Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

Equation for the risk-free asset that pays R_f is

$$U'(C_0) = R_f \delta E[U'(C_1)]$$

Since $U(C) = ln(C)$

$$U'(C_1) = \frac{1}{C_1}$$

$$U'(C_0) = \frac{1}{C_0}$$

and, since $U(C) = C^{\gamma}/\gamma$, so

$$\frac{1}{R_f} = \delta E\left[\left(\frac{C_0}{C_1}\right)^{1-\gamma}\right]$$

If there is only risk-free asset, then the equation is

$$R_f = \frac{1}{\delta}\left(\frac{C_1}{C_0}\right)^{1-\gamma}$$

there is non random labor income and C_1 is nonstocastic.

∴ Therefore when it has a high risk-free rate in the economy, then we expect consumption level of consumer is high as well.

Score.....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

Note that $\frac{\partial R_f}{\partial \frac{C_1}{C_0}} = \frac{1-\gamma}{\delta}\left(\frac{C_1}{C_0}\right)^{-\gamma}$

$= \underline{(1-\gamma)}\frac{R_f}{\frac{C_1}{C_0}}$

coefficient relative risk aversion

So that the intertemporal elasticity of substitution is

$$\epsilon \equiv \frac{R_f}{\frac{C_1}{C_0}} \frac{\partial \frac{C_1}{C_0}}{\partial R_f} = \frac{\partial \ln(C_1/C_0)}{\partial \ln(R_f)} = \frac{1}{1-\gamma} > 0$$

* Thus ϵ is the reciprocal of the coefficient of relative risk aversion

- When $0<\gamma<1$, $\epsilon>1$ and interest rate is falling, it decreases the individual's consumption more than one for one
also, when $\epsilon>1$, the substitution effect outweights the income effect.
- when $\gamma<0$, then $\epsilon<1$ and falling in interest rate, it decreases the individual's consumption less than one for one
also, for $\epsilon<1$, the income effect outweight the substitution effect.

Score.....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

The SDF of pricing kernal may differ across investor since there are the differences in random labor incom , causing distribution of C_1 to differ.

Since utility depend on real consumption , so P_i^N and X_i need to be deflated by price index to convert them to real quantities.

↳ let CPI_t is the consumer price index , so it becamel

$$\frac{P_i^N}{CPI_0} = E\left[\frac{\delta u'(C_1)}{u'(C_0)} \frac{X_i^N}{CPI_1} \right]$$

↳ also we difine $I_{ts} = CPI_s \div CPI_t$, so

$$P_i^N = E\left[\frac{1}{I_{01}} \frac{\delta u'(C_1)}{u'(C_0)} X_i^N \right]$$

$$= E[M_{01} X_i^N]$$

where M_{01} is the SDF for norminal returns, equal to the real pricing kernel , m_{01} , discounted at the rate of inflation between dates 0 and 1.

Score....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

Since:

$$U(c) = \ln c(c)$$

$$m_{01} = \delta U'(C_1/c_0)$$

$$var(X) = E(X^2) - E(X)^2$$

So

$$\frac{\sigma_{m_{01}}}{E[m_{01}]} = \sigma_{m_{01}} R_f$$

$$= \sqrt{e^{x} - 1}$$

$$= \sqrt{1 - X - 1}$$

$$= \sqrt{\cancel{1} - (r-1)^2 \sigma_c^2 - \cancel{1}}$$

$$= \sqrt{-(r-1)^2 \sigma_c^2} \quad //$$