



B.E. International Program

Faculty of Economics, Thammasat University



EE 320 Introductory Mathematical Economics (Section 046401)

Semester 1/2015

Quiz 4 (b) - Answers

Thursday November 12, 2015

Time: 11:10 – 11:25 hrs.

1. Consider the following production function

$$Q(K, L) = -K^2 - 2KL - 2L^2 + 8K + 12L.$$

Suppose that the output price is \$4 per unit, and the input prices for K and L are r and w , respectively.

a. (6 points) Find the levels of K^* and L^* that maximize the firm's profit.

$$\max_{K, L} \pi(K, L) = 4(-K^2 - 2KL - 2L^2 + 8K + 12L) - rK - wL$$

FONC:

$$\pi_K = 0 \rightarrow \pi_K = 4(-2K - 2L + 8) - r = 0 \quad -- (1)$$

$$\pi_L = 0 \rightarrow \pi_L = 4(-2K - 4L + 12) - w = 0 \quad -- (2)$$

$$\Rightarrow K^* = \frac{16-2r+w}{8} \text{ and } L^* = \frac{16+r-w}{8}$$

- b. (2 points) Verify the answer in part (a) by using the second-order sufficient conditions.

SOSC:

$$H = \begin{bmatrix} -8 & -8 \\ -8 & -16 \end{bmatrix}$$

$$\Rightarrow |H_1| = -8 < 0 \text{ \& } |H_2| = |H| = 128 - 64 = 64 > 0$$

$$\Rightarrow \pi(K^*, L^*) = \pi\left(\frac{16-2r+w}{8}, \frac{16+r-w}{8}\right) \text{ is a maximum profit.}$$

- c. (4 points) Determine the impact of changes in r on the optimal levels of inputs K^* and L^* (i.e. find $\frac{\partial K^*}{\partial r}$ and $\frac{\partial L^*}{\partial r}$), and evaluate the signs of both comparative statics derivatives.

$$\frac{\partial K^*}{\partial r} = \frac{\partial}{\partial r} \left(\frac{16 - 2r + w}{8} \right) = -\frac{1}{4} < 0$$

$$\frac{\partial L^*}{\partial r} = \frac{\partial}{\partial r} \left(\frac{16 + r - w}{8} \right) = \frac{1}{8} > 0$$