Lecture 10
Options (Part 2)

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FN 312 – INVESTMENTS

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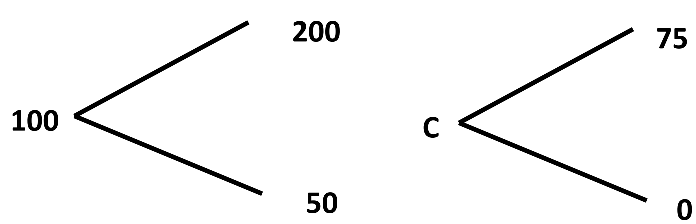
Factors Influencing Option Values: Calls

Factor	Effect on value
Stock price	increases
Exercise price	decreases
Volatility of stock price	increases
Time to expiration	increases
Interest rate	increases
Dividend Rate	decreases

Factors Influencing Option Values: Puts

Factor	Effect on value
Stock price	
Exercise price	
Volatility of stock price	
Time to expiration	
Interest rate	
Dividend Rate	

Binomial Option Pricing



Stock Price

Call Option Value
X = 125

Binomial Option Pricing

Alternative Portfolio

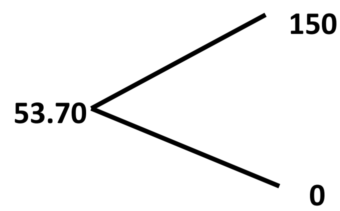
Buy 1 share of stock at $100

Borrow $46.30 (8% Rate)

Net outlay $53.70

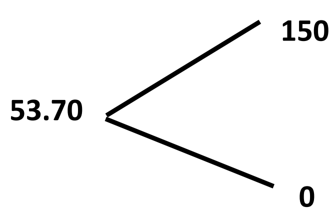
Payoff

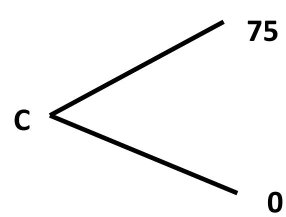
Value of Stock	**50**	**200**
Repay loan	**- 50**	**-50**
Net Payoff	**0**	**150**



Payoff Structure is exactly 2 times the Call

Binomial Option Pricing





Another View of Replication of Payoffs and Option Values

Alternative Portfolio - one share of stock and 2 calls written (X = 125)

Stock Value	50	200
Call Obligation	0	-150
Net payoff	50	50

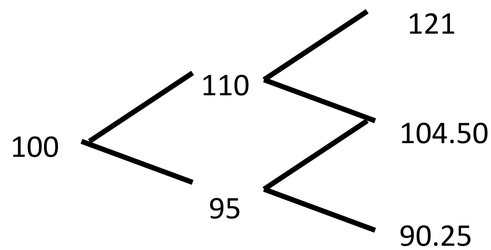
Hence 100 - 2C = 46.30 or C = 26.85

Hedge Ratio =

Generalizing the Two-State Approach

- Assume that we can break the year into two six-month segments
- In each six-month segment the stock could increase by 10% or decrease by 5%
- Assume the stock is initially selling at 100
- Possible outcomes
  - Increase by 10% twice
  - Decrease by 5% twice
  - Increase once and decrease once (2 paths)

Generalizing the Two-State Approach



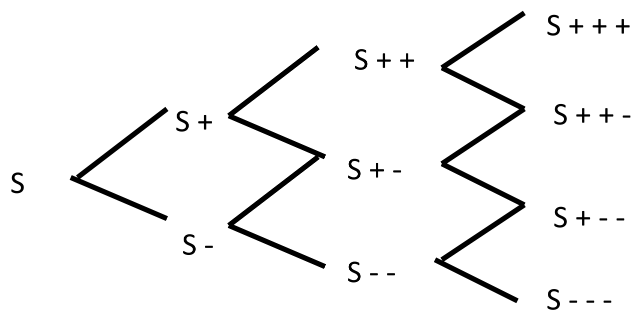
Example

Suppose that the risk-free interest rate is 5% per 6 month period and we wish to value a call option with exercise price $110 with the underlying stock price following the two-period price tree in the previous slide. Find the value of the call option.

Expanding to Consider Three Intervals

- Assume that we can break the year into three intervals
- For each interval the stock could increase by 5% or decrease by 3%
- Assume the stock is initially selling at 100

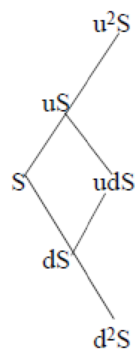
Expanding to Consider Three Intervals



Possible Outcomes with Three Intervals

Event	Probability	Stock Price	
3 **up**	**1/8**	**100 $(1.05)^3$**	**=115.76**
2 up 1 down	**3/8**	**100 $(1.05)^2$ (.97)**	**=106.94**
1 up 2 down	**3/8**	**100 (1.05) $(.97)^2$**	**= 98.79**
3 down	**1/8**	**100 $(.97)^3$ = 91.27**	

Black-Scholes Option Pricing



Binomial → lognormal (Black Scholes)

Black-Scholes Option Pricing

$$\mathbf{C_0 = S_0\, N(d_1) - Xe^{-rT}\, N(d_2)}$$

$$\mathbf{P_0 = Xe^{-rT}\, N(-d_2) - S_0\, N(-d_1)}$$

Where

$$d_1 = \frac{Ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$d_2 =$ $d_1 - \sigma\sqrt{T}$

$r =$ risk-free rate (annualized, continuously compounded)

$T =$ time to maturity in years

$\sigma =$ standard deviation of annualized continuously compounded rate of return on stock in decimal

Black-Scholes Option Valuation

N(.) = Cumulative Standard Normal distribution

Ln(.) = Natural logarithm

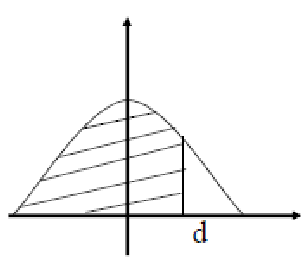
- Put price can also be obtained from put-call parity.

Recall from Statistics

Suppose Z is standard normal then

$P[Z \leq d] = N(d)$

Black Scholes Option Pricing



To get N(d) you can use the table of cumulative normal distribution or the function NORMSDIST in excel.

Call Option Example

Suppose S_0 = \$50 X = \$45

r = 6% σ = 20% T = 0.25 years

<u>Calculation</u>:

$$d_1 = \frac{\ln(\frac{50}{45}) + (0.06 + \frac{(0.20)^2}{2})\,0.25}{(0.20)\sqrt{0.25}}$$

$$= 1.2536$$

$$d_2 = 1.2536 - \sigma\sqrt{T} =$$

$$= 1.2536 - 0.1 = 1.1536$$

$N(d_1)$ = 0.8950

$N(d_2)$ = 0.8757

$$C = \underset{S}{50}(\underset{N(d_1)}{0.8950}) - \underset{X}{45}(\underset{e^{-rT}}{0.9851})\,(\underset{N(d_2)}{0.8757})$$

$$= 44.7500 - 38.8193$$

$$= 5.9307 \cong \$5.93$$

Binomial Tree versus Black-Scholes

		BS price	Binomial price	N
stock price	100	3.7146	3.0147	2
strike price	100	3.7146	3.347	4
volatility	0.15	3.7146	3.466	6
T	1 year	3.7146	3.563	10
Treasury	0.05	3.7146	3.663	30
Put option price ?		3.7146	3.699	100
		3.7146	3.7115	500

Binomial Tree versus Black-Scholes

- Black-Scholes is easy to use and quite accurate for European options, especially at the money options
- Black-Scholes cannot be used to value American options that have optimal early exercise while binomial tree can be modified to value American options
- Binomial tree method requires more programming and longer execution time for accuracy
- Binomial method is easy to modify for pricing exotic options

Assumptions in Black-Scholes

- Perfect Capital markets
- The stock pays no dividends until after option expiration date
- Both the interest rate and variance rate of the stock are constant and are known functions of time
- Stock prices are continuous, meaning that sudden extreme jumps such as those in the aftermath of an announcement of a takeover attempt are ruled out.

Implied Volatility

- Is the volatility that makes the option price calculated using Black-Scholes model equal to the market price of the option?
- How to obtain it?
  - Solve the non-linear Black-Scholes pricing equal fo the volatility given an option price
- Why do we care?
  - You want to know what the market consensus is for future volatility
  - Investors can then judge whether they think the actual stock standard deviation exceeds the implied volatility.
  - Used as an “investor fear gauge” as implied volatility is correlated with crisis

Adjusting the Black-Scholes Model for Dividends

- The call option formula applies to stocks that does not pay dividends
- One approach to incorporate dividends is to replace the stock price with a dividend adjusted stock price

Using the Black-Scholes Formula

Hedge ratio or delta

- The change in option price for a $1 increase in the stock price
- This is the slope of the curve (option value graphed as a function of the stock value) at the current stock price.
- Represents the number of stocks required to hedge against the price risk of holding one option
- For Black Scholes:

Option Elasticity

Percentage change in the option's value given a 1% change in the value of the underlying stock