1.)

3.2126

Student	Y_i	X_i	Y_i^2	X_i^2	$x_i-\bar{x}$	$y_i-\bar{y}$	x_iy_i	u_i	u_i^2	$(x_i-\bar{x})^2$
1	2.8	63	7.84	3969	-14.625	-0.4125	176.4	0.0862	0.0074	213.8906
2	3.4	72	11.56	5184	-5.625	0.1875	244.8	0.3793	0.1439	31.6406
3	3.0	78	9	6084	0.375	-0.2125	234	-0.2253	0.0508	0.1406
4	3.5	81	12.25	6561	3.375	0.2875	283.5	0.1724	0.0297	11.3906
5	3.6	87	12.96	7569	9.375	0.3875	313.2	0.0678	0.0046	87.8906
6	3.0	75	9	5625	-2.625	-0.2125	225	-0.1230	0.0151	6.8906
7	2.7	75	7.29	5625	-2.625	-0.5125	202.5	-0.4230	0.1789	6.8906
8	3.7	90	13.69	8100	12.375	0.4875	333	0.0655	0.0043	153.1406
$\bar{x}=77.625$ $\bar{y}=3.2125$	25.7 ΣY_i	621 ΣX_i	85.59 ΣY_i^2	48,717 ΣX_i^2	0 Σx_i	0 Σy_i	2,012.4 Σx_iy_i		0.4347 Σu_i^2	511.8748 Σx_i^2

$\bar{x}=\frac{\Sigma X_i}{n}$ $\quad \hat{\beta}_1=\bar{Y}-\beta_2\bar{X}$

$\bar{Y}=\frac{\Sigma Y_i}{n}$ $\quad \hat{\beta}_2=\frac{\Sigma x_iy_i}{\Sigma x_i^2}$ or $\hat{\beta}_2=\frac{n\Sigma X_iY_i-\Sigma X_i\Sigma Y_i}{n\Sigma X_i^2-(\Sigma X_i)^2}$

1.1) $$\hat{\beta}_2=\frac{8(2{,}012.4)-15{,}959.7}{8(48{,}717)-385{,}641}=\frac{16{,}099.2-15{,}959.7}{389{,}736-385{,}641}=\frac{139.5}{4095}=0.0341 \;\#$$

$$\hat{\beta}_1=3.2125-0.0341\,(77.625)=0.5655\#$$

$Y_i=0.5655+0.0341x_i+u_i$, for any given level of total econometric exampoints (x_i) the slope of estimated of GPA equal to 0.0341 , with an estimated intercept in y-axis at 0.5655 .

1.2) $\hat{Y}_i=\hat{\beta}_1+\hat{\beta}_2x_i$ $\quad Y_i=\hat{Y}_i+u_i$ $\quad Y_i=\hat{\beta}_1+\hat{\beta}_2x_i+u_i$ $\quad Y_i=\hat{Y}_i+\hat{u}_i$ $\quad Y_i-\hat{Y}_i=\hat{u}_i$

$\hat{Y}_1=0.5655+(0.0341)(63)=2.7138$

$\hat{Y}_2=0.5655+(0.0341)(72)=3.0207$

$\hat{Y}_3=0.5655+(0.0341)(78)=3.2253$

$\hat{Y}_4=0.5655+(0.0341)(81)=3.3276$

$\hat{Y}_5=0.5655+(0.0341)(87)=3.5322$

$\hat{Y}_6=0.5655+(0.0341)(75)=3.1230$

$\hat{Y}_7=0.5655+(0.0341)(75)=3.1230$

$\hat{Y}_8=0.5655+(0.0341)(90)=3.6345$

$\hat{u}_1=2.8-2.7138=0.0862$

$\hat{u}_2=3.4-3.0207=0.3793$

$\hat{u}_3=3.0-3.2253=-0.2253$

$\hat{u}_4=3.5-3.3276=0.1724$

$\hat{u}_5=3.6-3.5322=0.0678$

$\hat{u}_6=3.0-3.1230=-0.1230$

$\hat{u}_7=2.7-3.1230=-0.4230$

$\hat{u}_8=3.7-3.6345=0.0655$

$\sum \hat{u}_i = 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0678 - 0.1230 - 0.4230 + 0.0655$

$= -0.0001$

$\sum \hat{u}_i \approx 0$

1.3) $Var(\hat{\beta}_1) = \dfrac{\sigma_u^2 \sum x_i^2}{n \sum x_i^2}$ $\qquad \sigma_u^2 = \hat{\sigma}_u^2 = \dfrac{\sum u_i^2}{n-2} = \dfrac{0.4847}{6} = 0.0725$

$Var(\hat{\beta}_2) = \sigma_u^2 / \sum x_i^2$

$Var(\hat{\beta}_1) = \dfrac{(0.0725)(48,717)}{8(511.87487)} = \dfrac{3,531.9825}{4094.9984} = 0.8625$ #

$Var(\hat{\beta}_2) = (0.0725)/(511.8748) = 0.00014164 \approx 0.0001$ #

2.)

$\bar{x} = 20 \quad \bar{y} = 9.1$

X_i	Y_i	x_i^2	y_i^2	x_iy_i	$x_i - \bar{x}$	$(x_i-\bar{x})^2$	$y_i - \bar{y}$	$(y_i-\bar{y})^2$	x_iy_i
10	0	100	0	0	-10	100	-9.1	82.81	91
12	2	144	4	24	-8	64	-7.1	50.41	56.8
14	5	196	25	70	-6	36	-4.1	16.81	24.6
16	6	256	36	96	-4	16	-3.1	9.61	12.4
18	7	324	49	126	-2	4	-2.1	4.41	4.2
22	10	484	100	220	2	4	0.9	0.81	1.8
24	10	576	100	240	4	16	0.9	0.81	3.6
26	15	676	225	390	6	36	5.9	34.81	35.4
28	16	784	256	448	8	64	6.9	47.61	55.2
30	20	900	400	600	10	100	10.9	118.81	109
200	91	4,440	1,195	2,214	0	440	0	366.9	394

2.1)

$\hat{\beta}_2 = \dfrac{n\sum x_iy_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$ $\qquad \hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{x} = 9.1 - 0.8955(20)$

$= -8.81$

$= \dfrac{10(2214) - 18,200}{10(4440) - (40,000)} = \dfrac{3,940}{4440} = 0.8955$

$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i \,,\quad \hat{y}_i = -8.81 + 0.8955(x_i)$

The regression model of estimated $\hat{y}_i$ has a slope of 0.8955 , and y-intercept at point every increase in x_i by one unit increase -8.81 by 0.8955 unit

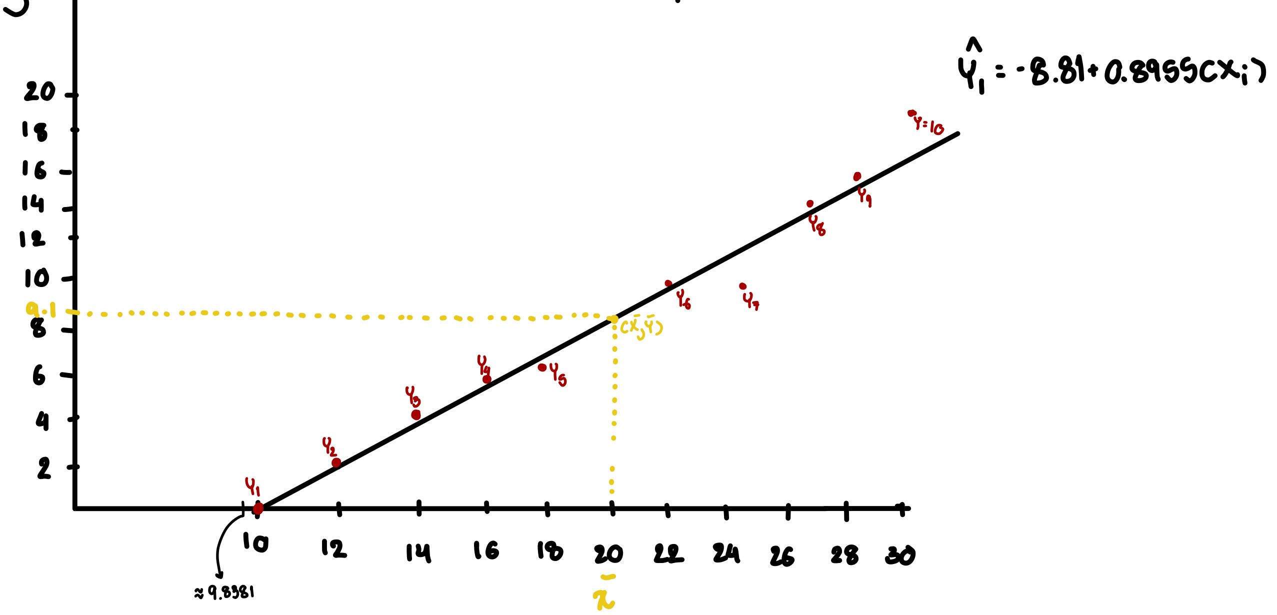
2.2

$\hat{Y}_1$	0.145	$\hat{u}_1$	-0.145	$\hat{u}_1^2$	0.0210
$\hat{Y}_2$	1.936	$\hat{u}_2$	0.064	$\hat{u}_2^2$	0.0041
$\hat{Y}_3$	3.727	$\hat{u}_3$	1.273	$\hat{u}_3^2$	1.6205
$\hat{Y}_4$	5.518	$\hat{u}_4$	0.482	$\hat{u}_4^2$	0.2323
$\hat{Y}_5$	7.309	$\hat{u}_5$	-0.309	$\hat{u}_5^2$	0.0955
$\hat{Y}_6$	10.891	$\hat{u}_6$	-0.891	$\hat{u}_6^2$	0.7939
$\hat{Y}_7$	12.682	$\hat{u}_7$	-2.682	$\hat{u}_7^2$	7.1931
$\hat{Y}_8$	14.473	$\hat{u}_8$	0.527	$\hat{u}_8^2$	0.2777
$\hat{Y}_9$	16.264	$\hat{u}_9$	-0.264	$\hat{u}_9^2$	0.0697
$\hat{Y}_{10}$	18.055	$\hat{u}_{10}$	1.945	$\hat{u}_{10}^2$	3.7830

$$\sum \hat{u}_i = -0.145+0.064+1.273+0.482-0.309-0.891-2.682+0.527-0.264+1.945 = 0$$

$$\sum u_i^2 = 14.0908$$

2.3



2.4) If $x_1 = 18$, $\hat{Y}_1 = -8.81 + 0.8955(18)$

$\hat{Y}_1 = 7.309$ #

2.5) $$\text{Var}(\hat{\beta}_1) = \frac{\sigma_w^2 \sum x_i}{n\sum x_i^2} = \frac{1.7614 \cdot 200}{10(440)} = \frac{352.28}{4400} = 0.0801 \text{ \#}$$

$$\text{Var}(\hat{\beta}_2) = \sigma_w^2 / \sum x_i^2 = 1.7614/440 = 0.0040 \text{ \#}$$

$$\sigma_w^2 = \frac{\sum u_i^2}{n-2} = \frac{14.0908}{8} = 1.7614 \text{ \#}$$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$\hat{\beta}_1 = \frac{\Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2} = \bar{y} - \hat{\beta}_2 \bar{x}$$

$\hat{\beta}_1$ is an unbiased estimator

$$E(\hat{\beta}_1) = E(\bar{y} - \hat{\beta}_2 \bar{x})$$

$$= E(\bar{y}) - \hat{\beta}_2 E(\bar{x})$$

$$= \beta_1 + \cancel{\beta_2 \bar{x}} - \cancel{\bar{x} \beta_2}$$

$$E(\hat{\beta}_1) = \beta_1$$