Q1) a) from $Q = f(K,L) = A(K^n + L^n)$

increase $K, L \rightarrow aK, aL$

$Q^n = A((aK)^n + (aL)^n)$

$Q^n = a^n (A(K^n + L^n))$

$Q^n = a^n Q^0$

$\therefore$ it will has a decreasing return to scale when $Q^n < Q^0$

, so $n < 1$, because a^n will decrease, then $Q^n < Q^0$

b) $MP_K : \frac{\partial Q}{\partial K} = AnK^{n-1}$

$\frac{\partial MP_K}{\partial K} = (n-1)AnK^{n-2}$

$n<1$ (from a) $\therefore (n-1)AnK^{n-2} < 0$

$\therefore$ from law of diminishing, K will go up and MP_K goes down. #

$MP_L : \frac{\partial Q}{\partial L} = AnL^{n-1}$

$\frac{\partial MP_L}{\partial L} = (n-1)AnL^{n-2}$

$n<1$ (from a) $\therefore (n-1)AnL^{n-2} < 0$

$\therefore$ from law of diminishing, K will go up and MP_K goes down. #

c) from $Q = A(K^n + L^n)$

$Q = AK^n + L^n$

$MPL = \frac{\partial Q}{\partial L} = nAL^{n-1}$

$MPK = \frac{\partial Q}{\partial L} = nAK^{n-1}$

$MRTS = \frac{MPL}{MPK}$

$MRTS = \frac{nAL^{n-1}}{nAK^{n-1}} = \left(\frac{L}{K}\right)^{n-1}$ #

d) $MRTS = \left(\frac{L}{K}\right)^{n-1}$

$\frac{\partial MRTS}{\partial L} = (n-1)\left(\frac{L^{n-2}}{K^{n-1}}\right)$

if $n<1$ from a) $\longrightarrow n-1 = \ominus$

$\frac{\partial MRTS}{\partial L} \Rightarrow$ if $L\uparrow \rightarrow MRTS\downarrow$

$L\downarrow \rightarrow MRTS\uparrow$

e) K; $Q = AK^n + AL^n$

$\frac{\partial Q}{\partial K} = nAK^{n-1}$

$\frac{\partial^2 Q}{\partial K^2} = n(n-1)AK^{n-2}$

from a) $n<1$, $n-1<0$

$\because$ the second order condition makes K negative

, so it is concave

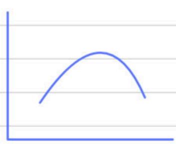
L; $Q = AK^n + AL^n$

$\frac{\partial Q}{\partial L} = nAL^{n-1}$

$\frac{\partial^2 Q}{\partial L^2} = n(n-1)AL^{n-2}$

$\ominus$

$\therefore$ it is concave



f) $Q^n = A((aK)^n + (aL)^n)$

$$\frac{dQ}{dt} = Ank^{n-1} \cdot dk + AnL^{n-1} \cdot dL$$

$$\frac{dk}{dt} = t+2$$

$$\frac{dL}{dt} = te^{t-1}$$

$$\frac{dQ}{dt} = Ank^{n-1}(t+2) + AnL^{n-1}(te^{t-1})$$

$\therefore$ Q is increasing.

g) $\left.\frac{dQ}{dt}\right|_{t=0}$; $k(t) = \cancel{\frac{1}{2}(0)^2} + \cancel{2(0)} + 3$ #

$= 3$

$L(t) = \cancel{e^0}^{1} + 3 = 4$ #

$\left.\frac{dQ}{dt}\right|_{t=0}$; $Ank^{n-1}(t+2) + AnL^{n-1}(te^{t-1})$; from f)

$Ank^{n-1}(2) + \cancel{AnL^{n-1}}(0)$

$k(t) = 3$; $2An(3)^{n-1}$

#

Q.2

b) Profit = TR - TC

find $TR = P \cdot Q$

$= (Q^{-\frac{1}{4}}) Q^{1}$

$= Q^{3/4}$ จาก $Q = K^{\frac{1}{3}} L^{\frac{1}{3}}$

$= \left[K^{\frac{1}{3}} L^{\frac{2}{3}} \right]^{\frac{3}{4}}$

$= K^{\frac{1}{4}} L^{2/4}$

find TC

Price of labor $= w$
Price of capital $= r$

$\left.\begin{array}{l}\text{Price of labor} = w \\ \text{Price of capital} = r\end{array}\right\} TC = wL + rK$

Profit $= K^{1/4} L^{2/4} - wL - rK$

c) Profit $= K^{\frac{1}{4}} L^{2/4} - WL - rK$

FOC; $\pi_K = \frac{1}{4} K^{-3/4} L^{2/4} - r = 0$

$$\frac{1}{4} K^{-3/4} L^{2/4} = r \quad — (1)$$

$\pi_L = \frac{2}{4} K^{\frac{1}{4}} L^{-\frac{2}{4}} - W = 0$

$$\frac{2}{4} K^{\frac{1}{4}} L^{-\frac{2}{4}} = W \quad — (2)$$

$\frac{(1)}{(2)}$; $$\frac{\frac{1}{4} K^{-\frac{3}{4}} L^{2/4}}{\frac{2}{4} K^{\frac{1}{4}} L^{-\frac{2}{4}}} = \frac{r}{W}$$

$$\frac{1}{2} \cdot K^{-\frac{3}{4}-\frac{1}{4}} L^{\frac{2}{4}-(-\frac{2}{4})} = \frac{r}{W}$$

$$\frac{1}{2} K^{-1} L = \frac{r}{W}$$

$$\frac{1}{2} \frac{L}{K} = \frac{r}{W}$$

$$L = \frac{2rK}{W}$$

Solution (2) $\frac{2}{4} \frac{K^{\frac{1}{4}}}{L^{2/4}} = W$

$$L^{2/4} = \frac{2}{4W} \cdot K^{\frac{1}{4}} \longrightarrow (1)$$

$$\frac{1}{4} K^{-3/4} \left(\frac{2}{4W} K^{\frac{1}{4}}\right) = r$$

$$\frac{2}{8W} K^{-2/4} = r$$

$$\frac{1}{4W} \cdot K^{-\frac{1}{2}} = r$$

$$K^{-\frac{1}{2}} = 4rW$$

$$\frac{1}{\sqrt{K}} = 4rW$$

$$\sqrt{K} = \frac{1}{4rW}$$

$$K = \frac{1}{16r^2W^2}$$

$$L^{\frac{2}{4}} = \frac{2}{4w} \cdot K^{\frac{1}{4}}$$

$$K = \frac{1}{16r^2w^2}$$

$$L^{\frac{1}{2}} = \frac{1}{2w}\left[\frac{1}{16r^2w^2}\right]^{\frac{1}{4}}$$

$$L = \frac{1}{4w^2}\left[\frac{1}{16r^2w^2}\right]^{\frac{2}{4}}$$

$$\therefore \quad K = \frac{1}{16r^2w^2} \qquad L = \frac{1}{4w^2}\left[\frac{1}{16r^2w^2}\right]^{\frac{2}{4}}$$

e) $\pi_K = \frac{1}{4} K^{-3/4} L^{2/4} - r$

$\pi_L = \frac{2}{4} K^{1/4} L^{-2/4} - w$

$$H = \begin{bmatrix} \pi_{KK} & \pi_{KL} \\ \pi_{LK} & \pi_{LL} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{4}\right)\left(-\frac{3}{4}\right) K^{-7/4} L^{2/4} & \left(\frac{1}{4}\right)\left(\frac{2}{4}\right) K^{-\frac{3}{4}} L^{-\frac{2}{4}} \\ \left(\frac{1}{4}\right)\left(\frac{2}{4}\right) K^{-\frac{3}{4}} L^{-\frac{2}{4}} & \left(\frac{2}{4}\right)\left(-\frac{2}{4}\right) K^{\frac{1}{4}} L^{-\frac{6}{4}} \end{bmatrix}$$

$\pi_{KK} = \left(\frac{1}{4}\right)\left(-\frac{3}{4}\right) K^{-7/4} L^{2/4}$

$\pi_{KL} = \left(\frac{1}{4}\right)\left(\frac{2}{4}\right) K^{-\frac{3}{4}} L^{-\frac{2}{4}}$

$\pi_{LK} = \left(\frac{1}{4}\right)\left(\frac{2}{4}\right) K^{-\frac{3}{4}} L^{-\frac{2}{4}}$

$\pi_{LL} = \left(\frac{2}{4}\right)\left(-\frac{2}{4}\right) K^{\frac{1}{4}} L^{-\frac{6}{4}}$

det

$$H_1 = \left| \left(\frac{1}{4}\right)\left(-\frac{3}{4}\right) K^{-\frac{7}{4}} L^{\frac{2}{4}} \right| < 0$$

$$\left| \left(\frac{2}{4}\right)\left(-\frac{2}{4}\right) K^{\frac{1}{4}} L^{-\frac{6}{4}} \right|$$

$$H = \begin{bmatrix} \left(\frac{1}{4}\right)\left(-\frac{3}{4}\right) K^{-7/4} L^{2/4} & \left(\frac{1}{4}\right)\left(\frac{2}{4}\right) K^{-\frac{3}{4}} L^{-\frac{2}{4}} \\ \left(\frac{1}{4}\right)\left(\frac{2}{4}\right) K^{-\frac{3}{4}} L^{-\frac{2}{4}} & \left(\frac{2}{4}\right)\left(-\frac{2}{4}\right) K^{\frac{1}{4}} L^{-\frac{6}{4}} \end{bmatrix}$$

$$H_2 = \left[\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right) K^{-7/4} L^{2/4}\right]\left[\left(\frac{2}{4}\right)\left(-\frac{2}{4}\right) K^{\frac{1}{4}} L^{-\frac{6}{4}}\right] - \left[\left(\frac{1}{4}\right)\left(\frac{2}{4}\right) K^{-3/4} L^{-\frac{2}{4}}\right]^2 > 0$$

$\therefore$ L and K Max

Q3

a) Cournot environment

$\text{Profit}_1 = P \cdot q_1 - C_1$

$= (100 - (q_1 + q_2))\, q_1 - 10q_1$

$= 100q_1 - q_1^2 - q_1 q_2 - 10q_1$

$\pi_1 = 90q_1 - q_1^2 - q_1 q_2$

Max π $\Rightarrow$ $\pi_1 = 2513.8$

$$\frac{d\pi}{dq_1} = 90 - 2q_1 - q_2 = 0$$

$q_1 = 45 - 0.5q_2$

$\text{Profit}_2 = P \cdot q_2 - C_2$

$= (100 - (q_1 + q_2))\, q_2 - q_2^2$

$= 100q_2 - q_1 q_2 - q_2^2 - q_2^2$

$\pi_2 = 100q_2 - q_1 q_2 - 2q_2^2$

$$\frac{\partial \pi_2}{\partial q_2} = 100 - q_1 - 4q_2 = 0$$

$\Rightarrow \pi_2 = 492.98$

$q_1 = 100 - 4q_2$

$\therefore$ $100 - 4q_2 = 45 - 0.5q_2$

$55 = 3.5q_2$

$q_2 = 15.7$

$q_1 = 37.2$

Total Quantity $= 15.7 + 37.2 = 52.9$

Price $= 100 - 52.9 = 47.1$

b) Total $\pi = \pi_1 + \pi_2$

$= p.Q - C_1 - C_2$

$= (100 - q_1 - q_2)(q_1 + q_2) - 10q_1 - q_2^2$

$= 100q_1 - q_1^2 - q_1q_2 + 100q_2 - q_1q_2 - q_2^2 - 10q_1 - q_2^2$

$= 90q_1 - 2q_1q_2 - 2q_2^2 - q_1^2 + 100q_2$

$\frac{d\pi}{dq_1} = 90 - 2q_2 - 2q_1 = 0$

$q_1 = 45 - q_2$ (1)

$\frac{d\pi}{dq_2} = -2q_1 - 4q_2 + 100 = 0$

$q_1 = 50 - 2q_2$ (2)

$\therefore$ $45 - q_2 = 50 - 2q_2$

$q_2 = 5$

$q_1 = 40$

$Q = 45$

$p = 55$

$\pi_1 = p.q_1 - C_1 = 1800$

$\pi_2 = p.q_2 - C_2 = 250$

c)

$\pi_1 = 2513.8$
$\pi_2 = 482.98$ } cournot

$\pi_1 = 1400$
$\pi_2 = 250$ } collusion

$\therefore$

As we can see π_1 (collusion) $<$ π_1 (cornot)

Firm 1 is not collusion (deviate)

π_2 (collusion) $<$ π_2 (cornot)

Firm 2 is not collusion (deviate)

Hence, collusion is not stable.

Q4

a) Market equilibrium A

Demand = Supply

$10 - 2Q_A = 1 + Q_A$

$3Q_A = 9$

$Q_A = 3 \quad , \quad P_A = 4$

Market equilibrium B

Demand = Supply

$20 - Q_B = 2 + 2Q_B$

$Q_B = 6$

$P_B = 14$

B) $t = p^D - p^S$

Market A

$t_A = [10 - 2Q_A] - [1 + Q_A]$

$t_A = 9 - 3Q_A$

$Q_A = \frac{9 - t_a}{3} \qquad P_A = \frac{1 + 9 - t_a}{3}$

Market B

$t_B = [20 - Q_B] - (2 + 2Q_B)$

$= 18 - 3Q_B$

$Q_B = \frac{18 - t_B}{3}$

$P_B = 2 + 2\frac{(18 - t_B)}{3}$

C) tax revenue $= t_A + t_B$

$\text{tax } R = t_A Q_A + t_B Q_B$

$\text{tax } R = t_a\left(\frac{9 - t_a}{3}\right) + tb\left(\frac{18 - tb}{3}\right)$

d) Max

$$\text{Tax } R = 3t_a - \frac{t_a^2}{3} + 6t_b - \frac{t_b^2}{3}$$

$$\frac{d\,\text{tax}R}{dt_a} = 3 - \frac{2}{3}t_a = 0$$

$$t_a = 4.5$$

$$\frac{d\,\text{tax}R}{dt_b} = 6 - \frac{2}{3}t_b = 0$$

$$t_b = 9$$

$\therefore$ tax $A = 4.5$

tax $B = 9$

Kritsanawit Boonkerd 6304640532

Date: No.:

Q5 Demand Equation $Q = 2000 + 4\sqrt{A} - 20P$

When A = dollar amount of expenditure on advertisement

P = unit price

Q = Quantity demanded

Advertising can boost the total demand in the market as potential customers get more information

Monopolist cost function is given by $C(Q,A) = C(Q) + A$ where $C(Q) = 2Q + 1000$

Consider the following problem.

a). Construct the profit function

$$\pi = TR - TC$$

When demand fn. $Q = 2000 + 4\sqrt{A} - 20P$

$$P = \frac{2000}{20} + \frac{4\sqrt{A}}{20} - \frac{Q}{20}$$

$$P = 100 + \frac{\sqrt{A}}{5} - \frac{Q}{20}$$

$$\pi = \left(100 + \frac{\sqrt{A}}{5} - \frac{Q}{20}\right)(Q) - (C(Q) + A)$$

$$= \left(100Q + \frac{\sqrt{A}}{5}Q - \frac{Q^2}{20}\right) - (2Q + 1000 + A)$$

$$\pi = 100Q + \frac{\sqrt{A}}{5}Q - \frac{Q^2}{20} - 2Q - 1000 - A$$

$$\pi = \frac{-Q^2}{20} + \left(100 + \frac{\sqrt{A}}{5} - 2\right)Q - (1000 + A)$$

$$\pi = \frac{-Q^2}{20} + \left(98 + \frac{\sqrt{A}}{5}\right)Q - (1000 + A)$$

b) Determine the optimal pricing P^*, optimal advertising A^* that maximizing profit

$$\frac{\partial \pi}{\partial Q} = \frac{-2Q}{20} + \left(98 + \frac{\sqrt{A}}{5}\right)$$

$$0 = \frac{-Q}{10} + 98 + \frac{\sqrt{A}}{5}$$

$$Q = 10\left(98 + \frac{\sqrt{A}}{5}\right) \quad - (1)$$

$$\frac{\partial \pi}{\partial A} = \frac{Q}{10\sqrt{A}} - 1$$

$$0 = \frac{Q}{10\sqrt{A}} - 1$$

Date: No.:

$$10\sqrt{A} = Q$$
$$Q = 10\sqrt{A} \quad — ②$$

① = ② ; $$10\left(98 + \frac{\sqrt{A}}{5}\right) = 10\sqrt{A}$$
$$980 + 2\sqrt{A} = 10\sqrt{A}$$
$$980 = 8\sqrt{A}$$
$$A^* = 15,006.25$$
$$Q^* = 1225$$
$$P^* = 100 + \frac{\sqrt{A}}{5} - \frac{Q}{20}$$
$$= 100 + \frac{\sqrt{15,006.25}}{5} - \frac{1225}{20}$$
$$P^* = 63.25$$

c. confirm with second-order differential test

1 st derivative $$\Pi = \frac{-Q^2}{20} + \left(98 + \frac{\sqrt{A}}{5}\right)Q - (1000 + A)$$
$$\frac{d\Pi}{dQ} = \frac{-Q}{10} + \left(98 + \frac{\sqrt{A}}{5}\right)$$

2nd derivative $$\frac{d^2\Pi}{dQ^2} = -\frac{1}{10} < 0$$

so, It is maximize profit at Q^*

Hessian - matrix

$$Q - 2\sqrt{A} = 980 \quad — ①$$
$$Q - 10\sqrt{A} = 0 \quad — ②$$

$$\begin{bmatrix} 1 & -2 \\ 1 & -10 \end{bmatrix} \begin{bmatrix} Q \\ \sqrt{A} \end{bmatrix} = \begin{bmatrix} 980 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -2 \\ 1 & -10 \end{vmatrix} = (-10) - (-2) = -8$$

$$\begin{vmatrix} 980 & -2 \\ 0 & -10 \end{vmatrix} = (-9800) - 0 = -9800$$

$$Q^* = \frac{-9800}{-8} = 1225$$

$$\begin{vmatrix} 1 & 980 \\ 1 & 0 \end{vmatrix} = 0 - (980) = -980$$

$$\sqrt{A} = \frac{-980}{-8}$$
$$A^* = 15,006.25$$