

Quiz 5

1. Find

$$\int \frac{\ln(2x)}{x^2} dx.$$

Solution: Consider $\int \frac{\ln(2x)}{x^2} dx = \int x^{-2} \ln(2x) dx$. Integration by parts:

- Let $u = \ln(2x)$ and $dv = x^{-2} dx$.
- Then $du = \frac{2}{2x} dx = \frac{1}{x} dx$ and
- $v = \int x^{-2} dx = \frac{x^{-1}}{-1} + C$. By setting $C = 0 \Rightarrow v = -\frac{1}{x}$.

$$\begin{aligned} \int \frac{\ln(2x)}{x^2} dx &= \int x^{-2} \ln(2x) dx = uv - \int v du \\ &= \ln(2x) \left[-\frac{1}{x} \right] - \int \left[-\frac{1}{x} \right] \frac{1}{x} dx \\ &= -\frac{\ln(2x)}{x} + \int x^{-2} dx \\ &= -\frac{\ln(2x)}{x} + \left[\frac{x^{-1}}{-1} \right] + C \\ &= -\frac{\ln(2x)}{x} - \frac{1}{x} + C \end{aligned}$$

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