Practice problem set 8

Semester 1/2019

Integration and its application in economics.

Question 1:

In the manufacture of a product, the marginal cost of producing x units is C'(x) and fixed cost are C(0). Find the total cost function C(x) when:

a. $C'(x) = 3x + 4$, $C(0) = 40$.

b. $C'(x) = ax + b$, $C(0) = C_0$.

Question 2

Let $K(t)$ denote the capital stock of an economy at time t. Then net investment at time t, denoted by $I(t)$, is given by the rate of increase $\frac{dK}{dt}$ of K(t).

a. If $I(t) = 3t^2 + 2t + 5, t \geq 0$, what is the total increase in the capital stock during the interval from t = 0 to t = 5?

b. If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time $t = t_0$ to $t = T$ when the investment function $I(t)$ is as in part (a).

Question 3:

Given the following demand and supply curves, compute the consumer and producer surplus.

a. Demand: $P = 200 - 0.2Q$; Supply: $P = 20 + 0.1Q$.

b. Demand: $P = \frac{6000}{Q+50}$; Supply: $P = Q + 10$.

Question 4

Suppose that the profit of a firm as a function of its output x is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, x > 0$$

a. Find the level of output that maximizes profit. Sketch the graph of f.

b. The actual output varies between 1000 and 3000 units. Compute the average profit $I = \frac{1}{2000}\int_{1000}^{3000} f(x)\, dx$.

Question 5:

Evaluate the following integrals by using integrations by substitution:

a. $\int_0^1 x\sqrt{1+x^2}\ dx$

b. $\int_1^e \frac{lny}{y} dy$

Question 6:

Evaluate the following integrals by using integrations by using integrations by parts (r $\neq 0$).

a. $\int_0^T bte^{-rt}\, dt$

b. $\int_0^T (a+bt)e^{-rt}\, dt$

Question 7:

a. Evaluate $\int_0^1 x^p(x^q + x^r + x^s)\,dx$ where $p, q, r, and\ s$ are positive numbers.

b. Let $F(x) = \int_0^x (t^2 + 2)\,dt$ and $G(x) = \int_0^{x^2} (t^2 + 2)\,dt$. Find F'(x) and G'(x).

Question 8:

Let the demand and supply of goods Q in a perfectly competitive market be the followings;

Demand Function : $P = 25 - Q^2$

Supply Function : $P = 2Q + 1$

a. Determine the consumer surplus as the equilibrium.
b. If the government imposes tax on consumers for \$4 per unit of production, calculate the deadweight loss.

Question 9:

Let $P = 274 - Q^2$ be the demand function in a monopoly market. Suppose further that marginal cost of the monopolist is given by $MC = 4 + 3Q$.

a. Determine consumer's surplus at the profit-maximizing production level.
b. Calculate deadweight loss under monopoly.

Question 10:

A company has MC = 80 where the demand function is P = 1400-6Q. At zero production, the company faces loss by \$1,500. Determine the maximum profit by using integral calculus and prove your answer.

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Semester 1/2019

Integration and its application in economics.

(Solution)

Question 1:

In the manufacture of a product, the marginal cost of producing x units is C'(x) and fixed cost are C(0). Find the total cost function C(x) when:

a. $C'(x) = 3x + 4$, $C(0) = 40$.

b. $C'(x) = ax + b$, $C(0) = C_0$.

Ans.

a. $C(x) = \frac{3}{2}x^2 + 4x + 40$

b. $C(x) = \frac{1}{2}ax^2 + bx + C_0$

Question 2

Let $K(t)$ denote the capital stock of an economy at time t. Then net investment at time t, denoted by $I(t)$, is given by the rate of increase $\frac{dK}{dt}$ of K(t).

a. If $I(t) = 3t^2 + 2t + 5, t \geq 0$, what is the total increase in the capital stock during the interval from t = 0 to t = 5?

Ans. $K(5) - K(0) = 175.$

b. If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time $t = t_0$ to $t = T$ when the investment function $I(t)$ is as in part (a).

Ans. $K(T) - K_0 = (T^3 - t_0^3) + (T^2 - t_0^2) + 5(T - t_0)$

Question 3:

Given the following demand and supply curves, compute the consumer and producer surplus.

a. Demand: $P = 200 - 0.2Q$; Supply: $P = 20 + 0.1Q$.

Ans. $(Q^*, P^*) = (600{,}80)$.

Consumer surplus = 36,000; Producer surplus = 18,000.

b. Demand: $P = \frac{6000}{Q+50}$; Supply: $P = Q + 10$.

Ans. $(Q^*, P^*) = (50{,}60)$.

Consumer surplus =6000ln100 -3000;

Producer surplus = 1250.

Question 4

Suppose that the profit of a firm as a function of its output x is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, x > 0$$

a. Find the level of output that maximizes profit. Sketch the graph of f.

Ans. $x = 1000\sqrt{3}$

b. The actual output varies between 1000 and 3000 units. Compute the average profit $I = \frac{1}{2000}\int_{1000}^{3000} f(x)dx$.

Ans. $I = 2000 - 1500ln(3) \approx 352$

Question 5:

Evaluate the following integrals by using integrations by substitution:

a. $\int_0^1 x\sqrt{1+x^2}\,dx$

Ans. Let $u = \sqrt{1+x^2}$. Thus, $\int_0^1 x\sqrt{1+x^2}\,dx = \int_1^{\sqrt{2}} u^2\,du = \frac{1}{3}(2\sqrt{2}-1)$

b. $\int_1^e \frac{lny}{y}dy$

Ans. Let u = ln(y). Thus, $\int_1^e \frac{lny}{y}dy = \frac{1}{2}$.

Question 6:

Evaluate the following integrals by using integrations by using integrations by parts (r $\neq 0$).

a. $\int_0^T bte^{-rt}\,dt$

Ans. $\int_0^T bte^{-rt}\,dt = br^{-2}[1-(1+rT)e^{-rT}]$

b. $\int_0^T (a+bt)e^{-rt}\,dt$

Ans. $\int_0^T (a+bt)e^{-rt}\,dt = ar^{-1}(1-e^{-rT}) + br^{-2}[1-(1+rT)e^{-rT}]$

Question 7:

a. Evaluate $\int_0^1 x^p(x^q + x^r + x^s)\,dx$ where $p, q, r, and\ s$ are positive numbers.

Ans: $\frac{1}{p+q+1} + \frac{1}{p+r+1} + \frac{1}{p+s+1}$

b. Let $F(x) = \int_0^x (t^{\,2} + 2)\,dt$ and $G(x) = \int_0^{x^2} (t^2 + 2)\,dt.$ Find F'(x) and G'(x).

Ans: $F(x) = x^3/3 + 2x$ and $G(x) = x^6/3 + 2x^2$
$F'(x) = x^2 + 2$ and $G'(x) = 2x^5 + 4x$

Question 8:

Let the demand and supply of goods Q in a perfectly competitive market be the followings;

Demand Function : $P = 25 - Q^2$

Supply Function : $P = 2Q + 1$

a. Determine the consumer surplus as the equilibrium.

CS = 42.67 and PS = 16

b. If the government imposes tax on consumers for $4 per unit of production, calculate the deadweight loss.

[DWL = 0.9]

Question 9:

Let $P = 274 - Q^2$ be the demand function in a monopoly market. Suppose further that marginal cost of the monopolist is given by $MC = 4 + 3Q$.

a. Determine consumer's surplus at the prot-maximizing production level.
   [Q = 9, P = 193, CS = 486];
b. Calculate deadweight loss under monopoly.

Q under perfect competition = 15. Thus, deadweight loss is 522.

Question 10:

A company has MC = 80 where the demand function is P = 1400-6Q. At zero production, the company faces loss by $1,500. Determine the maximum profit by using integral calculus and prove your answer.

[Q = 110 and maximum profit = 71,100] 71100