Assignment 2 Solution
Simultaneous Equations Model

Demand and Supply Equations

$$\ln S_t = \beta_{10} + \beta_{11}\ln P_{Dt} + \beta_{12}\ln P_{X2t} + \beta_{13}\ln P_{X3t} + \beta_{14}\ln P_{X4t} + \varepsilon_{1t} \quad (1)$$

$$\ln D_t = \beta_{20} + \beta_{21}\ln P_{Dt} + \beta_{22}\ln GDP_t + \varepsilon_{2t} \quad (2)$$

where: S_t = Domestic Supply at time t
D_t = Domestic Demand at time t
P_{Dt} = Domestic Price at time t = P_{Mt} + T_t
P_{Mt} = Import Price at time t
T_t = Tariff at time t
P_{X2t} = Price of Input 2 at time t
P_{X3t} = Price of Input 3 at time t
P_{X4t} = Price of Input 4 at time t
GDP_t = Gross Domestic Product at time t

Endogenous variables in this system include S_t, D_t, and P_{Dt}
Exogenous variables in this system include P_{X2t}, P_{X3t}, P_{X4t}, and GDP_t

From Data `Assignment 2.dta`:

1. State reduce form model of these system models.
2. Estimate reduce form model using OLS and prediction of the endogenous variables.
3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.
4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?
5. What does β_{21} and β_{22} mean?

1. State reduce form model of these system models.

$$\ln S_t = \pi_{10} + \pi_{11}\ln P_{X2t} + \pi_{12}\ln P_{X3t} + \pi_{13}\ln P_{X4t} + \pi_{14}\ln GDP_t + w_{1t}$$
$$\ln D_t = \pi_{20} + \pi_{21}\ln P_{X2t} + \pi_{22}\ln P_{X3t} + \pi_{23}\ln P_{X4t} + \pi_{24}\ln GDP_t + w_{2t}$$
$$\ln P_{Dt} = \pi_{30} + \pi_{31}\ln P_{X2t} + \pi_{32}\ln P_{X3t} + \pi_{33}\ln P_{X4t} + \pi_{34}\ln GDP_t + w_{3t}$$

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

Set up STATA analysis as time-series analysis

```
. tsset obs
        time variable:  obs, 1986 to 2007
                delta:  1 unit
```

Generate Variable in ln-form

```
. g lnst=ln(st)

. g lndt=ln(dt)

. g pd=pm+t

. g lnpd=ln(pd)

. g lnpx2=ln(px2)

. g lnpx3=ln(px3)

. g lnpx4=ln(px4)

. g lngdp=ln(gdp)
```

Estimate Reduced Form model for Supply model

```
. reg lnst lnpx2 lnpx3 lnpx4 lngdp

      Source |       SS       df       MS              Number of obs =      22
-------------+------------------------------           F(  4,    17) =   37.32
       Model |  4.64569724     4  1.16142431           Prob > F      =  0.0000
    Residual |  .529104674    17  .031123804           R-squared     =  0.8978
-------------+------------------------------           Adj R-squared =  0.8737
       Total |  5.17480192    21  .246419139           Root MSE      =  .17642

------------------------------------------------------------------------------
        lnst |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lnpx2 |  -.4503744   .1515961    -2.97   0.009    -.7702142   -.1305347
       lnpx3 |  -.9242052   .2783356    -3.32   0.004    -1.511442   -.3369685
       lnpx4 |  -.3883793   .4222332    -0.92   0.371    -1.279214    .5024549
       lngdp |   .3438812   .1913463     1.80   0.090    -.0598242    .7475865
       _cons |   24.65741   5.309757     4.64   0.000      13.4548    35.86002
------------------------------------------------------------------------------
```

Predict yhat for Supply model

```
. predict lnsthat, xb
```

Estimate Reduced Form model for Demand model

```
. reg lndt lnpx2 lnpx3 lnpx4 lngdp

      Source |       SS       df       MS              Number of obs =      22
-------------+------------------------------           F(  4,    17) =   26.43
       Model |   3.4026552     4  .850663799           Prob > F      =  0.0000
    Residual |   .54721789    17  .032189288           R-squared     =  0.8615
-------------+------------------------------           Adj R-squared =  0.8289
       Total |  3.94987309    21  .188089195           Root MSE      =  .17941

------------------------------------------------------------------------------
        lndt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lnpx2 |  -.4887365   .1541691    -3.17   0.006    -.8140049   -.1634682
       lnpx3 |  -.7243134   .2830597    -2.56   0.020    -1.321517   -.1271097
       lnpx4 |   -.577921   .4293997    -1.35   0.196    -1.483875    .3280333
       lngdp |   .1265855    .194594     0.65   0.524    -.2839719    .5371429
       _cons |   27.18614   5.399879     5.03   0.000     15.79339    38.57889
------------------------------------------------------------------------------
```

Predict yhat for Supply model

```
. predict lndthat, xb
```

Estimate Reduced Form model for Domestic Price model

```
. reg lnpd lnpx2 lnpx3 lnpx4 lngdp

      Source |       SS       df       MS              Number of obs =      22
-------------+------------------------------           F(  4,    17) =    6.76
       Model |   .17707359     4  .044268398           Prob > F      =  0.0019
    Residual |  .111247189    17  .006543952           R-squared     =  0.6142
-------------+------------------------------           Adj R-squared =  0.5234
       Total |  .288320779    21  .013729561           Root MSE      =  .08089
```

```
------------------------------------------------------------------------------
        lnpd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lnpx2 |   .1318015   .0695123     1.90   0.075    -.0148567    .2784596
       lnpx3 |   .0939842    .127627     0.74   0.472    -.1752851    .3632535
       lnpx4 |   .4939641   .1936093     2.55   0.021     .0854842    .9024439
       lngdp |   .1632779   .0877392     1.86   0.080    -.0218357    .3483914
       _cons |    2.87652   2.434717     1.18   0.254    -2.260283    8.013322
------------------------------------------------------------------------------
```

<u>*Predict yhat for Domestic Price model</u>

```
. predict lnpdhat, xb
```

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.

```
. reg lnst lnpdhat lnpx2 lnpx3 lnpx4

      Source |       SS       df       MS              Number of obs =      22
-------------+------------------------------           F(  4,    17) =   37.32
       Model |  4.64569773     4  1.16142443           Prob > F      =  0.0000
    Residual |  .529104183    17  .031123775           R-squared     =  0.8978
-------------+------------------------------           Adj R-squared =  0.8737
       Total |  5.17480192    21  .246419139           Root MSE      =  .17642

------------------------------------------------------------------------------
        lnst |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnpdhat |   2.106112   1.171903     1.80   0.090    -.3663879    4.578612
       lnpx2 |   -.727963   .1840856    -3.95   0.001     -1.11635   -.3395762
       lnpx3 |  -1.122146   .2824139    -3.97   0.001    -1.717988   -.5263052
       lnpx4 |  -1.428722   .4751381    -3.01   0.008    -2.431176   -.4262679
       _cons |   18.59912   8.546622     2.18   0.044     .5673274    36.63092
------------------------------------------------------------------------------

. reg lndt lnpdhat lngdp

      Source |       SS       df       MS              Number of obs =      22
-------------+------------------------------           F(  2,    19) =   44.99
       Model |  3.26129847     2  1.63064924           Prob > F      =  0.0000
    Residual |  .688574614    19  .036240769           R-squared     =  0.8257
-------------+------------------------------           Adj R-squared =  0.8073
       Total |  3.94987309    21  .188089195           Root MSE      =  .19037

------------------------------------------------------------------------------
        lndt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnpdhat |  -2.574157   .5697943    -4.52   0.000     -3.76675   -1.381563
       lngdp |   .5212927   .1344816     3.88   0.001     .2398194     .802766
       _cons |   35.93498   7.189835     5.00   0.000     20.88648    50.98347
------------------------------------------------------------------------------
```

<u>*Alternative Command 2sls</u>

```
. ivregress 2sls lnst lnpx2 lnpx3 lnpx4 (lnpd= lnpx2 lnpx3 lnpx4 lngdp),  first

First-stage regressions
-----------------------

                                                Number of obs     =         22
                                                F(   4,      17)  =       6.76
                                                Prob > F          =     0.0019
                                                R-squared         =     0.6142
                                                Adj R-squared     =     0.5234
                                                Root MSE          =     0.0809

------------------------------------------------------------------------------
        lnpd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lnpx2 |   .1318015   .0695123     1.90   0.075    -.0148567    .2784596
       lnpx3 |   .0939842    .127627     0.74   0.472    -.1752851    .3632535
       lnpx4 |   .4939641   .1936093     2.55   0.021     .0854842    .9024439
       lngdp |   .1632779   .0877392     1.86   0.080    -.0218357    .3483914
```

```
       _cons |    2.87652   2.434717     1.18   0.254    -2.260283    8.013322
------------------------------------------------------------------------------


Instrumental variables (2SLS) regression          Number of obs   =         22
                                                  Wald chi2(4)    =      55.22
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.6424
                                                  Root MSE        =     .29004

------------------------------------------------------------------------------
        lnst |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnpd |    2.10611   1.926677     1.09   0.274    -1.670107    5.882327
       lnpx2 |  -.7279628   .3026471    -2.41   0.016     -1.32114   -.1347853
       lnpx3 |  -1.122146    .464304    -2.42   0.016    -2.032165    -.212127
       lnpx4 |  -1.428722   .7811544    -1.83   0.067    -2.959757    .1023125
       _cons |   18.59914   14.05113     1.32   0.186    -8.940569    46.13886
------------------------------------------------------------------------------
Instrumented:  lnpd
Instruments:   lnpx2 lnpx3 lnpx4 lngdp

. ivregress 2sls lndt lngdp (lnpd= lnpx2 lnpx3 lnpx4 lngdp)

Instrumental variables (2SLS) regression          Number of obs   =         22
                                                  Wald chi2(2)    =     178.41
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.8982
                                                  Root MSE        =      .1352

------------------------------------------------------------------------------
        lndt |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnpd |  -2.574157   .4046743    -6.36   0.000    -3.367304    -1.78101
       lngdp |   .5212921   .0955104     5.46   0.000     .3340951     .708489
       _cons |   35.93499   5.106302     7.04   0.000     25.92682    45.94316
------------------------------------------------------------------------------
Instrumented:  lnpd
Instruments:   lngdp lnpx2 lnpx3 lnpx4
```

4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), ols inst(lnpx2 lnpx3 lnpx4
lngdp)

Multivariate regression
----------------------------------------------------------------------
Equation          Obs  Parms        RMSE    "R-sq"      F-Stat        P
----------------------------------------------------------------------
lnst               22      4    .1652258    0.9103       43.14   0.0000
lndt               22      2    .1391259    0.9069       92.53   0.0000
----------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnst         |
        lnpd |  -1.111835   .4515147    -2.46   0.019    -2.027549   -.1961207
       lnpx2 |  -.4189546   .1431634    -2.93   0.006    -.7093034   -.1286059
       lnpx3 |  -.9424196   .2585266    -3.65   0.001    -1.466736   -.4181034
       lnpx4 |   -.521346   .3441643    -1.51   0.139    -1.219344    .1766516
       _cons |    41.4946   3.661911    11.33   0.000      34.0679     48.9213
-------------+----------------------------------------------------------------
lndt         |
        lnpd |  -2.181329   .2946999    -7.40   0.000    -2.779008    -1.58365
       lngdp |   .5776586   .0887536     6.51   0.000      .397658    .7576593
       _cons |   31.03578   3.761201     8.25   0.000     23.40771    38.66385
------------------------------------------------------------------------------

. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 2sls inst(lnpx2 lnpx3 lnpx4
lngdp)

Two-stage least-squares regression
```

```
------------------------------------------------------------------------
Equation          Obs  Parms        RMSE    "R-sq"     F-Stat        P
------------------------------------------------------------------------
lnst               22      4     .329951    0.6424      10.67   0.0000
lndt               22      2    .1454858    0.8982      77.04   0.0000
------------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnst         |
        lnpd |    2.10611   2.191774     0.96   0.343    -2.339013    6.551233
       lnpx2 |  -.7279628   .3442892    -2.11   0.041    -1.426214   -.0297119
       lnpx3 |  -1.122146    .528189    -2.12   0.041    -2.193363   -.0509293
       lnpx4 |  -1.428722   .8886357    -1.61   0.117    -3.230959    .3735147
       _cons |   18.59914   15.98447     1.16   0.252    -13.81886    51.01715
-------------+----------------------------------------------------------------
lndt         |
        lnpd |  -2.574157   .4354519    -5.91   0.000    -3.457295    -1.69102
       lngdp |   .5212921   .1027745     5.07   0.000     .3128558    .7297283
       _cons |   35.93499   5.494663     6.54   0.000      24.7913    47.07868
------------------------------------------------------------------------------
Endogenous variables:  lnst lnpd lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
------------------------------------------------------------------------------

. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls inst(lnpx2 lnpx3 lnpx4
lngdp)

Three-stage least-squares regression
------------------------------------------------------------------------
Equation          Obs  Parms        RMSE    "R-sq"       chi2        P
------------------------------------------------------------------------
lnst               22      4    .2963642    0.6266      57.47   0.0000
lndt               22      2     .135203    0.8982     178.41   0.0000
------------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnst         |
        lnpd |   2.171576   1.926095     1.13   0.260    -1.603501    5.946652
       lnpx2 |  -.7990055   .2985983    -2.68   0.007    -1.384247   -.2137635
       lnpx3 |  -1.329743   .4560002    -2.92   0.004    -2.223487   -.4359989
       lnpx4 |  -1.171403    .775654    -1.51   0.131    -2.691657     .348851
       _cons |   17.84948   14.04122     1.27   0.204    -9.670808    45.36976
-------------+----------------------------------------------------------------
lndt         |
        lnpd |  -2.574157   .4046743    -6.36   0.000    -3.367304    -1.78101
       lngdp |   .5212921   .0955104     5.46   0.000     .3340951     .708489
       _cons |   35.93499   5.106302     7.04   0.000     25.92682    45.94316
------------------------------------------------------------------------------
Endogenous variables:  lnst lnpd lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
------------------------------------------------------------------------------

. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls ireg3 inst(lnpx2 lnpx3
lnpx4 lngdp)

Iteration 1:   tolerance =   .1059484
Iteration 2:   tolerance =  .04569793
Iteration 3:   tolerance =  .01846611
Iteration 4:   tolerance =  .00725496
Iteration 5:   tolerance =  .00281814
Iteration 6:   tolerance =  .00108981
Iteration 7:   tolerance =  .00042072
Iteration 8:   tolerance =  .00016231
Iteration 9:   tolerance =   .0000626
Iteration 10:   tolerance =  .00002414
Iteration 11:   tolerance =  9.310e-06
Iteration 12:   tolerance =  3.590e-06
Iteration 13:   tolerance =  1.384e-06
Iteration 14:   tolerance =  5.339e-07

Three-stage least-squares regression, iterated
------------------------------------------------------------------------
Equation          Obs  Parms        RMSE    "R-sq"       chi2        P
```

```
-----------------------------------------------------------------------
lnst              22     4    .3022006    0.6117     54.83   0.0000
lndt              22     2     .135203    0.8982    178.41   0.0000
-----------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnst         |
        lnpd |   2.212666   2.005956     1.10   0.270    -1.718936    6.144268
       lnpx2 |  -.8435967   .3049354    -2.77   0.006    -1.441259   -.2459342
       lnpx3 |  -1.460044   .4623671    -3.16   0.002    -2.366267   -.5538216
       lnpx4 |  -1.009892   .7998393    -1.26   0.207    -2.577548     .557764
       _cons |   17.37893   14.61488     1.19   0.234    -11.26571    46.02357
-------------+----------------------------------------------------------------
lndt         |
        lnpd |  -2.574157   .4046743    -6.36   0.000    -3.367304    -1.78101
       lngdp |   .5212921   .0955104     5.46   0.000     .3340951     .708489
       _cons |   35.93499   5.106302     7.04   0.000     25.92682    45.94316
------------------------------------------------------------------------------
Endogenous variables:  lnst lnpd lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
------------------------------------------------------------------------------
```

Concerning on the asymptotic property, 3SLS or I3SLS is the most appropriated model. Because if there exists correlation of the error terms across equations, 3SLS or I3SLS will lead to a more asymptotically efficient estimators. However, if there exists specification error problem in any model(s), the specification error will spread throughout the whole system.

5. What does β_{21} and β_{22} mean?
   β_{21} represents price elasticity of quantity demand.
   β_{22} represents income elasticity of quantity demand.