

1. State reduce form model of these system models.

$$\ln S_t = \pi_{10} + \pi_{11} \ln P_{X2t} + \pi_{12} \ln P_{X3t} + \pi_{13} \ln P_{X4t} + \pi_{14} \ln GDP_t + w_{1t}$$

$$\ln D_t = \pi_{20} + \pi_{21} \ln P_{X2t} + \pi_{22} \ln P_{X3t} + \pi_{23} \ln P_{X4t} + \pi_{24} \ln GDP_t + w_{2t}$$

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

```
. tsset obs
      time variable: obs, 1986 to 2007
      delta: 1 unit
```

```
. g lnst=ln(st)
. g lndt=ln(dt)
. g pd=pm+t
. g lnpx2=ln(px2)
. g lnpx3=ln(px3)
. g lnpx4=ln(px4)
. g lngdp=ln(gdp)
```

```
. reg lnst lnpx2 lnpx3 lnpx4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	4.64569724	4	1.16142431	F(4, 17)	=	37.32
Residual	.529104674	17	.031123804	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lnpx2	-.4503744	.1515961	-2.97	0.009	-.7702142 -.1305347
lnpx3	-.9242052	.2783356	-3.32	0.004	-1.511442 -.3369685
lnpx4	-.3883793	.4222332	-0.92	0.371	-1.279214 .5024549
lngdp	.3438812	.1913463	1.80	0.090	-.0598242 .7475865
_cons	24.65741	5.309757	4.64	0.000	13.4548 35.86002

```
. predict lnsthat, xb
```

```
. reg lndt lnpx2 lnpx3 lnpx4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	3.4026552	4	.850663799	F(4, 17)	=	26.43
Residual	.54721789	17	.032189288	Prob > F	=	0.0000
				R-squared	=	0.8615
				Adj R-squared	=	0.8289
Total	3.94987309	21	.188089195	Root MSE	=	.17941

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lnpx2	-.4887365	.1541691	-3.17	0.006	-.8140049 -.1634682
lnpx3	-.7243134	.2830597	-2.56	0.020	-1.321517 -.1271097
lnpx4	-.577921	.4293997	-1.35	0.196	-1.483875 .3280333
lngdp	.1265855	.194594	0.65	0.524	-.2839719 .5371429
_cons	27.18614	5.399879	5.03	0.000	15.79339 38.57889

```
. predict lndthat, xb
```

```
. reg lnpd lnp2 lnp3 lnp4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	.17707359	4	.044268398	F(4, 17)	=	6.76
Residual	.111247189	17	.006543952	Prob > F	=	0.0019
				R-squared	=	0.6142
				Adj R-squared	=	0.5234
Total	.288320779	21	.013729561	Root MSE	=	.08089

lnpd	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnp2	.1318015	.0695123	1.90	0.075	-.0148567	.2784596
lnp3	.0939842	.127627	0.74	0.472	-.1752851	.3632535
lnp4	.4939641	.1936093	2.55	0.021	.0854842	.9024439
lngdp	.1632779	.0877392	1.86	0.080	-.0218357	.3483914
_cons	2.87652	2.434717	1.18	0.254	-2.260283	8.013322

```
. predict lnpdhat, xb
```

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.

```
. reg linst lnpdhat lnp2 lnp3 lnp4
```

Source	SS	df	MS	Number of obs	=	22
Model	4.64569773	4	1.16142443	F(4, 17)	=	37.32
Residual	.529104183	17	.031123775	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879	4.578612
lnp2	-.727963	.1840856	-3.95	0.001	-1.11635	-.3395762
lnp3	-1.122146	.2824139	-3.97	0.001	-1.717988	-.5263052
lnp4	-1.428722	.4751381	-3.01	0.008	-2.431176	-.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274	36.63092

```
. reg lndt lnpdhat lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	3.26129847	2	1.63064924	F(2, 19)	=	44.99
Residual	.688574614	19	.036240769	Prob > F	=	0.0000
				R-squared	=	0.8257
				Adj R-squared	=	0.8073
Total	3.94987309	21	.188089195	Root MSE	=	.19037

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675	-1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194	.802766
_cons	35.93498	7.189835	5.00	0.000	20.88648	50.98347

4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS.

Concerning on the asymptotic property, which model is the most appropriated model? Why?

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), ols inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344	.1766516
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 2sls inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Two-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	10.67	0.0000
lndt	22	2	.1454858	0.8982	77.04	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	2.10611	2.191774	0.96	0.343	-2.339013	6.551233
lnpx2	-.7279628	.3442892	-2.11	0.041	-1.426214	-.0297119
lnpx3	-1.122146	.528189	-2.12	0.041	-2.193363	-.0509293
lnpx4	-1.428722	.8886357	-1.61	0.117	-3.230959	.3735147
_cons	18.59914	15.98447	1.16	0.252	-13.81886	51.01715
lndt						
lnpd	-2.574157	.4354519	-5.91	0.000	-3.457295	-1.69102
lngdp	.5212921	.1027745	5.07	0.000	.3128558	.7297283
_cons	35.93499	5.494663	6.54	0.000	24.7913	47.07868

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls inst (lnpx2 lnpx3 lnpx4 lngdp)
```

Three-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.2963642	0.6266	57.47	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpd	2.171576	1.926095	1.13	0.260	-1.603501	5.946652
lnpx2	-.7990055	.2985983	-2.68	0.007	-1.384247	-.2137635
lnpx3	-1.329743	.4560002	-2.92	0.004	-2.223487	-.4359989
lnpx4	-1.171403	.775654	-1.51	0.131	-2.691657	.348851
_cons	17.84948	14.04122	1.27	0.204	-9.670808	45.36976
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls ireg3 inst (lnpx2 lnpx3 lnpx4 lngdp)
```

```
Iteration 1: tolerance = .1059484
Iteration 2: tolerance = .04569793
Iteration 3: tolerance = .01846611
Iteration 4: tolerance = .00725496
Iteration 5: tolerance = .00281814
Iteration 6: tolerance = .00108981
Iteration 7: tolerance = .00042072
Iteration 8: tolerance = .00016231
Iteration 9: tolerance = .0000626
Iteration 10: tolerance = .00002414
Iteration 11: tolerance = 9.310e-06
Iteration 12: tolerance = 3.590e-06
Iteration 13: tolerance = 1.384e-06
Iteration 14: tolerance = 5.339e-07
```

Three-stage least-squares regression, iterated

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpd	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

Hausman Test

```
. reg lnst lnpx lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
Model	4.71070935	4	1.17767734	F(4, 17)	=	43.14
Residual	.464092568	17	.027299563	Prob > F	=	0.0000
				R-squared	=	0.9103
				Adj R-squared	=	0.8892
Total	5.17480192	21	.246419139	Root MSE	=	.16523

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpx	-1.111835	.4515147	-2.46	0.025	-2.064448	-.1592222
lnpx2	-.4189546	.1431634	-2.93	0.009	-.7210029	-.1169063
lnpx3	-.9424196	.2585266	-3.65	0.002	-1.487863	-.3969762
lnpx4	-.521346	.3441643	-1.51	0.148	-1.247469	.2047773
_cons	41.4946	3.661911	11.33	0.000	33.76865	49.22056

```
. estimates store ols
```

```
. ivregress 2sls lnst lnpx4 (lnpx2=lnpx3 lnpx4 lngdp)
```

Instrumental variables (2SLS) regression	Number of obs	=	22
	Wald chi2(3)	=	213.72
	Prob > chi2	=	0.0000
	R-squared	=	0.9131
	Root MSE	=	.14301

lnst	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnpx2	1.324509	.2971871	4.46	0.000	.7420325	1.906985
lnpx3	-.3078559	.5016916	-0.61	0.539	-1.291153	.6754415
lnpx4	.6456652	.5662349	1.14	0.254	-.4641348	1.755465
_cons	-7.224838	8.946038	-0.81	0.419	-24.75875	10.30908

Instrumented: lnpx2
 Instruments: lnpx4 lnpx3 lngdp

```
. estimates store twostage
```

```
. hausman twostage ols
```

	Coefficients			
	(b) twostage	(B) ols	(b-B) Difference	sqrt(diag(V_b-V_B)) S.E.
lnpx2	-.3078559	-.4189546	.1110987	.4808312
lnpx4	.6456652	-.521346	1.167011	.4496364

b = consistent under Ho and Ha; obtained from ivregress
 B = inconsistent under Ha, efficient under Ho; obtained from regress

Test: Ho: difference in coefficients not systematic

$$\begin{aligned} \text{chi2}(2) &= (b-B)'[(V_b-V_B)^{-1}](b-B) \\ &= 9.75 \\ \text{Prob}>\text{chi2} &= 0.0076 \end{aligned}$$

Since p-value which equals to 0.0076 is less than 0.05, we reject null hypothesis. This means the model with OLS has endogeneity biased and we should run two stage least square instead of OLS.

5. What do β_{21} and β_{22} mean?

β_{21} represents price elasticity of quantity demanded.

β_{22} represents percentage change of domestic demanded to one percentage change to GDP