

EE320 (1/2017)

INTRODUCTORY MATHEMATICAL ECONOMICS

MATHEMATICS AND ECONOMIC RELATIONS

Recap

- Last time, we started talking about the structure of a mathematical economic model, which generally consists of a **systems of equations**.
- Also, to analyze an economic problem, we need to first determine what are **dependent** or **independent variables** to be included in the model.
- So, today's lecture will start with a review of **set theory**, the concepts of **relation and functions**, and discuss **possible forms of functions** that can be used to characterize the *behavioral* equations.

Topics

- Review of set theory
- Relations and functions
 - Ordered pairs and Cartesian products
 - Relations and Functions
 - Inverse Function
- Types of functions
 - Constant functions
 - Polynomial functions
 - Rational functions
 - Non-algebraic functions
 - Functions of two or more independent variables

Set Notation

- A *set* is a collection of distinct objects (e.g. numbers, persons, food items, etc.).
- The objects in a set are called the *elements* of the set.
- Two ways of writing a set:
 - By enumeration:
 $A = (7, 11, 13, 17, 19)$
 $S = \{\text{apple, blackberry, computer}\}$
 - By description:
 $I = \{x \mid x \text{ is a positive integer}\}$
 $J = \{x \mid 5 < x < 20\}$

Set Notation (cont'd)

- A *finite* set: A set with a finite number of elements
- An *infinite* set: A set with infinite number of elements

Examples:

$$B = \{3, 6, 9\}$$

$$C = \{x \mid 10 < x < 29\}$$

$$D = \{x \mid x \text{ is a rational number}\}$$

Questions: Which of the sets above are finite or infinite?

→ B is a finite set. C and D are infinite sets.

- \in indicates membership in a set:

E.g. $9 \in B$ $9 \notin C$

Relationships between Sets

- **Set equality**
 - e.g. If $A = \{2, 5, r, t\}$ and $B = \{2, r, 5, t\}$, then $A = B$.
- **Subset:** $T \subset S$ iff $x \in T$ implies $x \in S$.
 - e.g. If $S = \{2, 3, 4, 6\}$ and $T = \{3, 4\}$, then $T \subset S$.
 - A **proper subset** of S is any subset that does not contain *all* the elements of S .
 - Every set is a subset of itself.
- **Null set or empty set:** $\{ \}$ or Φ .
 - Every set contains the null set.
- **Disjoint sets:**
 - e.g. Sets A and T are disjoint.

Operations on Sets

- **Intersection:** $C = A \cap B$

$$C = \{a \mid a \in A \wedge a \in B\}$$

- **Union:** $C = A \cup B$

$$C = \{a \mid a \in A \vee a \in B\}$$

- **Complement of a set:** \tilde{A}

$$\tilde{A} = \{x \mid x \in U \wedge x \notin A\}$$

- **Example:** Let $U = \{1,2,3,4,5,6,7,8\}$, $A = \{3,4,5,6\}$, and $B = \{5,6,7,8\}$.

$$A \cap B = \{5,6\}$$

$$A \cup B = \{3,4,5,6,7,8\}$$

$$\tilde{A} = \{1,2,7,8\}$$

Laws of Set Operations

- Commutative law:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

- Associative law:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive law:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Ordered Pairs and Cartesian Products

- Ordered pairs
 - Unordered pair : $\{a, b\} = \{b, a\} \rightarrow$ Ordering does not matter.
 - *Ordered pair*: The ordering of a and b matters.
 $(a, b) \neq (b, a)$, unless $a = b$.
- Example: $(h, w) =$ (height in cm., weight in kg.) of students in class
e.g. $(h, w) = (165, 54)$. Would writing $(54, 165)$ make any senses?
- Given two sets x and y , the *Cartesian product* is the set of all the possible ordered pairs with the first element taken from set x and the second element taken from set y :

$$x \times y = \{(a, b) \mid a \in x \wedge b \in y\}$$

Relations and Functions

- Any subset of the Cartesian product $x \times y$ will constitute a *relation* between y and x . Given an x value, one or more y values will be specified by that relation.
- Example: $\{(x, y) \mid y = 2x\}$ and $\{(x, y) \mid y \geq x\}$

Relations and Functions (cont'd)

- A function is a special case of a relation, in which for each x value there exists only *one* corresponding y value.
- Thus, a *function* is a set of ordered pairs with the property that any x value uniquely determines a y value.

$$y = f(x)$$

where x is the *argument* of the function (i.e. *independent* variable) and y is the *value* of the function (i.e. *dependent* variable).

- Alternative names of a function: a *mapping* or *transformation*:

$$f: x \rightarrow y$$

- **Domain**: The set of all permissible values that x can take.
- **Range**: The set of all images (y value into which an x value is mapped)

Example: Domain and Range

- The total cost C of a firm is a function of its daily output Q :
 $C = 300 + 4Q$.
- Suppose that there is a capacity limit of 200 units of output per day.
- What are the domain and range of the cost function?

Domain =

Range:

→ Range =

Inverse Function

- Let $y = f(x)$ be an invertible function, and f maps X to Y .
→ The **inverse** function $f^{-1}(x) = g(y)$ maps Y to X .
- Example: Given $y = f(x) = 3x + 2$, what is f^{-1} ?
→ The inverse function:
- Example:
Suppose a demand function is given by $Q_d = 100 - 0.2P$. Derive the **inverse demand function**.
→

Constant Function

- A **constant function** is a function whose range consists of only one element:

$$y = f(x) = y_0, \quad \text{where } y_0 \text{ is a constant.}$$

- Example: In a cost function of a firm, the total cost (TC) includes only a fixed cost (F), say \$50. Thus, the cost function can be written as: $TC = F = 50$.
- Graph

Polynomial Functions

- General form of a polynomial function of a variable x :

$$y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where $a_0 \dots a_m$ are the coefficients and the value of n is the degree of the polynomial function.

- Subclass of polynomial functions:

- $n=0$: $y = a_0$ [constant function]
- $n=1$: $y = a_0 + a_1x$ [linear function]
- $n=2$: $y = a_0 + a_1x + a_2x^2$ [quadratic function]
- $n=3$: $y = a_0 + a_1x + a_2x^2 + a_3x^3$ [cubic function]

Linear Function: $y = a_0 + a_1x$

- Example: $TC = FC + VC = a + bQ$

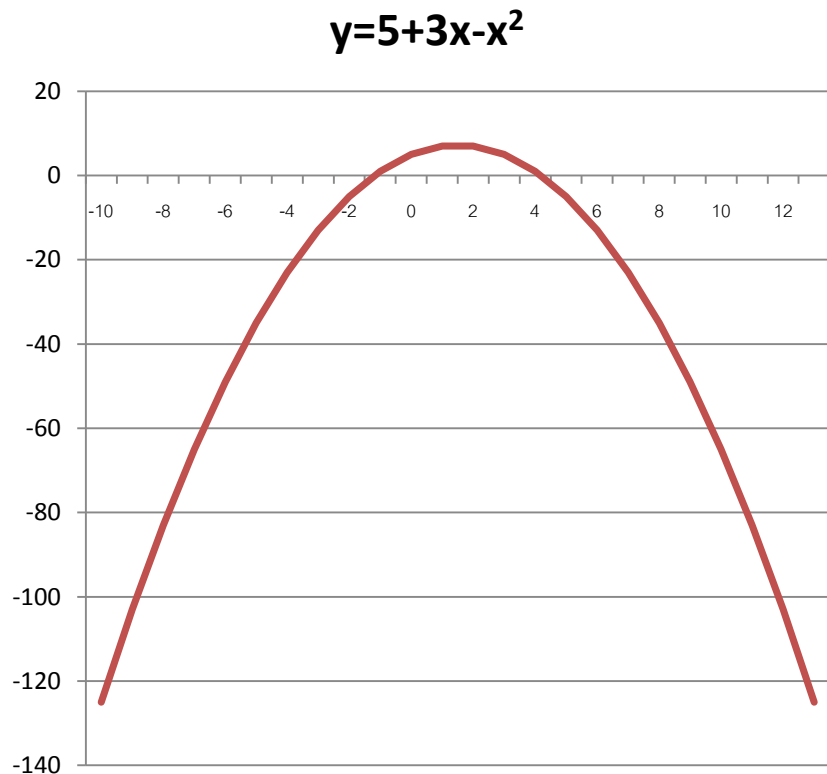
Example: A Demand Function

- Given a demand function $Q = 10 - P$, derive an inverse demand function and draw the graph.

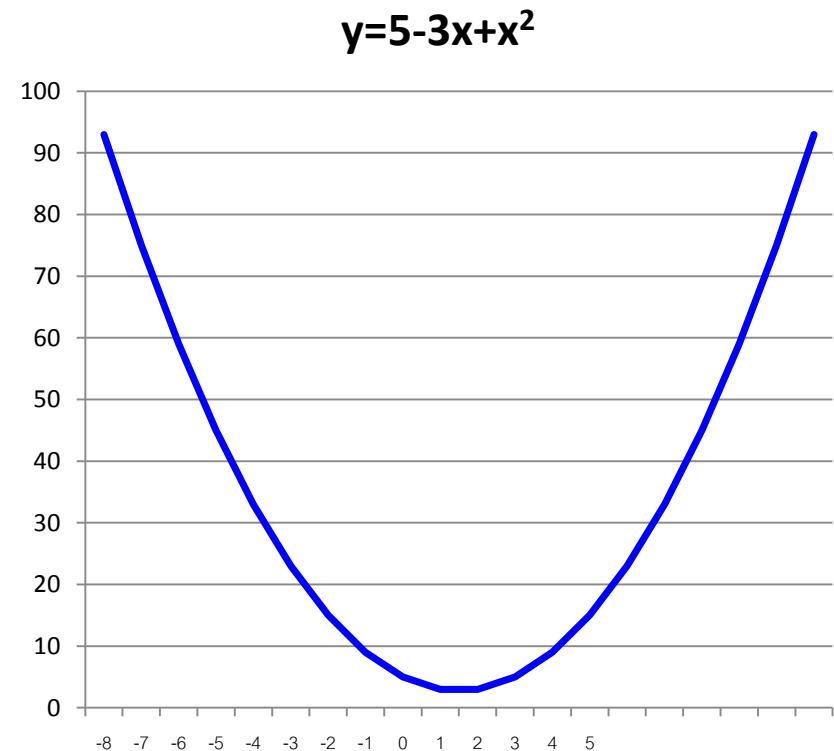
A Side Note on Price Elasticity of Demand

Quadratic Function: $y = a_0 + a_1x + a_2x^2$

- Case of $a_2 < 0$



- Case of $a_2 > 0$



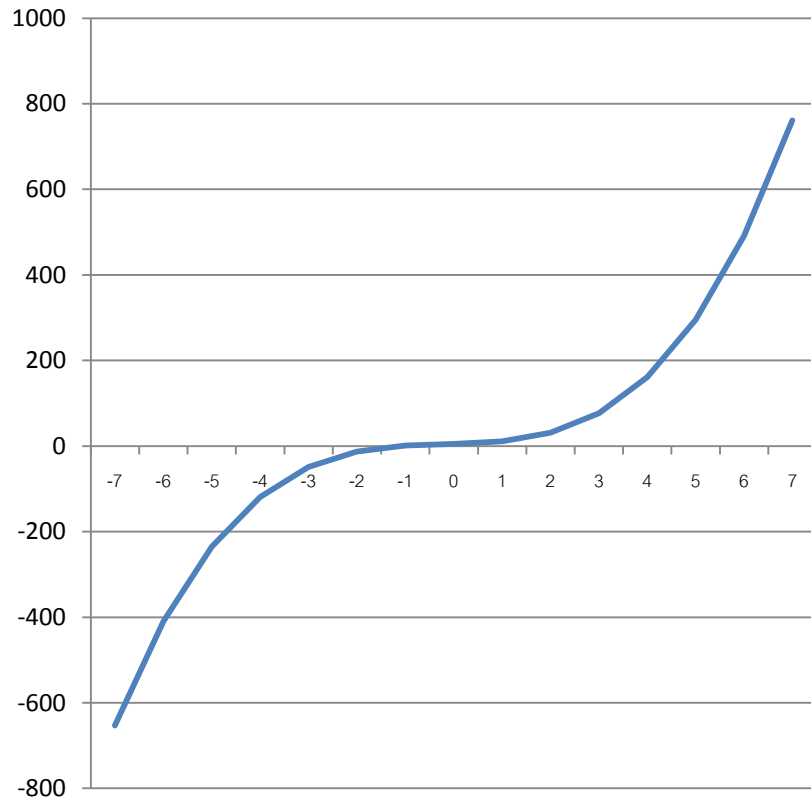
Example: A Total Revenue Function

- Given a demand function $Q = 10 - P$, derive a total revenue function and draw the graph.

Cubic Function: $y = a_0 + a_1x + a_2x^2 + a_3x^3$

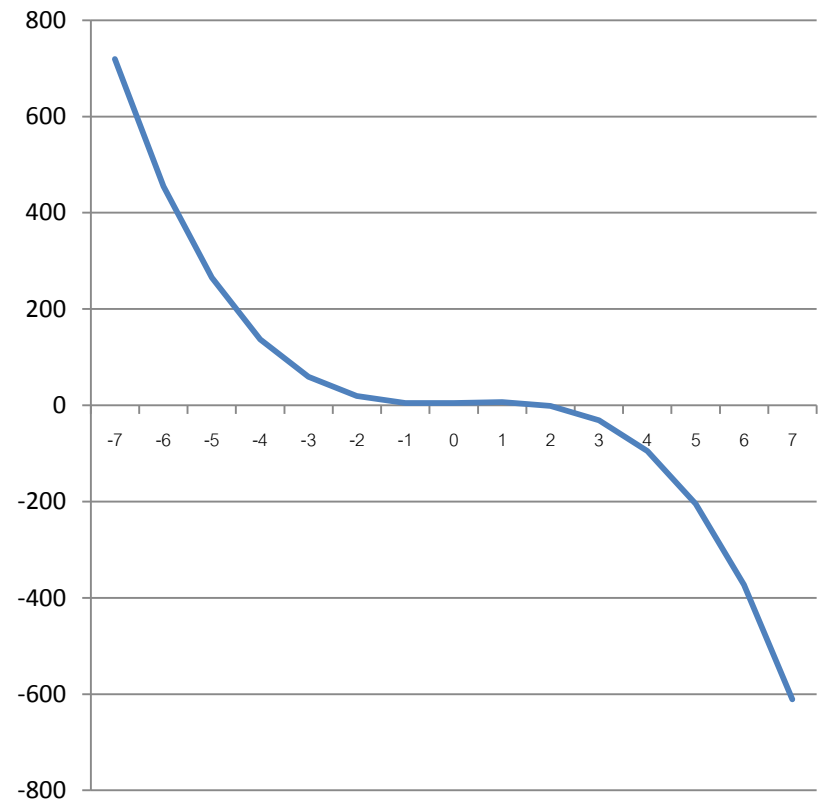
- Case of $a_3 > 0$

$$y = 5 + 3x + x^2 + 2x^3$$



- Case of $a_3 < 0$

$$y = 5 + 3x + x^2 - 2x^3$$



Rational Functions

- A *rational function* is a function expressed as a ratio of two polynomials in the variable x : $y = f(x)/g(x)$, where $g(x) \neq 0$.

- Special case: A *rectangular hyperbola*:

$$y = a/x \quad \text{or} \quad xy = a.$$

- Applications:

- A demand curve $Q = f(P)$ with constant total expenditure PQ , where the price elasticity is unitary.
- An average fixed cost (AFC) curve: $AFC = f(Q) = c/Q$, where c is a constant. Here, total fixed cost is a constant because $TFC = c$.

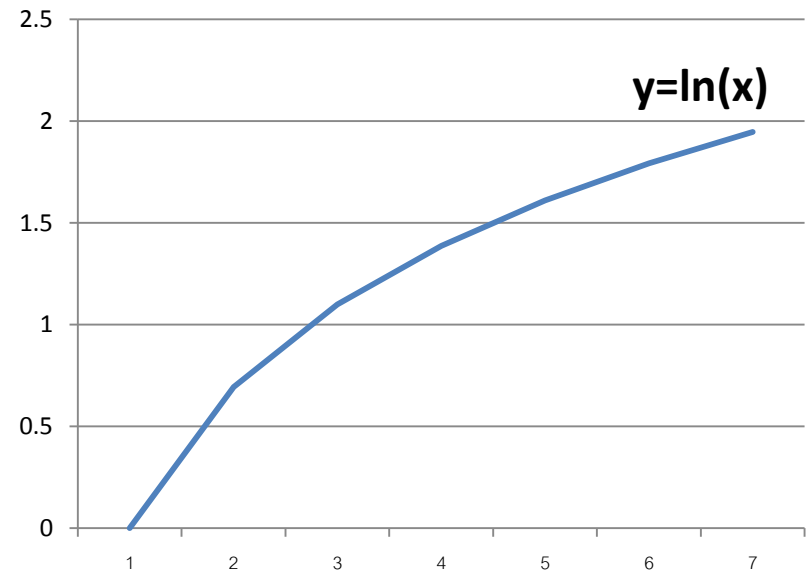
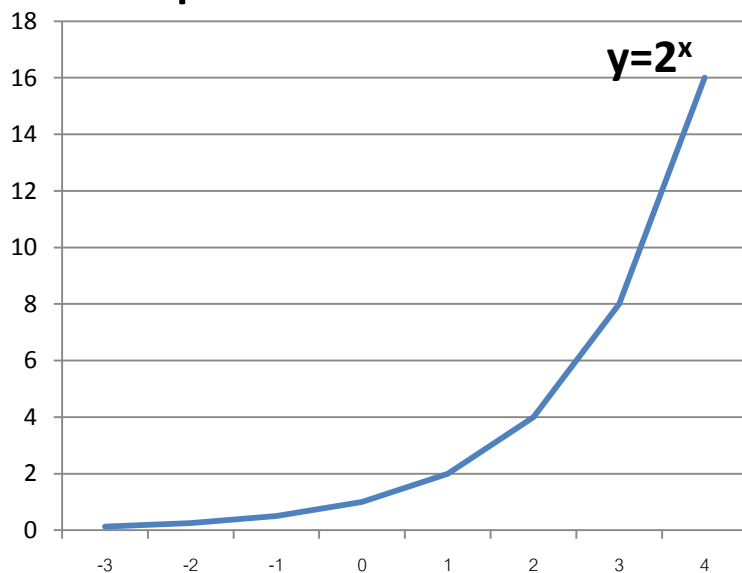
Example: Rectangular hyperbola

- Suppose $TFC = 50$.

Nonalgebraic Function

- A *nonalgebraic* function is a function that *cannot* be expressed in terms of polynomials and/or roots of polynomials.
 - **Exponential function:** $y = b^x$
 - **Logarithm function:** $y = \log_b x$

- **Graphs**



Example: Exponential Function

Functions of Multiple Independent Variables

- Given two independent variables x and y , a function of uniquely determined (dependent) variable z is:

$$z = g(x, y),$$

where the domain of the function will be some subset of the points in the xy plane.

- The association between the three variables are summarized by the ordered triple (x, y, z) .
- Examples:
 - Production function:
 - Utility function:

Example: Production function in a 3-dimension space