

MA217 Calculus for Social Science II

QUIZ I Saturday 18th September 2010: 9.00-9.30 (30 mins)

There is 1 question. **Show all your calculation steps and simplify your answers where possible.** Full mark of 20 can be obtained without completing the whole question (total marks available are 21).

1. If $\underline{\mathbf{A}} = \begin{bmatrix} 1 & 2 & a_{13} \\ 2 & 5 & 7 \\ -3 & -6 & -10 \end{bmatrix}$, a linear system $\begin{matrix} x_1 & +2x_2 & +a_{13}x_3 & = & 1 \\ 2x_1 & +5x_2 & +7x_3 & = & 9 \\ -3x_1 & -6x_2 & -10x_3 & = & b_3 \end{matrix}$, and

$$\underline{\mathbf{C}} = \begin{bmatrix} 1 & 2 & 3 & 1 & 6 & 2 \\ 4 & 5 & 6 & 2 & 0 & 0 \\ 9 & 2 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- [2 marks] Using the row operations to determine the value of a_{13} that makes $\underline{\mathbf{A}}$ non-invertible or singular.
- [2 marks] If $\underline{\mathbf{A}}$ is non-invertible, determine the value of b_3 that will make the linear system inconsistent.
- [1 mark] If $\underline{\mathbf{A}}$ is non-invertible, determine the value of b_3 that will make the linear system have infinite solutions and identify the solution.
- [2 marks] From the inconsistent linear system (b) and the linear system with infinite solutions (c), try to determine x_1 using Cramer's rule. It is not possible to calculate x_1 but what conclusion can you draw from this.
- [4 marks] If $a_{13} = 3$, find $\underline{\mathbf{A}}^{-1}$ and show that your answer is correct.
- [2 marks] If $a_{13} = 3$, use $\underline{\mathbf{A}}^{-1}$ obtained from (e) to solve the linear system when $b_3 = 8$.
- [2 marks] If $a_{13} = 3$, determine $|\underline{\mathbf{A}}|$ and $|\underline{\mathbf{A}}^{-1}|$.
- [4 marks] Determine $|\underline{\mathbf{C}}|$.
- [2 marks] If $a_{13} = 3$, from (g) and (h) determine $\left| \left\{ (\underline{\mathbf{A}} \underline{\mathbf{C}}^T \underline{\mathbf{A}}^{-1})^T \right\}^{-1} \right|$.