

From Data Assignment 2.dta:

1. State reduce form model of these system models.

$$\ln S_t = \beta_{10} + \beta_{11} \ln P_{D_t} + \beta_{12} \ln P_{X2_t} + \beta_{13} \ln P_{X3_t} + \beta_{14} \ln P_{X4_t} + \varepsilon_{1t}$$

$$\ln D_t = \beta_{20} + \beta_{21} \ln P_{D_t} + \beta_{22} \ln GDP_t + \varepsilon_{2t}$$

Assume $\ln S_t = \ln D_t$

$$\beta_{10} + \beta_{11} \ln P_{D_t} + \beta_{12} \ln P_{X2_t} + \beta_{13} \ln P_{X3_t} + \beta_{14} \ln P_{X4_t} + \varepsilon_{1t} = \beta_{20} + \beta_{21} \ln P_{D_t} + \beta_{22} \ln GDP_t + \varepsilon_{2t}$$

$$\beta_{11} \ln P_{D_t} - \beta_{21} \ln P_{D_t} = (\beta_{20} - \beta_{10}) - (\beta_{12} \ln P_{X2_t}) - (\beta_{13} \ln P_{X3_t}) - (\beta_{14} \ln P_{X4_t}) + (\beta_{22} \ln GDP_t) + (\varepsilon_{2t} - \varepsilon_{1t})$$

$$\ln P_{D_t} = \frac{(\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{12} \ln P_{X2_t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{13} \ln P_{X3_t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{14} \ln P_{X4_t})}{\beta_{11} - \beta_{21}} + \frac{(\beta_{22} \ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}}$$

$$\ln P_{D_t} = \left(\frac{\beta_{20} - \beta_{10}}{\beta_{11} - \beta_{21}} \right) - \frac{(\beta_{12} \ln P_{X2_t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{13} \ln P_{X3_t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{14} \ln P_{X4_t})}{\beta_{11} - \beta_{21}} + \frac{(\beta_{22} \ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}}$$

→ Plug in $\ln P_{D_t}$ in $\ln S_t$

$$\ln S_t = \beta_{10} + \beta_{11} \left(\frac{\beta_{20} - \beta_{10}}{\beta_{11} - \beta_{21}} \right) - \frac{(\beta_{12} \ln P_{X2_t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{13} \ln P_{X3_t})}{\beta_{11} - \beta_{21}} - \frac{(\beta_{14} \ln P_{X4_t})}{\beta_{11} - \beta_{21}} + \frac{(\beta_{22} \ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} + \beta_{12} \ln P_{X2_t} + \beta_{13} \ln P_{X3_t} + \beta_{14} \ln P_{X4_t} + \varepsilon_{1t}$$

$$\begin{aligned}
 &= \beta_{10} + \frac{\beta_{11}(\beta_{20} - \beta_{21})}{\beta_{11} - \beta_{21}} - \frac{\beta_{11}\beta_{21}}{\beta_{11} - \beta_{21}} \ln P_{2t} - \frac{\beta_{11}\beta_{13}}{\beta_{11} - \beta_{21}} \ln P_{3t} \\
 &\quad - \frac{\beta_{11}\beta_{14}}{\beta_{11} - \beta_{21}} \ln P_{4t} + \frac{\beta_{11}\beta_{22}}{\beta_{11} - \beta_{21}} \ln GDP + \frac{\beta_{11}(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} \\
 &\quad + \beta_{12} \ln P_{2t} + \beta_{13} \ln P_{3t} + \beta_{14} \ln P_{4t} + \varepsilon_{1t}
 \end{aligned}$$

where $\pi_{10} = \frac{\beta_{10} + \beta_{11}(\beta_{20} - \beta_{21})}{\beta_{11} - \beta_{21}}$ $\pi_{11} = \frac{-\beta_{11}\beta_{21}}{\beta_{11} - \beta_{21}} + \beta_{12}$

$\pi_{12} = \frac{-\beta_{11}\beta_{13}}{\beta_{11} - \beta_{21}} + \beta_{13}$ $\pi_{13} = \frac{-\beta_{11}\beta_{14}}{\beta_{11} - \beta_{21}} + \beta_{14}$

$\pi_{14} = \frac{\beta_{11}\beta_{22}}{\beta_{11} - \beta_{21}}$ $W_{1t} = \frac{\beta_{11}(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} + \varepsilon_{1t}$

→ Plug in $\ln P_{2t}$ in $\ln D_t$

$$\begin{aligned}
 \ln D_t &= \beta_{10} + \frac{\beta_{11}(\beta_{20} - \beta_{21})}{\beta_{11} - \beta_{21}} - \frac{\beta_{12} \ln P_{2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{13} \ln P_{3t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{14} \ln P_{4t}}{\beta_{11} - \beta_{21}} \\
 &\quad + \left(\frac{\varepsilon_{2t} - \varepsilon_{1t}}{\beta_{11} - \beta_{21}} \right) + \beta_{12} \ln GDP_t + \varepsilon_{1t}
 \end{aligned}$$

$$\begin{aligned}
 &= \beta_{10} + \frac{\beta_{11}(\beta_{20} - \beta_{21})}{\beta_{11} - \beta_{21}} - \frac{\beta_{12}\beta_{12}(\ln P_{2t})}{\beta_{11} - \beta_{21}} - \frac{\beta_{12}\beta_{13}(\ln P_{3t})}{\beta_{11} - \beta_{21}} \\
 &\quad - \frac{\beta_{12}\beta_{14}(\ln P_{4t})}{\beta_{11} - \beta_{21}} + \frac{\beta_{12}\beta_{22}(\ln GDP_t)}{\beta_{11} - \beta_{21}} + \frac{\beta_{12}(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} \\
 &\quad + \beta_{12} \ln GDP_t + \varepsilon_{1t}
 \end{aligned}$$

where $\pi_{20} = \beta_{20} + \frac{\beta_{21}(\beta_{20} - \beta_{21})}{\beta_{11} - \beta_{21}}$ $\pi_{21} = \frac{-\beta_{21}\beta_{12}}{\beta_{11} - \beta_{21}}$

$\pi_{22} = \frac{-\beta_{21}\beta_{13}}{\beta_{11} - \beta_{21}}$ $\pi_{23} = \frac{-\beta_{21}\beta_{14}}{\beta_{11} - \beta_{21}}$ $\pi_{24} = \frac{\beta_{21}\beta_{12}}{\beta_{11} - \beta_{21}} + \beta_{20}$ $w_{2t} = \frac{\beta_{21}(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}}$

$\pi_{30} = \frac{\beta_{30} - \beta_{31}}{\beta_{11} - \beta_{21}}$ $\pi_{33} = \frac{\beta_{32}}{\beta_{11} - \beta_{21}}$ $\pi_{32} = \frac{-\beta_{33}}{\beta_{11} - \beta_{21}}$

$\pi_{33} = \frac{-\beta_{34}}{\beta_{11} - \beta_{21}}$ $\pi_{34} = \frac{\beta_{32}}{\beta_{11} - \beta_{21}}$ $w_{3t} = \frac{\varepsilon_{2t} - \varepsilon_{1t}}{\beta_{11} - \beta_{21}}$

To reduce form model of these system model

$\ln S_t = \pi_{10} + \pi_{11}(\ln P_{x2t}) + \pi_{12}(\ln P_{x3t}) + \pi_{13}(\ln P_{x4t}) + \pi_{14}(\ln GDP)_t$
 $\ln P_t = \pi_{20} + \pi_{21}(\ln P_{x2t}) + \pi_{22}(\ln P_{x3t}) + \pi_{23}(\ln P_{x4t}) + \pi_{24}(\ln GDP) + w_{2t}$
 $\ln P_{Dt} = \pi_{30} + \pi_{31}(\ln P_{x2t}) + \pi_{32}(\ln P_{x3t}) + \pi_{33}(\ln P_{x4t}) + \pi_{34}(\ln GDP) + w_{3t}$

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

```
. g pd = pm+ t
. g lnst = ln(st)
. g lndt = ln(dt)
. g lnpx2 = ln(px2)
. g lnpx3 = ln(px3)
. g lnpx4 = ln(px4)
. g lngdp = ln(gdp)
. reg lnst lnpx2 lnpx3 lnpx4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	4.64569724	4	1.16142431	F(4, 17)	=	37.32
Residual	.529104674	17	.031123804	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpx2	-.4503744	.1515961	-2.97	0.009	-.7702142	-.1305347
lnpx3	-.9242052	.2783356	-3.32	0.004	-1.511442	-.3369685
lnpx4	-.3883793	.4222332	-0.92	0.371	-1.279214	.5024549
lngdp	.3438812	.1913463	1.80	0.090	-.0598242	.7475865
_cons	24.65741	5.309757	4.64	0.000	13.4548	35.86002

```
. predict lnsthat
(option xb assumed; fitted values)
```

```
. reg lndt lnp2 lnp3 lnp4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	3.4026552	4	.850663799	F(4, 17)	=	26.43
Residual	.54721789	17	.032189288	Prob > F	=	0.0000
				R-squared	=	0.8615
				Adj R-squared	=	0.8289
Total	3.94987309	21	.188089195	Root MSE	=	.17941

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnp2	-.4887365	.1541691	-3.17	0.006	-.8140049	-.1634682
lnp3	-.7243134	.2830597	-2.56	0.020	-1.321517	-.1271097
lnp4	-.577921	.4293997	-1.35	0.196	-1.483875	.3280333
lngdp	.1265855	.194594	0.65	0.524	-.2839719	.5371429
_cons	27.18614	5.399879	5.03	0.000	15.79339	38.57889

```
. predict lndthat
```

(option **xb** assumed; fitted values)

```
. reg lnpd lnp2 lnp3 lnp4 lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	.17707359	4	.044268398	F(4, 17)	=	6.76
Residual	.111247189	17	.006543952	Prob > F	=	0.0019
				R-squared	=	0.6142
				Adj R-squared	=	0.5234
Total	.288320779	21	.013729561	Root MSE	=	.08089

lnpd	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnp2	.1318015	.0695123	1.90	0.075	-.0148567	.2784596
lnp3	.0939842	.127627	0.74	0.472	-.1752851	.3632535
lnp4	.4939641	.1936093	2.55	0.021	.0854842	.9024439
lngdp	.1632779	.0877392	1.86	0.080	-.0218357	.3483914
_cons	2.87652	2.434717	1.18	0.254	-2.260283	8.013322

```
. predict lnpdhat
```

(option **xb** assumed; fitted values)

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.

. reg lnst lnpdhat lnp2 lnp3 lnp4

Source	SS	df	MS	Number of obs	=	22
Model	4.64569773	4	1.16142443	F(4, 17)	=	37.32
Residual	.529104183	17	.031123775	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879	4.578612
lnp2	-.727963	.1840856	-3.95	0.001	-1.11635	-.3395762
lnp3	-1.122146	.2824139	-3.97	0.001	-1.717988	-.5263052
lnp4	-1.428722	.4751381	-3.01	0.008	-2.431176	-.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274	36.63092

. reg lndt lnpdhat lngdp

Source	SS	df	MS	Number of obs	=	22
Model	3.26129847	2	1.63064924	F(2, 19)	=	44.99
Residual	.688574614	19	.036240769	Prob > F	=	0.0000
				R-squared	=	0.8257
				Adj R-squared	=	0.8073
Total	3.94987309	21	.188089195	Root MSE	=	.19037

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675	-1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194	.802766
_cons	35.93498	7.189835	5.00	0.000	20.88648	50.98347

4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?

```
. reg3 (lnst lnpd lnp2 lnp3 lnp4) (lndt lnpd lngdp), ols
```

Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344	.1766516
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

```
. reg3 (lnst lnpd lnp2 lnp3 lnp4) (lndt lnpd lngdp), 2sls nodfk inst(lnpx2 lnp3 lnp4 lngdp)
```

Two-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	13.81	0.0000
lndt	22	2	.1454858	0.8982	89.20	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpd	2.10611	1.926677	1.09	0.282	-1.801371	6.013591
lnpx2	-.7279628	.3026471	-2.41	0.021	-1.34176	-.114166
lnpx3	-1.122146	.464304	-2.42	0.021	-2.063798	-.180494
lnpx4	-1.428722	.7811544	-1.83	0.076	-3.012977	.1555325
_cons	18.59914	14.05113	1.32	0.194	-9.897873	47.09616
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.394875	-1.75344
lngdp	.5212921	.0955104	5.46	0.000	.327588	.7149961
_cons	35.93499	5.106302	7.04	0.000	25.57893	46.29105

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnp2 lnp3 lnp4 lngdp

```
. reg3 (lnst lnpg lnpx2 lnpx3 lnpx4) (lndt lnpg lngdp), 3sls ireg3 nodfk inst(lnpx2 lnpx3 lnpx4 lngdp)
```

```
Iteration 1: tolerance = .1059484
Iteration 2: tolerance = .04569793
Iteration 3: tolerance = .01846611
Iteration 4: tolerance = .00725496
Iteration 5: tolerance = .00281814
Iteration 6: tolerance = .00108981
Iteration 7: tolerance = .00042072
Iteration 8: tolerance = .00016231
Iteration 9: tolerance = .0000626
Iteration 10: tolerance = .00002414
Iteration 11: tolerance = 9.310e-06
Iteration 12: tolerance = 3.590e-06
Iteration 13: tolerance = 1.384e-06
Iteration 14: tolerance = 5.339e-07
```

Three-stage least-squares regression, iterated

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpg	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
lndt						
lnpg	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpg lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

According to the result, 2SLS is more appropriate model since OLS would give us bias, inconsistent, and inefficiency due to the endogeneity problem in the model. Since we don't know whether there is specification error or not, 3SLS and I3SLS might give us inconsistent. We use the techniques of instrumental variable and reduce form to obtain consistent and efficiency by solving the endogeneity problem in 2SLS.

5. What do β_{21} and β_{22} mean?

β_{21} represents the price elasticity of domestic demand where we capture the effect of price change. For example, when domestic price change by 1%, how much the demand will change holding other things constant.

β_{22} represents the income elasticity of demand where we capture the effect of income change holding other things constant.