FN312 Investment Lecture 4
Return and Risk from the Historical Record

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Outline

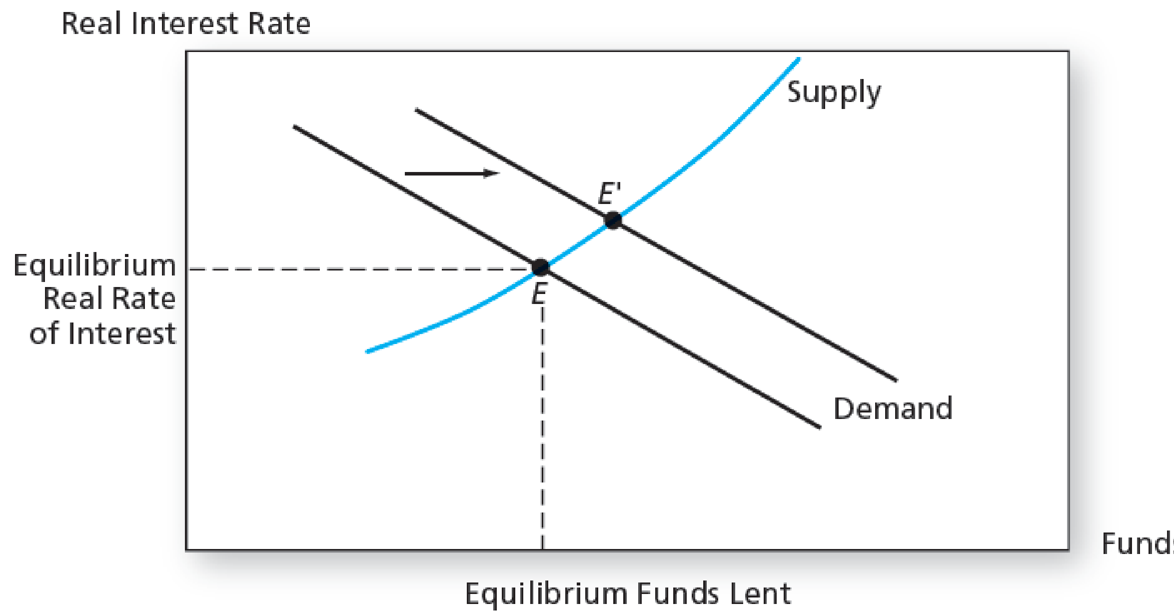
- Interest rate determinants
- Rates of return for different holding periods
- Risk, risk premiums, and estimation
- Normal distribution
  - Deviation from normality and risk estimation
- *Historic returns on risky portfolios**

Reading:
Chapter 5

Interest rate determinants

Interest Rate Determinants

- Supply
  - Households
- Demand
  - Businesses
- Government's net demand
  - Federal Reserve actions



Real versus Nominal Interest Rates

- ***Nominal* interest rate:** Growth rate of your money
- ***Real* interest rate:** Growth rate of your purchasing power

$$r_{nom} = \text{ Nominal Interest Rate}$$
$$r_{real} = \text{ Real Interest Rate}$$
$$i = \text{Inflation Rate}$$

$$r_{real} = \frac{r_{nom} - i}{1 + i}$$

$Note: \ Approximately$

=>

$Types \ of \ growth?$

- If $E(i)$ denotes current expectations of inflation, then we get the Fisher Equation:

Example: Real versus Nominal Interest Rates

Ex13: During a period of severe inflation, a bond offered a nominal HPR of 80% per year. The inflation rate was 70% per year.

a. What was the real HPR on the bond over the year?

b. Compare this real HPR to the approximation $r_{real} \approx r_{nom} - i$

Taxes and the Real Interest Rate

- Tax liabilities are based on nominal income

$$r_{nom} = \text{ Nominal Interest Rate}$$
$$r_{real} = \text{ Real Interest Rate}$$
$$i = \text{Inflation Rate}$$
$$t = \text{ Tax Rate}$$

- The after-tax real rate falls as the inflation rises

Taxes and the Real Interest Rate: Example

$$r_{nom} = 7\%$$

$$i = 3.5\%$$

$$t = 40\%$$

Rates of return for different holding periods

Rates of Return for Different Holding Periods

- Zero Coupon Bond:
  - Par = $100
  - Maturity = T
  - Price = P
  - Total risk free return

$$r_f(T) = \frac{100}{P(T)} - 1$$

Suppose prices of zero-coupon Treasuries with $100 face value and various maturities are as follows. We find the total return of each security by using Equation 5.6:

Horizon, T	Price, $P(T)$	$[100/P(T)] - 1$	Total Return for Given Horizon
Half-year	$97.36	100/97.36 – 1 = 0.0271	$r_f(0.5) = 2.71\%$
1 year	$95.52	100/95.52 – 1 = 0.0469	$r_f(1) = 4.69\%$
25 years	$23.30	100/23.30 – 1 = 3.2918	$r_f(25) = 329.18\%$

Effective Annual Rate (EAR) and Annual Percentage Rate (APR)

- Effective Annual Rate (EAR): Percentage increase in funds invested over a 1-year horizon

$$1 + \text{EAR} = \left[1 + r_f(T)\right]^{1/T}$$

- Annualized Percentage Rate (APR): Annualizing using simple interest

$$\text{APR} = \frac{(1 + \text{EAR})^T - 1}{T}$$

Example: APR versus EAR

18. Consider these long-term investment data:

- The price of a 10-year \$100 par value zero-coupon inflation-indexed bond is \$84.49.
- A real-estate property is expected to yield 2% per quarter (nominal) with a SD of the (effective) quarterly rate of 10%.

  a. Compute the annual rate of return on the real (i.e., inflation-indexed) bond.

  b. Compute the continuously compounded annual risk premium on the real-estate investment.

Example: APR versus EAR

Compounding Period	T	EAR = $[1 + r_f(T)]^{1/T} - 1 = .058$		APR = $r_f(T)*(1/T) = .058$	
		$r_f(T)$	APR = $[(1 + \text{EAR})^\wedge T - 1]/T$	$r_f(T)$	EAR = $(1 + \text{APR}*T)^\wedge(1/T) - 1$
1 year	1.0000	.0580	.05800	.0580	.05800
6 months	0.5000	.0286	.05718	.0290	.05884
1 quarter	0.2500	.0142	.05678	.0145	.05927
1 month	0.0833	.0047	.05651	.0048	.05957
1 week	0.0192	.0011	.05641	.0011	.05968
1 day	0.0027	.0002	.05638	.0002	.05971
Continuous			$r_{cc} = \ln(1 + \text{EAR}) = .05638$		EAR = $\exp(r_{cc}) - 1 = .05971$

Risk, risk premiums, and estimation

Risk and Risk Premiums

- Rates of return: Single period

$$\text{HPR} = \frac{E(P_1) - P_0 + E(D_1)}{P_0}$$

  - HPR = Holding period return
  - P_0 = Beginning price
  - $E(P_1)$ = Expected Ending price
  - $E(D_1)$ = Expected Dividend during period one

Rates of Return: Single Period Example

Expected Ending Price =	$110
Beginning Price =	$100
Expected Dividend =	$4

Expected Return and Standard Deviation

- Expected returns

$$E(r) = \sum_{s} p(s) \times r(s)$$

  - $p(s)$ = Probability of a state
  - $r(s)$ = Return if a state occurs
  - s = State

Scenario Returns: Example

State	Prob. of State	*r* in State
Excellent	.25	0.3100
Good	.45	0.1400
Poor	.25	-0.0675
Crash	.05	-0.5200

$$E(r) = (.25)\times(.31)+(.45)\times(.14)+(.25)\times(-.0675)+(0.05)\times(-0.52)$$

$$E(r) =$$

Expected Return and Standard Deviation

- Variance (VAR):

$$\sigma^2 = \sum_s p(s) \times \left[r(s) - E(r) \right]^2$$

- Standard Deviation (STD):

$$\text{STD} = \sqrt{\sigma^2}$$

Scenario VAR and STD: Example

- Example VAR calculation:

$$\sigma^2 = .25\times(.31 - 0.0976)^2 + .45\times(.14 - .0976)^2$$
$$+ .25\times(-0.0675 - 0.0976)^2 + .05\times(-.52 - .0976)^2$$
$$=$$

- Example STD calculation:

$$\sigma =$$

$$=$$

Use the following scenario analysis for Stocks X and Y to answer CFA Problems 3 through 6 (round to the nearest percent).

	Bear Market	Normal Market	Bull Market
Probability	0.2	0.5	0.3
Stock X	−20%	18%	50%
Stock Y	−15%	20%	10%

3. What are the expected rates of return for Stocks X and Y?
4. What are the standard deviations of returns on Stocks X and Y?
5. Assume that of your $10,000 portfolio, you invest $9,000 in Stock X and $1,000 in Stock Y. What is the expected return on your portfolio?
6. Probabilities for three states of the economy and probabilities for the returns on a particular stock in each state are shown in the table below.

State of Economy	Probability of Economic State	Stock Performance	Probability of Stock Performance in Given Economic State
Good	0.3	Good	0.6
		Neutral	0.3
		Poor	0.1
Neutral	0.5	Good	0.4
		Neutral	0.3
		Poor	0.3
Poor	0.2	Good	0.2
		Neutral	0.3
		Poor	0.5

What is the probability that the economy will be neutral *and* the stock will experience poor performance?

Time Series Analysis of Past Rates of Return

- True means and variances are unobservable
  - Possible scenarios like the one in the examples are unknown

- Means and variances must be estimated

Returns Using Arithmetic and Geometric Averaging

- Arithmetic Average: $$E(r) = \sum_{s=1}^{n} p(s) r(s) = \frac{1}{n} \sum_{s=1}^{n} r(s)$$

- Geometric (Time-Weighted) Average:
  - Terminal value of the investment: $TV_n = (1 + r_1)(1 + r_2)...(1 + r_n)$
  - Geometric Average: $g = TV_n^{1/n} - 1$

Estimating Variance and Standard Deviation

- Estimated Variance
    - Expected value of squared deviations

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{s=1}^{n}\left[r(s) - \bar{r}\right]^2$$

- Unbiased estimated standard deviation (?)

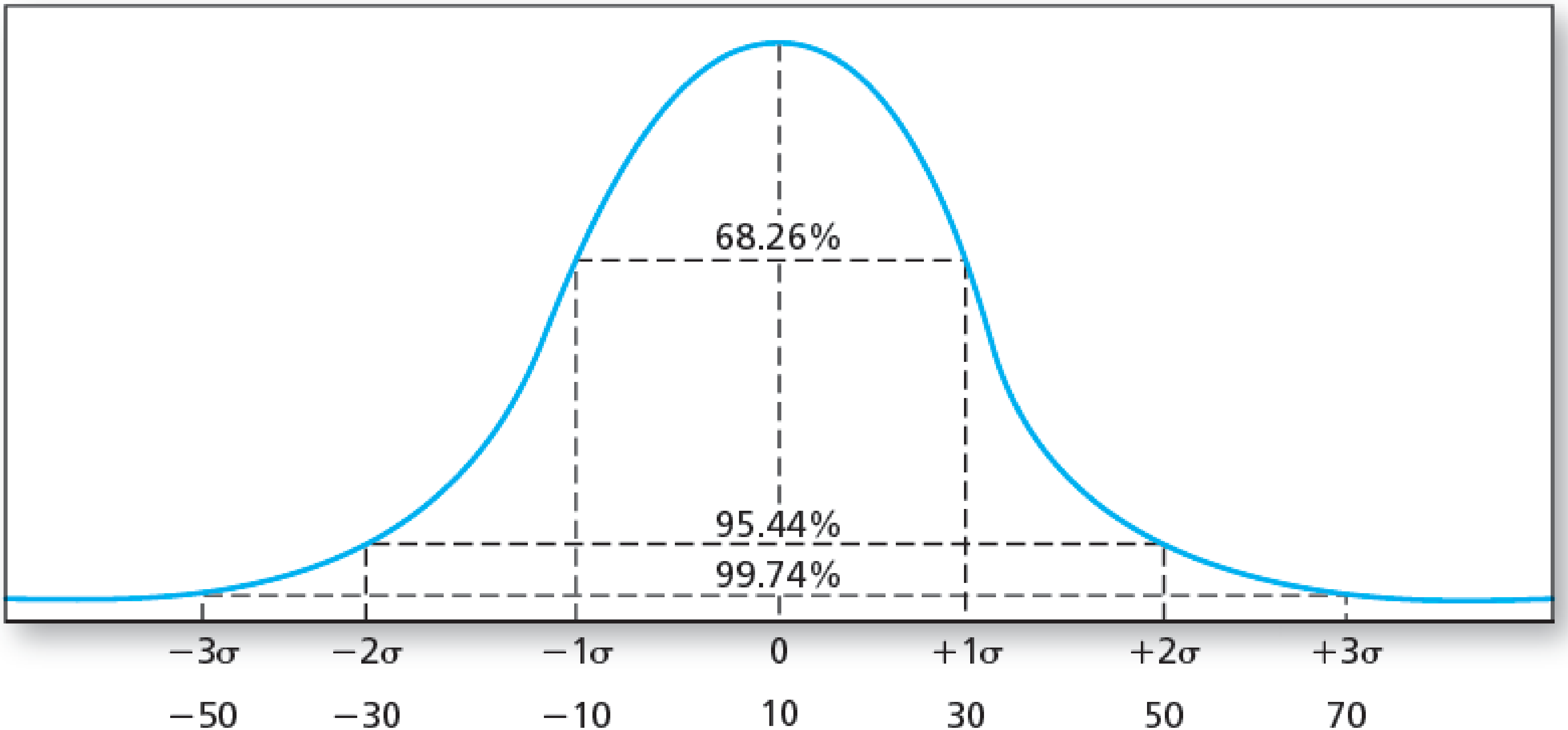
The Reward-to-Volatility (Sharpe) Ratio

- ***Excess Return:*** The difference in any particular period between the actual rate of return on a risky asset and the actual risk-free rate
- ***Risk Premium:*** The difference between the expected HPR on a risky asset and the risk-free rate

- *Sharpe Ratio* = $$\frac{\text{Risk premium}}{\text{SD of excess returns}}$$

Normal distribution

The Normal Distribution



Mean = 10%, SD = 20%

The Normal Distribution

- Investment management is easier with normal returns:
  - Symmetric Returns

    >> Standard deviation is a good measure of risk

    >> Portfolio returns will be as well

  - **Only mean and standard deviation** needed to estimate future scenarios
  - **Pairwise** correlation coefficients summarize the dependence of returns across securities

What if excess returns are not normally distributed?

- STD is no longer a complete measure of risk
  - ***Skew:*** Ratio of the average cubed deviations from the sample average, also called the third moment

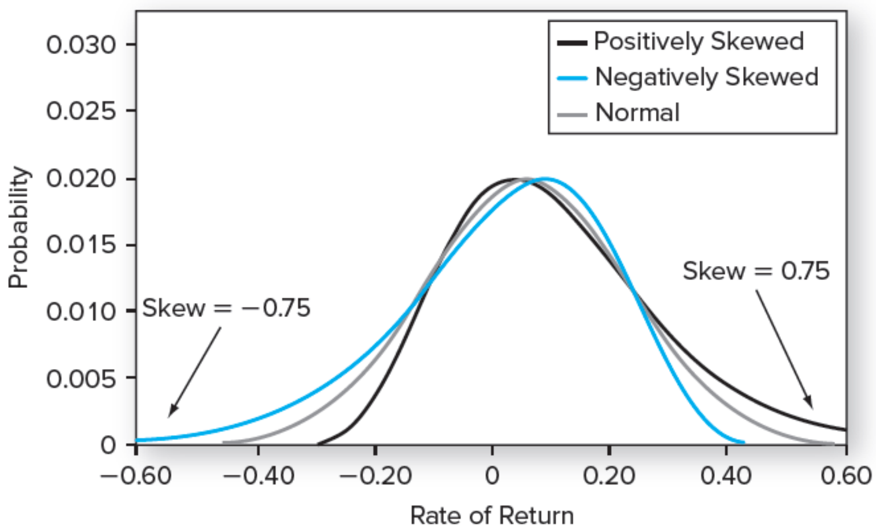
$$Skew = Average\left[\frac{(R-\bar{R})^3}{\hat{\sigma}^3}\right]$$

  - ***Kurtosis:*** The likelihood of extreme values on either side of the mean

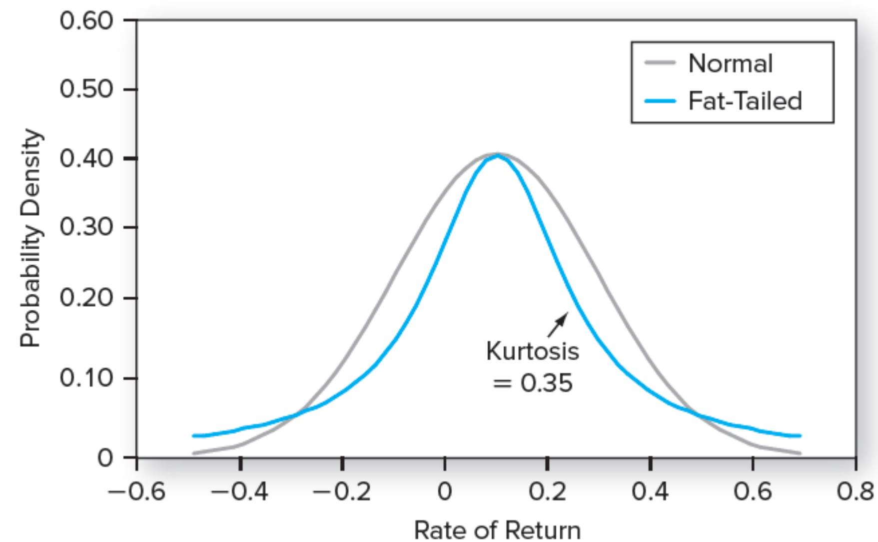
$$Kurtosis = Average\left[\frac{(R-\bar{R})^4}{\hat{\sigma}^4}\right]-3$$

- Sharpe ratio is not a complete measure of portfolio performance

Normal and Skewed Distributions



Normal and Fat-Tailed Distributions



Normality and Risk Measures

- *Value at Risk (VaR)*
  - Loss corresponding to a very low percentile of the entire return distribution, such as the fifth or first percentile return
- *Expected Shortfall (ES)*
  - Also called conditional tail expectation (CTE), focuses on the *expected* loss in the worst-case scenario (left tail of the distribution)
  - More conservative measure of downside risk than VaR

Question?