STACKELBERG'S MODEL (1934) OR LEADER-FOLLOWER GAME

TIMING OF EVENTS

1. FIRM 1 (LEADER) CHOOSES q_1.
2. FIRM 2 (FOLLOWER) OBSERVES q_1 AND THEN CHOOSES ITS OWN q_2
3. PAYOFFS ARE REALIZED

Q: IS THERE ANY ADVANTAGE OF BEING MOVED FIRST?

Q: HOW WOULD THE EQUILIBRIUM MARKET PRICE AND PRODUCTION LEVELS COMPARE TO THE STATIC COURNOT EQUILIBRIUM PRICE AND OUTPUT LEVELS?

WE SOLVE THE GAME BACKWARD:

THE SECOND-PERIOD GAME

FIRM 2 MOVES AND CHOOSES q_2 TO MAXIMIZE ITS PROFIT, TAKING INTO A/C FIRM 1'S QUANTITY PRODUCED, q_1 AS GIVEN.

$$\boxed{BR_2(q_1) = \frac{a-c_2}{2b} - \frac{1}{2}q_1}$$

$\Rightarrow$ FROM $P(Q) = a - bQ$ $(= q_1 + q_2)$

$TC_i(q_i) = c_i q_i$

$i = 1, 2$

WHERE $c_1, c_2 > 0$

THE FIRST-PERIOD GAME

FIRM 1 CAN CALCULATE $BR_2(q_1)$ IN THE SAME WAY AS FIRM 2 DOES. THUS, FIRM 1 IS ABLE TO CALCULATE HOW FIRM 2 WILL BEST REPLY TO ITS CHOICE OF OUTPUT LEVEL.

KNOWING THAT, FIRM 1 CHOOSE q_1^* TO

$$\max_{q_1} \pi_1^s = P(q_1 + BR_2(q_1)) \cdot q_1 - c\,q_1$$

$$= \left[a - b\left(q_1 + \left(\frac{a-c}{2b} - \frac{q_1}{2}\right)\right)\right] q_1 - c\,q_1$$

SOLVING F.O.N.C. GIVES

$$\boxed{q_1^s = \frac{a-c}{2b} = \frac{3}{2}q_1^c > q_1^c}$$

NOTE:

$$q_1^c = \frac{a - 2c_1 + c_2}{3b}$$

OR IF $c_1 = c_2$,

$$q_1^c = \frac{a-c}{3b}$$

FIRM 1 (LEADER) PRODUCES MORE THAN ITS OUTPUT LEVEL IN COURNOT'S MARKET STRUCTURE.

SUBSTITUTING $q_1^s = \frac{a-c}{2b}$ INTO $BR_2(q_1^s)$ GIVES

$$\boxed{q_2^s = \frac{a-c}{4b} = \frac{3}{4}q_2^c < q_2^c}$$

$\left| \; q_2 \qquad \overline{\;4b\;} \qquad \overline{\;4\;}\, q_2 \qquad - \qquad q_2 \; \right|$

FIRM 2 (FOLLOWER) PRODUCES LESS THAN ITS OUTPUT IN COURNOT'S MARKET STRUCTURE !

NEXT,

$$p^s = \frac{a+3c}{4} < \frac{a+2c}{3} = p^c$$

$$Q^s = \frac{3(a-c)}{4b} > \frac{2(a-c)}{3b} = Q^c$$

PROPOSITION #1 : A SEQUENTIAL-MOVES QUANTITY GAME YIELDS A HIGHER AGGREGATE OUTPUT LEVEL AND A LOWER MARKET PRICE THAN THE STATIC COURNOT MARKET STRUCTURE.

$$\pi_1^s = \frac{(a-c)^2}{8b} > \pi_1^c$$

$$\pi_2^s = \frac{(a-c)^2}{16b} < \pi_2^c$$

SIMPLE MODELS FOR TWO DIFFERENTIATED PRODUCTS

CONSIDER A TWO-FIRM INDUSTRY PRODUCING TWO DIFFERENTIATED PRODUCTS INDEXED BY $i = 1, 2$. ASSUME THAT PRODUCTION IS COSTLESS. (TO SIMPLIFY THE ANALYSES)

FOLLOWING DIXIT (1979) AND SINGH AND VIVES (1984), WE ASSUME THAT

$p = f(q)$

$$p_1 = \alpha - \beta q_1 - \gamma q_2 \quad \text{AND} \quad p_2 = \alpha - \gamma q_1 - \beta q_2$$

REPRESENTS "OWN-PRICE EFFECT"

CROSS-PRICE EFFECT

ASSUMPTION *** $\rightarrow$ $\underbrace{\beta^2 > \gamma^2}$, $\beta > 0$

- EFFECT OF CHANGE IN q_1 ON p_1 IS STRONGER THAN EFFECT OF CHANGE IN q_2 ON p_1.
- EFFECT OF CHANGE IN q_2 ON p_2 IS STRONGER THAN EFFECT OF CHANGE IN q_1 ON p_2.

W/ MATRIX ALGEBRA & CRAMER'S RULE

DIRECT DEMAND FUNCTION

DIRECT DEMAND FUNCTION

EFFECT OF CHANGE IN q_1 ON P_2.

$Q = f(P)$

DIRECT DEMAND FUNCTION — $\boxed{q_1 = a - bp_1 + cp_2}$ AND DIRECT DEMAND FUNCTION — $\boxed{q_2 = a + cp_1 - bp_2}$

WHERE $a \equiv \dfrac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2}$, $b \equiv \dfrac{\beta}{\beta^2 - \gamma^2}$, $c \equiv \dfrac{\gamma}{\beta^2 - \gamma^2}$

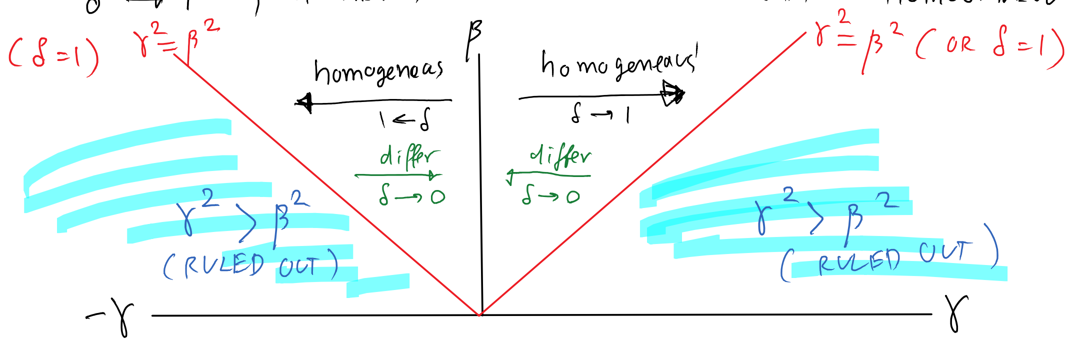
HOW TO MEASURE DEGREE OF DIFFERENTIATION ?

LET'S DEFINE $\delta \equiv \dfrac{\gamma^2}{\beta^2}$

WHERE δ = THE BRAND'S MEASURE OF DIFFERENTIATION.

IF $\delta \to 0$, IT MEANS BECOMING MORE DIFFERENTIATED (THEN $\gamma^2 \to 0$)

IF $\delta \to 1$, IT MEANS BECOMING TOWARDS HOMOGENEOUS (THEN $\gamma^2 \to \beta^2$)



- A MOVEMENT TOWARD THE CENTER $\equiv$ PRODUCTS BECOMING MORE DIFFERENTIATED
- A MOVEMENT TOWARD THE RED LINE $\equiv$ PRODUCTS ARE BECOME MORE HOMOGENEOUS