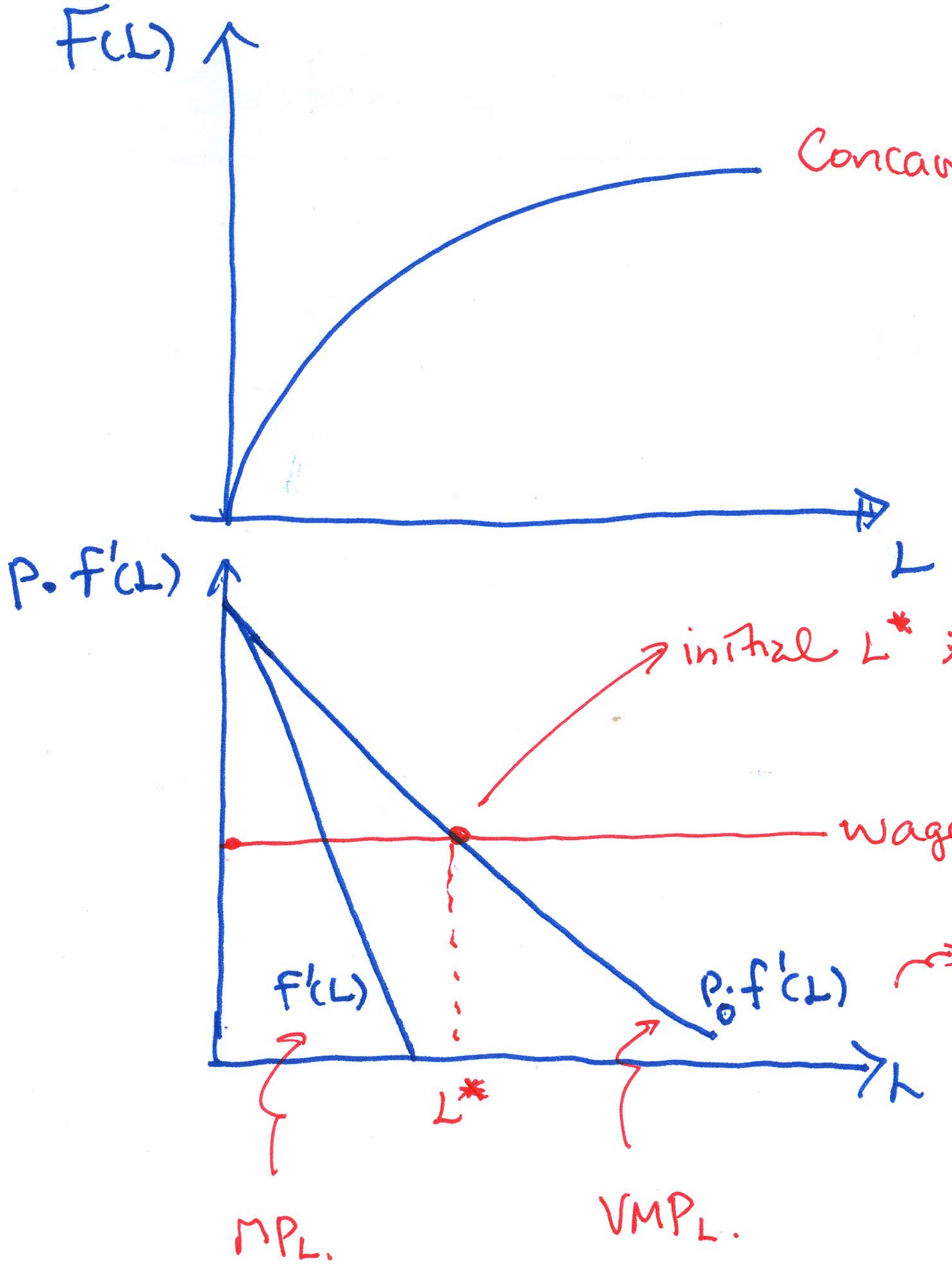
If $dw = 0$ then

$$dL^* = -\frac{f'(L^*)}{p \cdot f''(L^*)} \cdot dp$$

Same Logic as before

$$\frac{dL^*}{dp} = -\frac{\overbrace{f'(L^*)}^{>0}}{p \cdot \underbrace{f''(L^*)}_{<0}} > 0$$

$$\frac{(-)\cdot(+)}{(+)\cdot(-)} > 0$$



$VMP_L = W$

$\rightsquigarrow F'(L)$ is decreasing because f is concave

If $P_0 \to P_1$ $(P_1 > P_0)$

- $P_0 f'(L)$ rotate forward
- Fix "W" $\to L^*$ increase.

If $wage_0 \to wage_1$ $(W_1 > W_0)$

- VMP_L fixed
- Wage line shift up
- Downsize L^* to squeeze out the productivity.

System of Equation Model

Demand $\quad Q = D(P, y)$

Price, income

Supply $\quad Q = S(P, w)$

wage

$\left.\begin{matrix} Q^* \\ P^* \end{matrix}\right\}$ f^n of (y, w) in the Equ^{m}.

? $\dfrac{\partial Q^*}{\partial y}$

? $\dfrac{\partial Q^*}{\partial w}$

? $\dfrac{\partial P^*}{\partial y}$

? $\dfrac{\partial P^*}{\partial w}$

"First" $\longrightarrow$

$$\left.\begin{array}{l} Q^* = D(p^*, y) \\ Q^* = S(p^*, \cancel{y}\,W) \end{array}\right\}$$

Q^*, p^* simultaneously solve the model (in the level form)

Consider things in the total changes form

Demand : $dQ^* = d(D(p^*, y))$

$= D_p\, dp^* + D_y \cdot dy$ — ①

Supply : $dQ^* = \cancel{d} \; d(S(p^*, \cancel{y}\,W))$

$= S_p \cdot dp^* + S_w\, dw.$ — ②

$dQ^*; dp^*$ simultaneously solve the 2 equations at the same time.

$$dQ^* - D_p\, dp^* = D_y \cdot dy$$

$$dQ^* - S_p\, dp^* = S_w\, dw.$$

$$\begin{pmatrix} 1 & -D_p \\ 1 & -S_p \end{pmatrix} \begin{pmatrix} dQ^* \\ dp^* \end{pmatrix} = \begin{pmatrix} D_y\, dy \\ S_w\, dw \end{pmatrix}$$

$$A \cdot x = B$$

- $X^* = A^{-1} \cdot B \quad (|A| \neq 0)$
- Apply Cramer's rule for X^*

$$|A| = D_p - S_p$$

$$\therefore \quad dQ^* = \frac{\begin{vmatrix} D_y dy & -D_p \\ S_w dw & -S_p \end{vmatrix}}{|A|}$$

$$dQ^* = \frac{1}{|A|} \cdot \left(D_p S_w dw - S_p D_y dy \right).$$

$$\frac{dQ^*}{dw} = \frac{1}{|A|} \cdot D_p \cdot S_w \qquad \frac{dQ^*}{dy} = \frac{-S_p D_y}{|A|}. \qquad |A| < 0$$

Sign? normally (i) $S_p > 0$; (ii) $D_p < 0$ $\therefore$

" S_w ; D_y " ? if (iii) $S_w < 0$ and (iv) $D_y > 0$

$dQ^*/dw < 0$; $\dfrac{dQ^*}{dy} > 0$

What about dp^*?

$$dp^* = \frac{\begin{vmatrix} 1 & D_y\,dy \\ 1 & S_w\,dw \end{vmatrix}}{|A|}$$

$$= \frac{1}{|A|} \cdot \left(-D_y\,dy + S_w\,dw \right)$$

$$\frac{dp^*}{dw} = \frac{S_w}{|A|} \qquad \frac{dp^*}{dy} = -\frac{D_y}{|A|}$$

$$\left.\begin{array}{ll} \text{(iii)} & S_w < 0 \\ \text{(iv)} & D_y > 0 \end{array}\right\} \rightarrow \frac{dp^*}{dw} > 0 \quad ; \quad \frac{dp^*}{dy} > 0$$

Demand $\quad F^1(Q, P; y, w) = 0 \quad [\overbrace{Q - D(p, w)}^{F^1} = 0]$.

Supply $\quad F^2(Q, P; y, w) = 0 \quad [\overbrace{Q - S(p, w)}^{F^2} = 0]$

$$dF^1 = F^1_Q\, dQ + F^1_p\, dp + F^1_y\, dy + F^1_w\, dw = 0$$

$$dF^2 = F^2_Q\, dQ + F^2_p\, dp + F^2_y\, dy + F^2_w\, dw$$

$$\underbrace{\begin{bmatrix} F^1_Q & F^1_p \\ F^2_Q & F^2_p \end{bmatrix}}_{D_{Endo} F} \begin{bmatrix} dQ \\ dp \end{bmatrix} = - \underbrace{\begin{bmatrix} F^1_y & F^1_w \\ F^2_y & F^2_w \end{bmatrix}}_{D_{EXO} F} \begin{bmatrix} dy \\ dw \end{bmatrix}$$

$$d\,Endos = -[D_{Endo} F]^{-1} [D_{EXO} F] \cdot d\,EXos$$

$$dy = d\{C + I + G_0\} = dC + dI + dG_0$$

$$dC = dC(y) \rightarrow C'(y)\,dy \qquad 0 < C'(.) < 1$$

$$dI = dI(y) \rightarrow I'(y)\cdot dy \qquad 0 < I'(.) < 1$$

$$G = G_0$$

$$\boxed{\frac{dy}{dG_0};\ \frac{dC}{dG_0};\ \frac{dI}{dG_0}} = ?$$

$$dy = dc + dI + dG_0$$

$$dc = c'(y) \cdot dy$$

$$dI = I'(y) \cdot \cancel{dI}\, dy$$

3 to~~tal~~ ~~Differtial~~ Equations in the total differential form:

$$dy - dc - dI = dG_0$$

$$-c'(y)dy + dc + 0 \cdot dI = 0$$

$$-I'(y)dy + 0 \cdot dc + dI = 0$$

$$\underbrace{\begin{pmatrix} 1 & -1 & -1 \\ -c' & 1 & 0 \\ -I' & 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} dy \\ dc \\ dI \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} dG_0 \\ 0 \\ 0 \end{pmatrix}}_{B}$$

$$|A| = 1 - I' - c'$$

$$dy = \frac{\begin{vmatrix} dG_0 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}{|A|} = \frac{dG_0}{|A|}$$

$$\cancel{\frac{dy}{dA}} \quad \frac{dy}{dG_0} = \frac{1}{|A|} = \frac{1}{1 - I' - c'}$$

$$\underbrace{\begin{pmatrix} 1 & -1 & -1 \\ -c' & 1 & 0 \\ -I' & 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} dy \\ dc \\ dI \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} dG_0 \\ 0 \\ 0 \end{pmatrix}}_{B}$$

$$|A| = 1 - I' - c'$$

$$dy = \frac{\begin{vmatrix} dG_0 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}{|A|} = \frac{dG_0}{|A|}$$

$$\cancel{\frac{dy}{dA}} \quad \frac{dy}{dG_0} = \frac{1}{|A|} = \frac{1}{1 - I' - c'}$$