Solution: Quiz 4

1. Find the solution set for
$$|1+x| \ \geq \ \left|\frac{x}{|x|}-1\right|.$$

Solution:

From $|x| = \begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$, for $x \neq 0$, we have $\frac{x}{|x|} = \begin{cases} \frac{x}{-x}=-1, & x<0 \\ \frac{x}{x}=1, & x>0 \end{cases}$.

That is, $x \neq 0$ and we will consider 2 cases.

- Case (i): For $x<0$, $\frac{x}{|x|} = \frac{x}{-x} = -1$, $\left|\frac{x}{|x|}-1\right| = |-1-1| = 2$ and the inequality becomes
$$|1+x| \ \geq \ 2 \qquad \Leftrightarrow \qquad 1+x \leq -2 \quad \text{or} \quad 1+x \geq 2 \qquad \Leftrightarrow \qquad x \leq -3 \quad \text{or} \quad x \geq 1$$
That is, the solution in this case is $x \in \{(-\infty,-3]\cup[1,\infty)\} \cap (-\infty,0) = (-\infty,-3]$.
- Case (ii): For $x>0$, $\frac{x}{|x|} = \frac{x}{x} = 1$, $\left|\frac{x}{|x|}-1\right| = |1-1| = 0$ and the inequality becomes
$$|1+x| \ \geq \ 0 \qquad \Rightarrow \qquad x \in (-\infty,\infty) = \mathbb{R}$$
since the absolute value is always bigger than or equal to zero. That is, the solution in this case is $x \in \mathbb{R} \cap (0,\infty) = (0,\infty)$.

From cases (i) and (ii), the solution set is $(-\infty,-3]\cup(0,\infty)$. ■