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$$\left[\begin{array}{cccc|c} 3 & -6 & -3 & 0 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 6 & -12 & 2 & R & C \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & R-16 & C-14 \end{array} \right]$$

Inconsistent $R=16$ and $C \neq 14$

d)

$$C=14$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\underline{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \quad c_1, c_2 \in \mathbb{R}$$

② a) a basis for $\text{Nul } A$ is

$$\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

b) a basis for column space A ($\text{Col } A$) is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 28 \end{bmatrix} \right\}$$

$$\underline{x} = \begin{bmatrix} 3 \\ 8 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad c_1, c_2 \in \mathbb{R}$$

Past exam solutions

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$$\left[\begin{array}{cccc|c} 3 & -6 & -3 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 6 & -12 & 2 & R & C \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & R-16 & C-14 \end{array} \right]$$

Inconsistent $R=16$ and $C \neq 14$

d)

$$C=14$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\underline{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \quad c_1, c_2 \in \mathbb{R}$$

② a) a basis for $\text{Nul } A$ is

$$\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

b) a basis for Column space A ($\text{Col } A$) is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 28 \end{bmatrix} \right\}$$

c) $\underline{x} = \begin{bmatrix} 3 \\ 8 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad c_1, c_2 \in \mathbb{R}$

3)

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 3 & 1 \\ -2 & -1 & 4 \end{bmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - (-1)R_1}} \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 1 & 6 \end{bmatrix} \xrightarrow{R_3 - (-1)R_2} \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

Check

$$\underline{L} \underline{U} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 3 & 1 \\ -2 & -1 & 4 \end{bmatrix}$$

$$\underline{L} \underline{U} \underline{x} = \begin{bmatrix} 2 \\ -3 \\ 11 \end{bmatrix} \quad \text{let } \underline{U} \underline{x} = \underline{y}$$

~~$$\underline{L} \underline{U} \underline{x} = \underline{y}$$~~

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 11 \end{bmatrix}$$

$$y_1 = 2$$

$$y_2 = -7$$

$$y_3 = 6$$

$$\underline{U} \underline{x} = \begin{bmatrix} 2 \\ -7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 6 \end{bmatrix}$$

$$x_3 = 3$$

$$x_2 = -2$$

$$x_1 = 0$$

b) Yes, because all rows contain pivots.

c) No, columns 4, 5, 6 and 7 in B are dependent and row operations preserve linear dependence and independence of columns. Hence, columns 4, 5, 6, 7 of A are dependent.

d) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

e) $\left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right\}$

Ⓐ $(AX=0) \cdot 17-11=6$

$(A^T y=0) \cdot 64-11=53$

Ⓑ 3 $3,2,1$

Ⓒ False