Solution to Homework Assignment #3

Important Remarks before reading the following solution

<u>Let</u>

Bundle A be the optimal bundle before the price change (or initial bundle)

Bundle B be the decomposition bundle (or substitution bundle)

Bundle C be the optimal bundle after the price change (or final bundle)

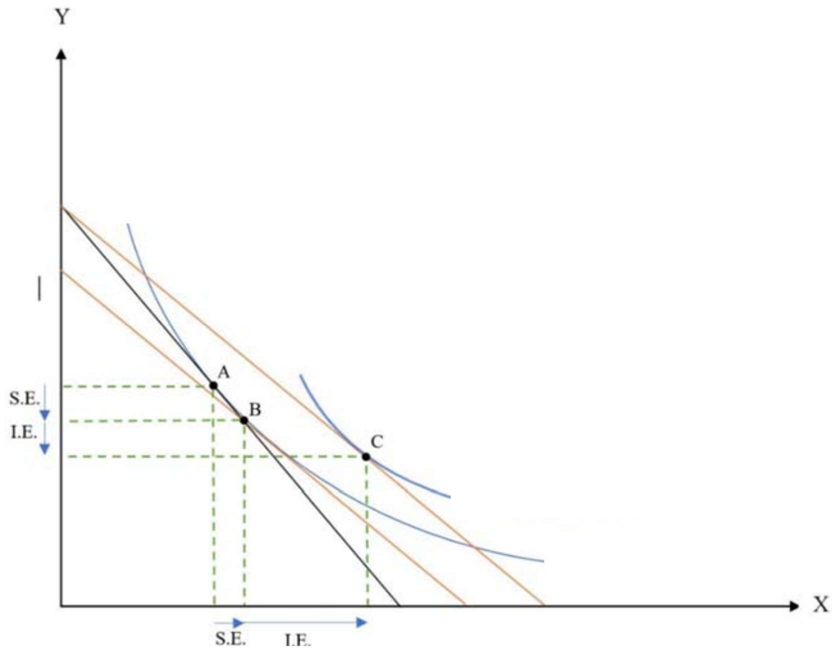
Px' be the new price

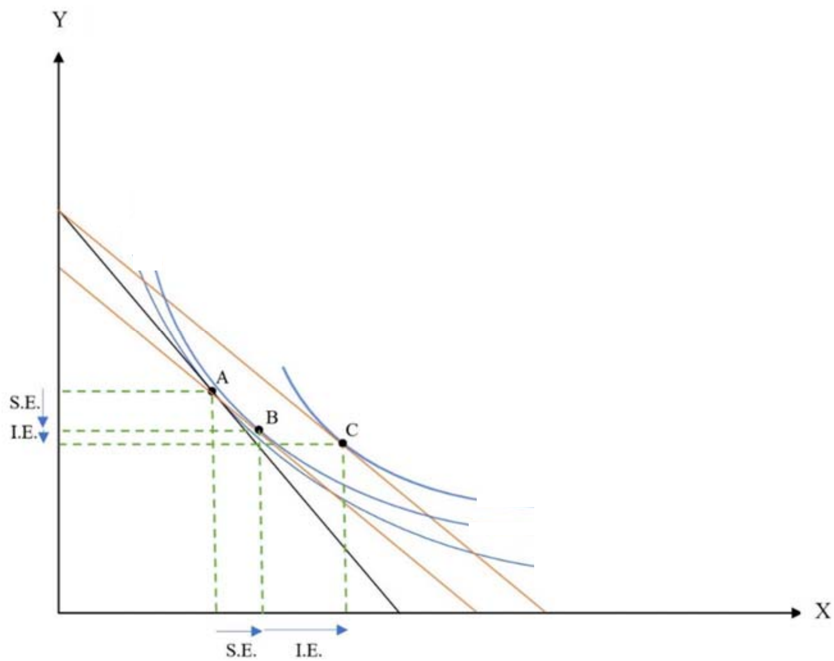
I' be the (Slutsky) compensated income or the income of the pivoted BL.

Bundle E (for EV calculation) be the optimal bundle (at the old price) that gives the new utility as if the price had changed.

Question 1 Sec 1 – Px falls

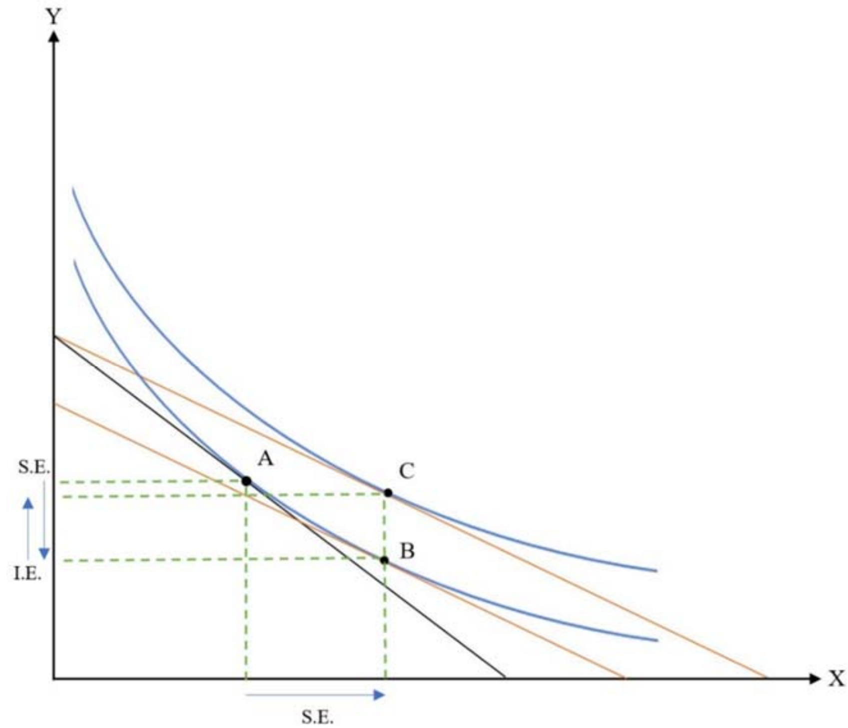
a)

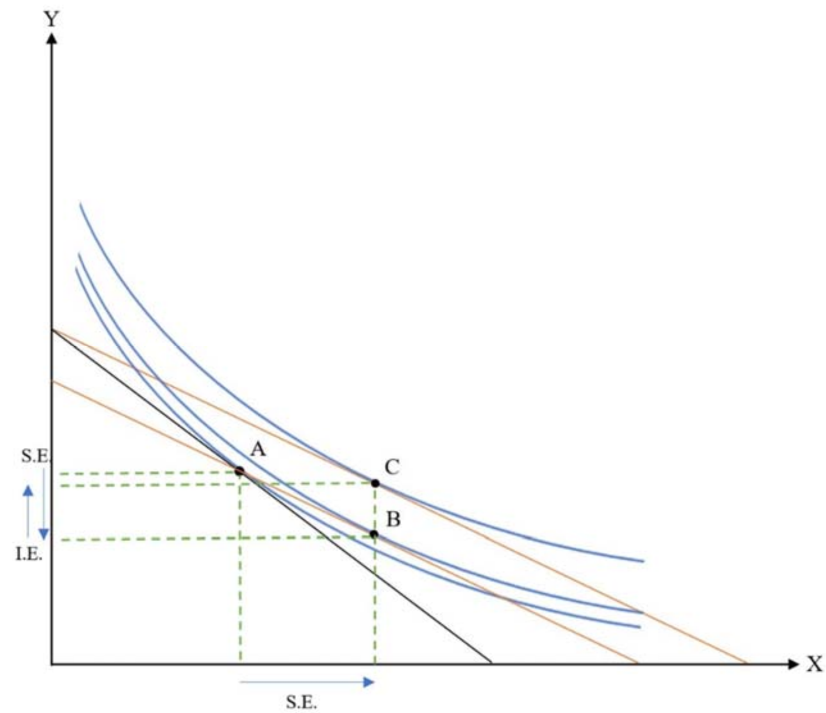




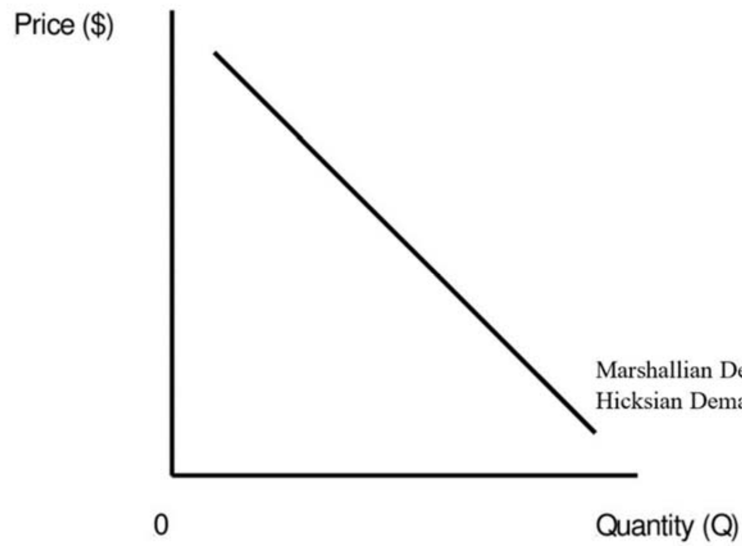
The initial consumption is basket A. Substitution effect moves consumption basket to basket B, more X less Y. X is normal good, income effect increases amount of X consumed. Y is inferior good, income effect decreases amount of Y consumed (change from basket B to C).

b)

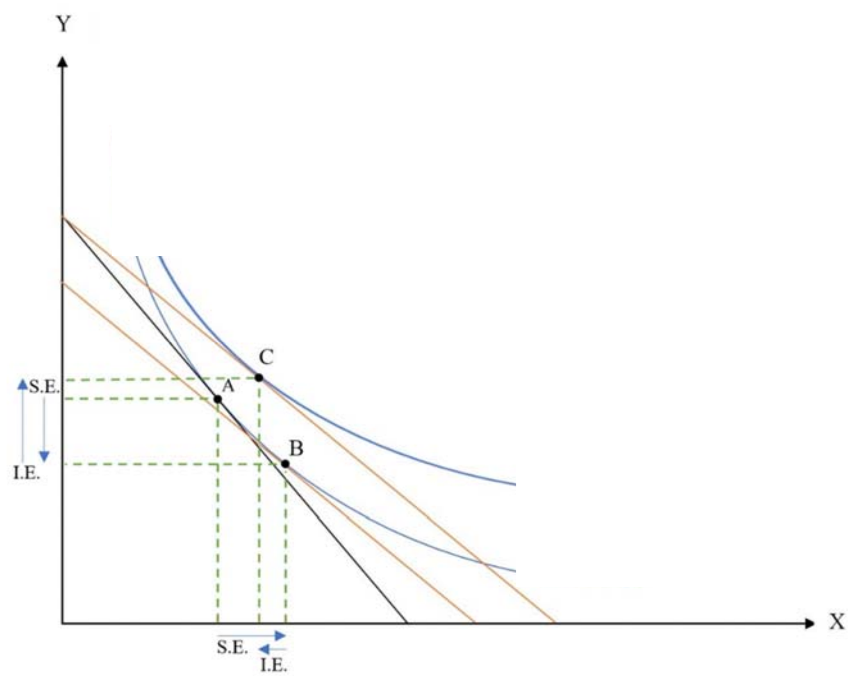


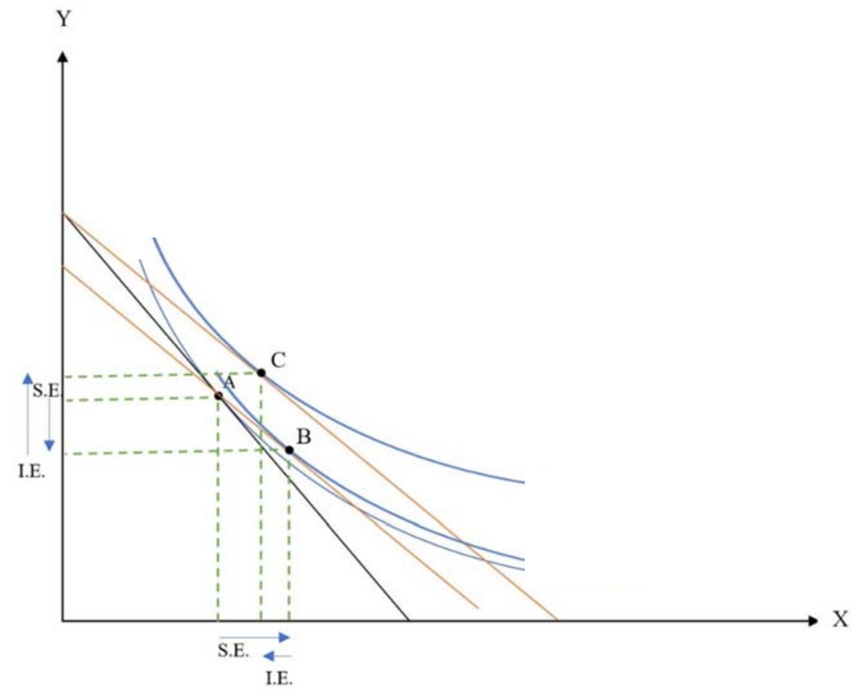


Assuming Y is normal good. Substitution effect move consumption basket from A to B, more X less Y. Income affect only affect amount of Y, Y increases (from B to C).

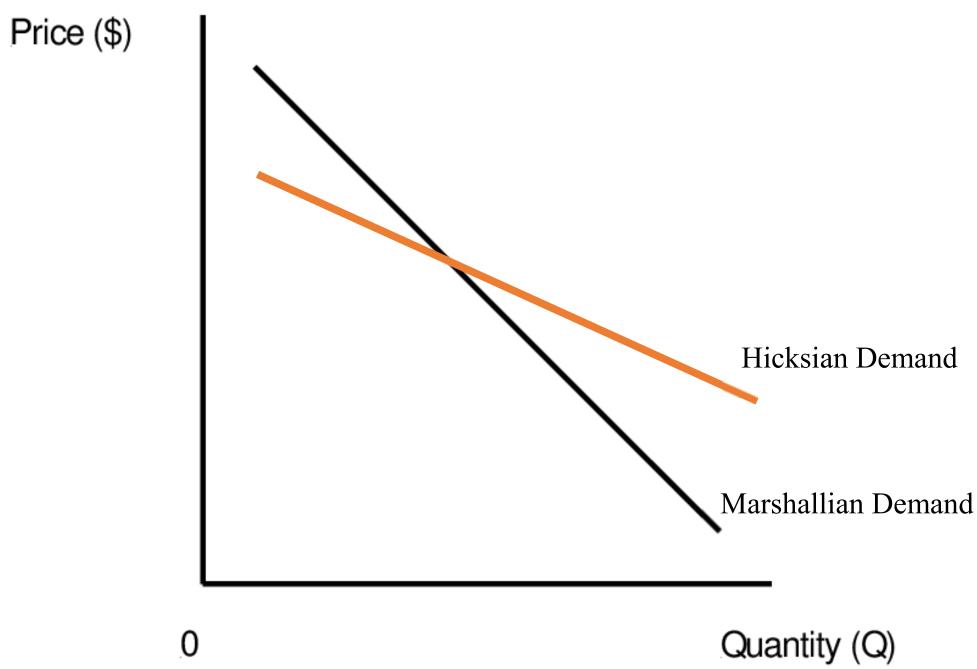

Price ($)
Marshallian Demand,
Hicksian Demand
0
Quantity (Q)

c)

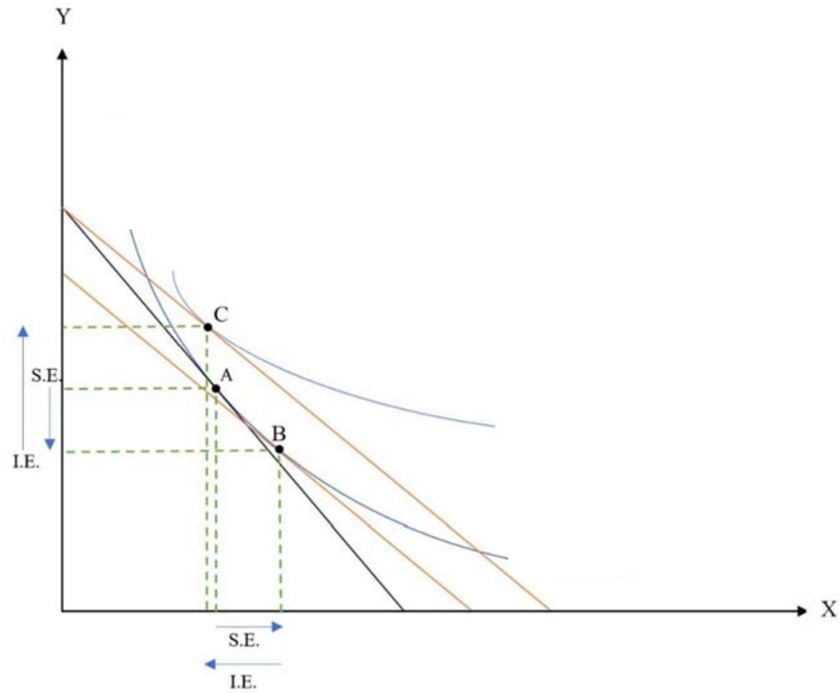

Y
C
S.E.
A
B
I.E.
X
S.E.
I.E.

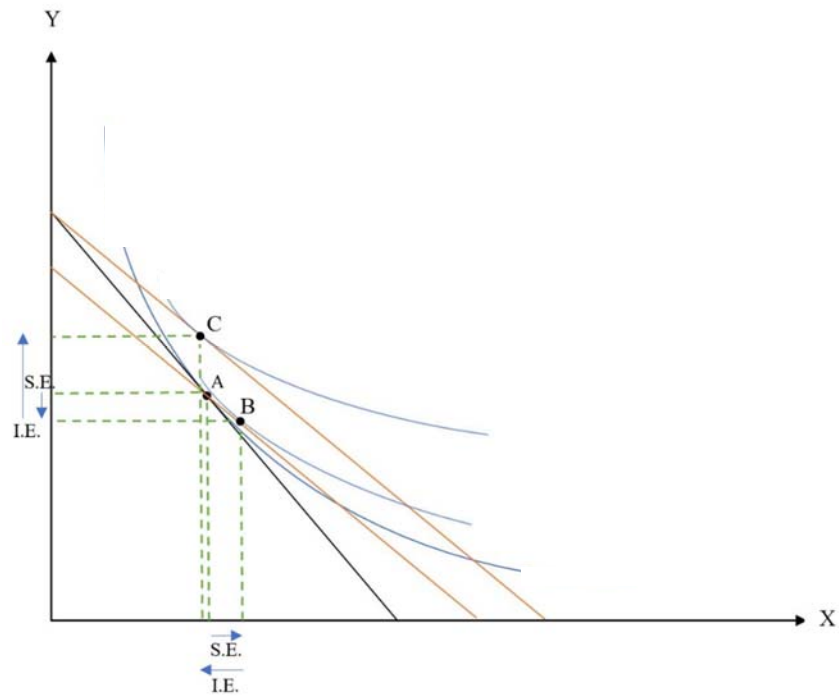


Assuming Y is normal good. Substitution effect moves consumption basket from A to B, more X less Y. Income effect moves consumption basket from B to C. Since X is inferior good, X at C is less than X at B. Substitution effect stills dominate income effect.

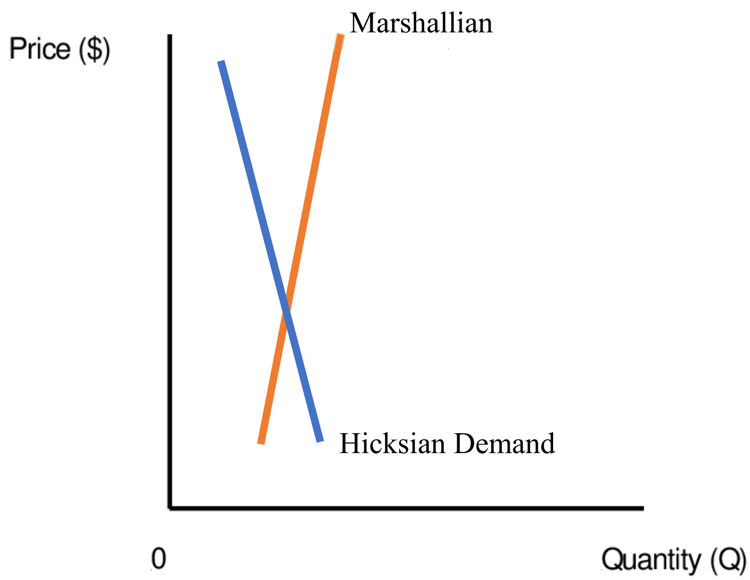


d)



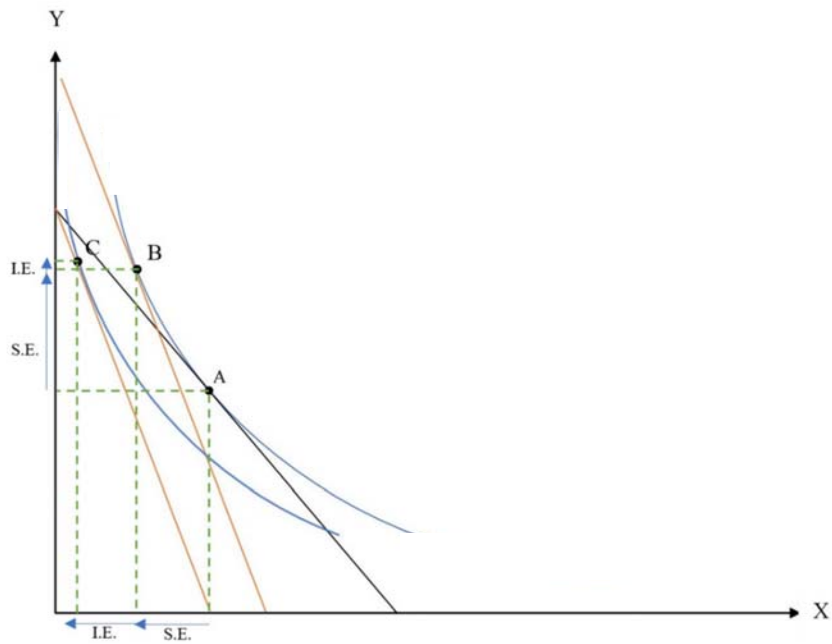


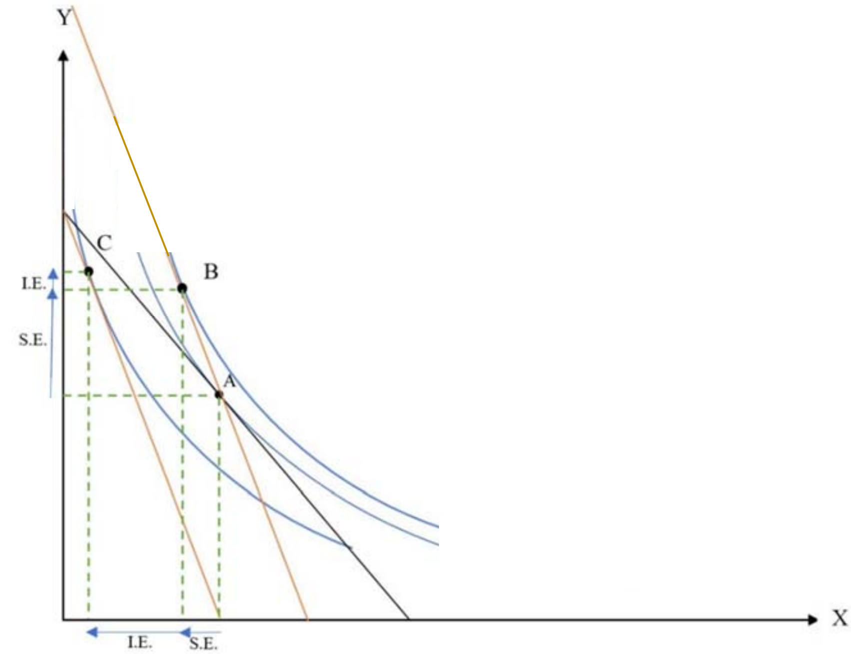
Assuming Y is normal good. Substitution effect moves consumption basket from A to B, more X less Y. Income effect moves consumption basket from B to C. Since X giffen good, Income effect dominate substitution effect. After Px declines, this consumer demands less X.



Question 1 Sec 2 – Px rises

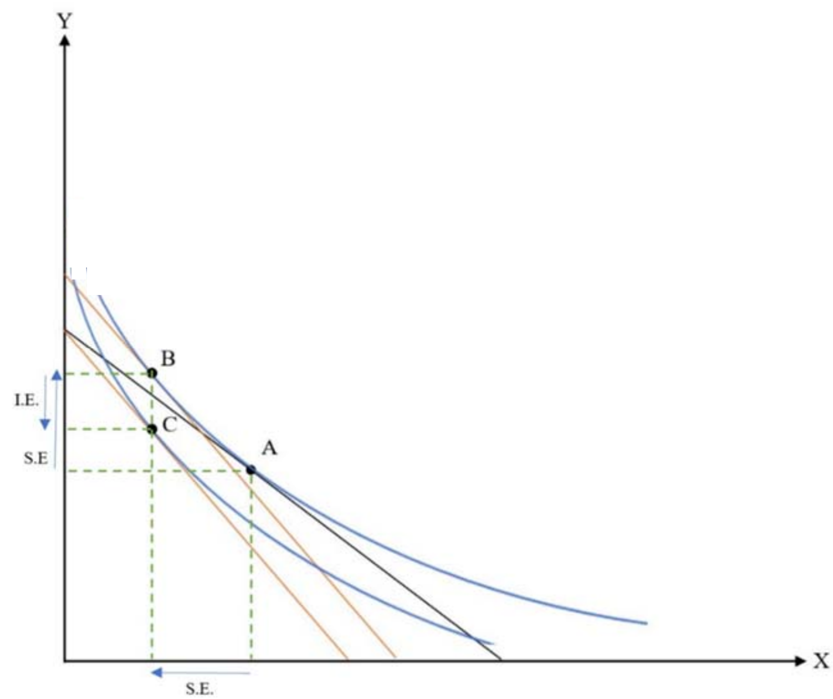
a)

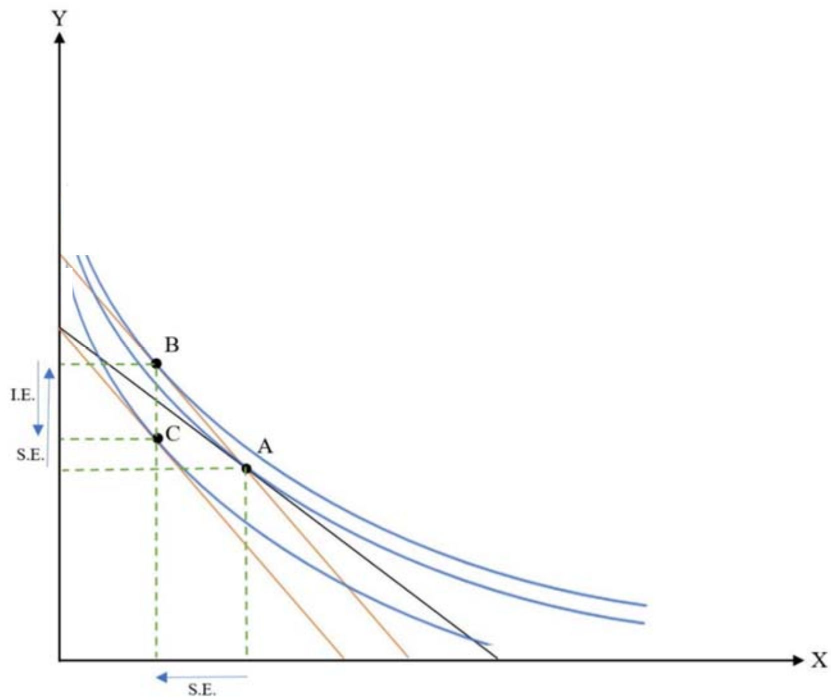




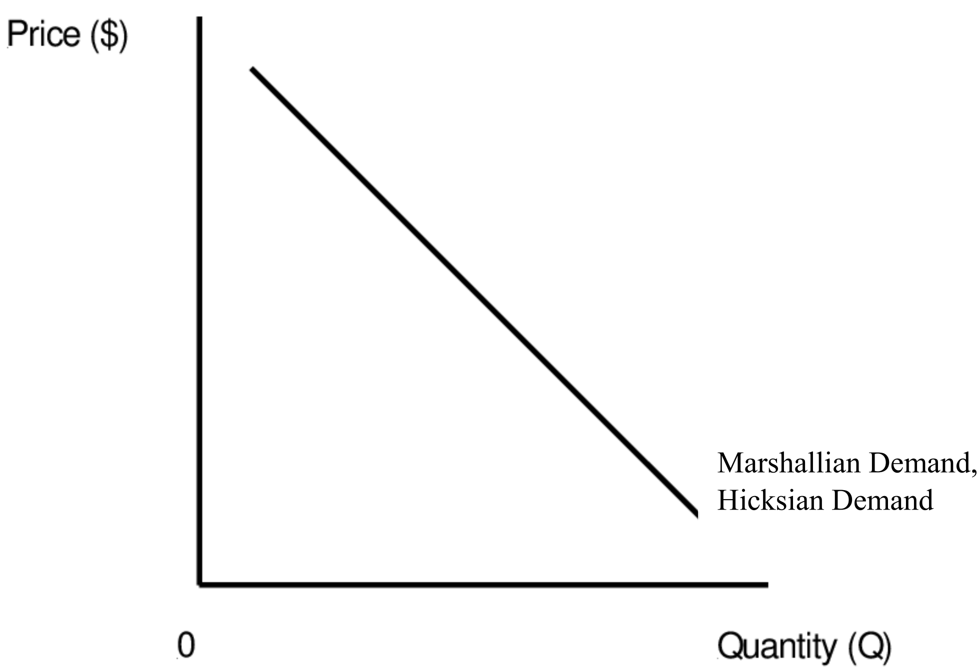
The initial consumption is basket A. Substitution effect moves consumption basket to basket B, less X more Y. X is normal good, income effect decreases amount of X consumed. Y is inferior good, income effect increases amount of Y consumed (change from basket B to C).

b)

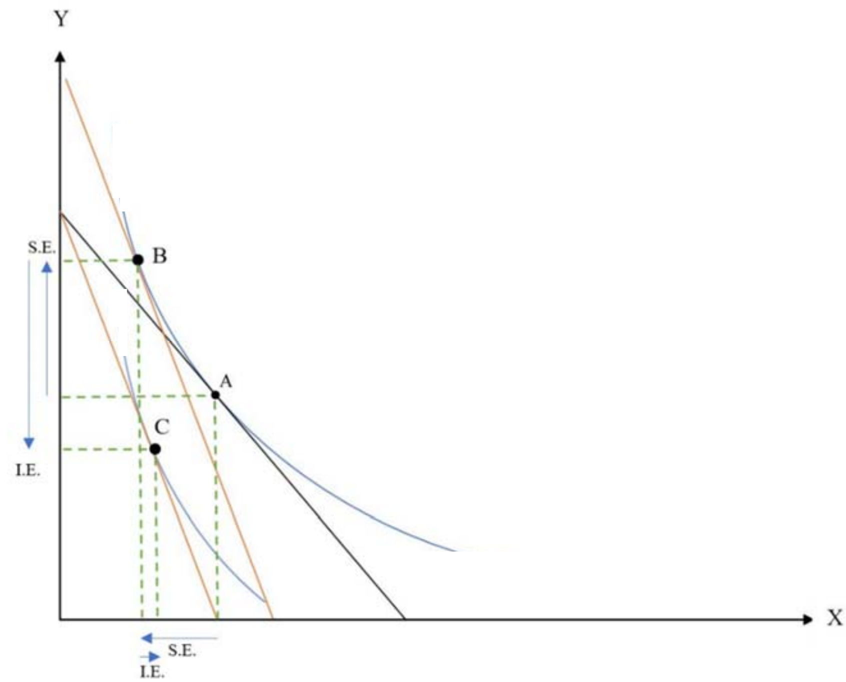


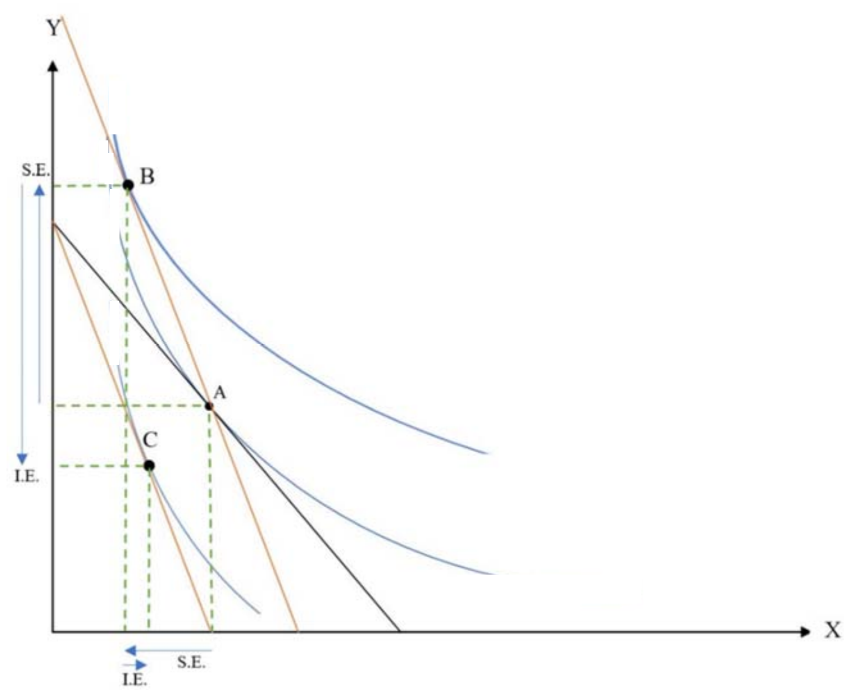


Assuming Y is normal good. Substitution effect move consumption basket from A to B, less X more Y. Income affect only affect amount of Y, Y decreases (from B to C).

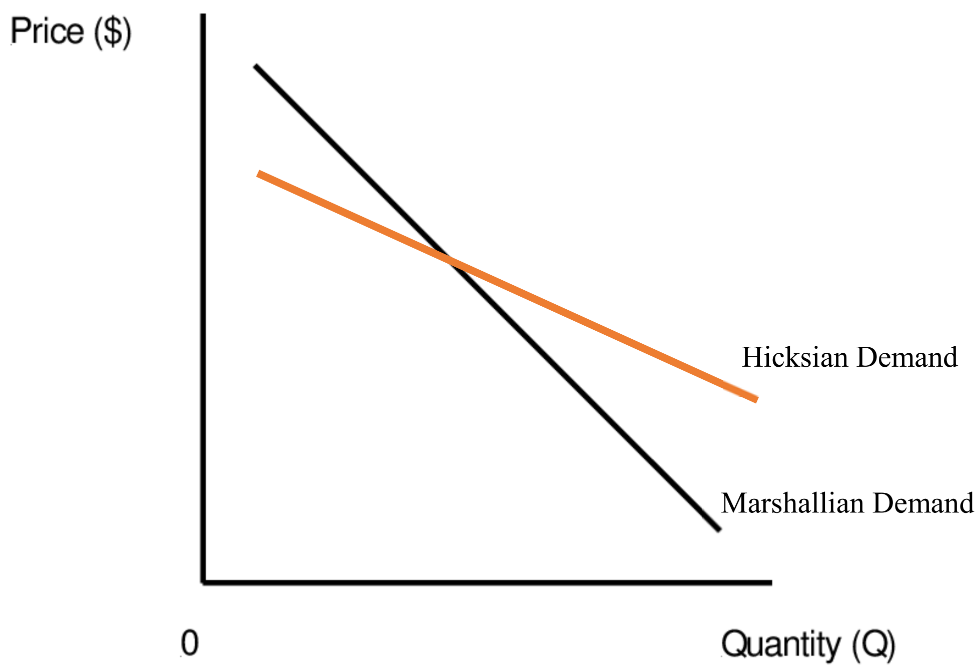


c)

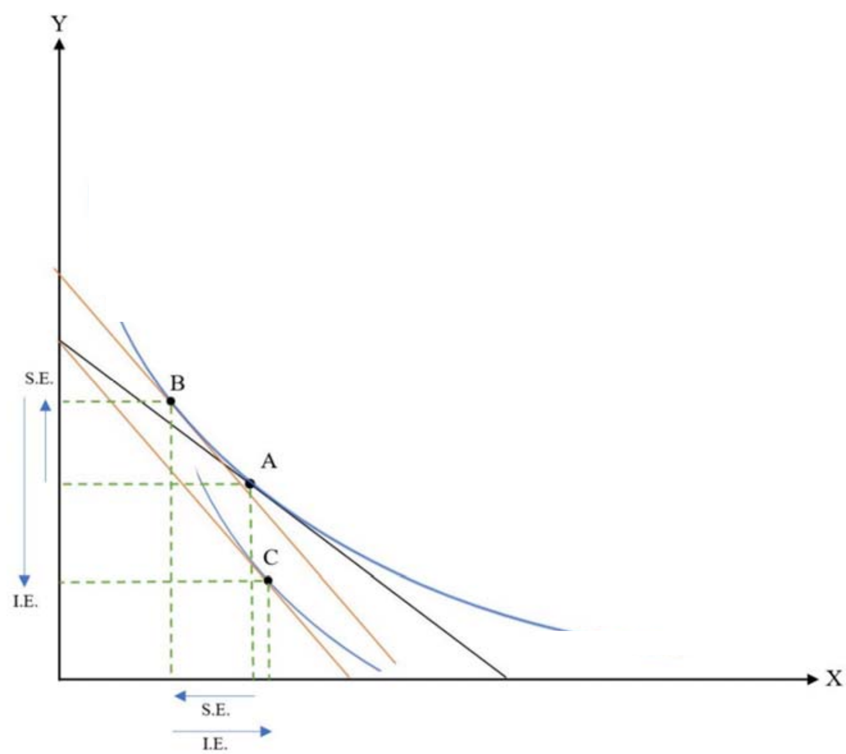




Assuming Y is normal good. Substitution effect moves consumption basket from A to B, less X more Y. Income effect moves consumption basket from B to C. Since X is inferior good, X at C is more than X at B. Substitution effect stills dominate income effect.


Price ($)
Hicksian Demand
Marshallian Demand
0
Quantity (Q)

d)


Y
S.E.
B
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X
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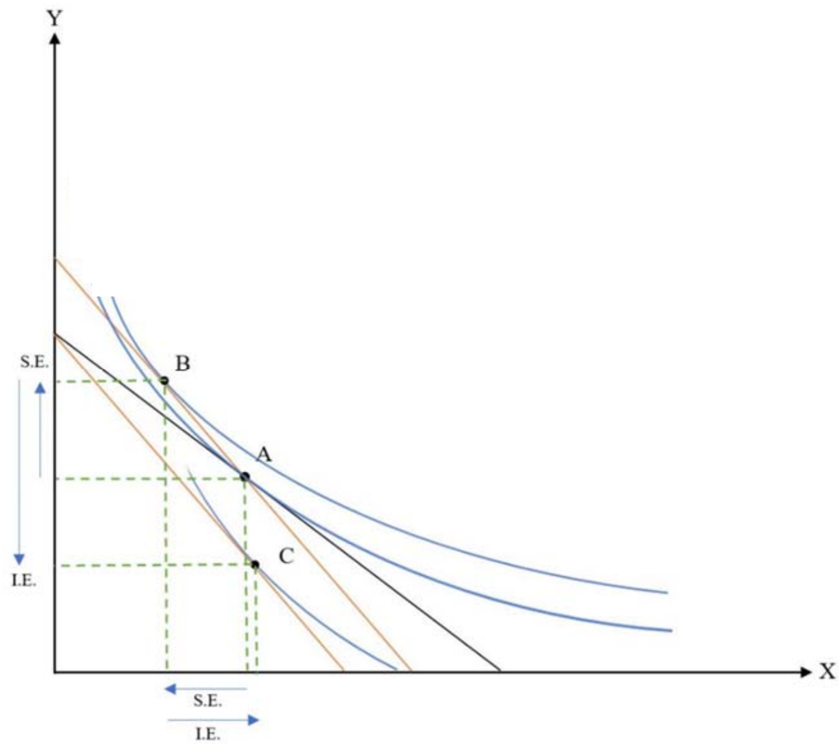
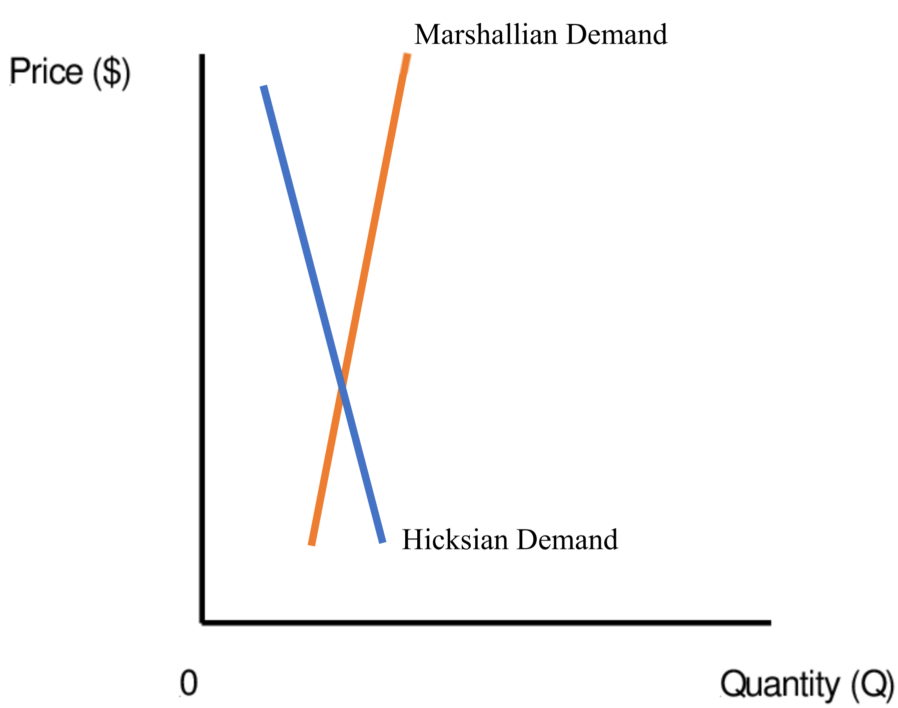
I.E.



Assuming Y is normal good. Substitution effect moves consumption basket from A to B, more X less Y. Income effect moves consumption basket from B to C. Since X giffen good, Income effect dominate substitution effect. After Px rises, this consumer demands more X.



Question 2

Note that MUx = Y, MUy = X, Px = 4, Px' = 9, Py = 1, I = 72.

a) To derive the demand, we use optimality condition.

   MUx/MUy = Px/Py
   Y/X = Px/1 (we do not substitute Px when we derive the demand!)
   Y = PxX (we need to eliminate Y, so we will use the BL)

   From the BL: PxX + PyY = I, we get
   2PxX = I >>> **X = I/2Px** (this is the demand for X)
   (we do not substitute I when we derive the demand too!)

b) dX/dI > 0, so X is normal.

c) dX/dPx < 0, so X is ordinary.

d) Find SE and IE, using the Hicksian Approach.

   At Bundle A, Px = 4 and I = 72. We use the demand equation to find Bundle A.
   $X_A = 72/2(4) = 72/8 = 9$
   $Y_A = I – PxX_A = 72 – 4(9) = 36$

   At Bundle C, Px = 9 and I = 72. We use the demand equation to find Bundle C.
   $X_C = 72/2(9) = 72/18 = 4$
   $Y_C = I – PxX_C = 72 – 9(4) = 36$

   To find the Decomposition (Substitution) Bundle B:

   We know that U(Bundle A) = U(Bundle B).

   U(Bundle A) = $X_AY_A = 9(36) = 324$ = U(Bundle B)

   Thus, $X_BY_B = 324$.

   We also know that MUx/MUy = Px'/Py holds at Bundle B.

   Thus, Y/X = 9/1 >>> $Y_B = 9X_B$.

   Solving gives $9(X_B)^2 = 324$

   $X_B = 6$ $Y_B = 9X_B = 54$

   $SE_{Hicks} = X_B – X_A = 6 – 9 = – 3$.

   $IE_{Hicks} = X_C – X_B = 4 – 6 = – 2$.

e) Find SE and IE, using the Slutskian Approach.

To find the Decomposition (Substitution) Bundle B, the Slutskian Approach will "adjust" the income or "compensate" the consumer so that the purchasing power of the consumer does not change and that he can afford his original Bundle A with no "leftover" income.

- **In the case of price rise, the consumer can no longer afford Bundle A (this means his purchasing power falls), so he will be compensated by "getting some extra money".**
- **In the case of price fall, the consumer can afford Bundle A and will have "leftover" money (this means his purchasing power rises), so he will be compensated by "having some money TAKEN from him".**

Since this is the case of price rise, the consumer will get some money so that Bundle A can still be affordable after the price change.

Pivoted BL: $Px'X_A + PyY_A = I'$ (where I' = 72 + extra money)

$9(9) + 1(36) = 81 + 36 = 117 = I'$

Bundle B is the optimal bundle at Px' and I'. We can use the demand equation:

$X_B = 117/2(9) = 6.5$ $\quad\quad$ $Y_B = 117 – 6.5(9) = 58.5$

$SE_{Slutsky} = X_B – X_A = 6.5 – 9 = – 2.5$

$IE_{Slutsky} = X_C – X_B = 4 – 6.5 = – 2.5$

f) The income required to compensate the consumer after the price change is 117 – 72 = 45.

Alternatively, this can be obtained by $\Delta I = \Delta Px(X_A) = (9 – 4)(9) = 45$.

g) Find CV and EV. **(NOTE THAT we use Hicksian Bundle B)**

CV and EV measures the change in utility, measured in monetary terms, as a result of the price change.

***** There are many ways to find CV and EV. In this solution, the approach will be based on the extra sheet. *****

CV is the change in income required to buy the optimal bundles before and after the price change, at the <u>original</u> utility.

CV = income required to buy Bundle A – income required to buy Bundle B

EV is the change in income required to buy the optimal bundle before and after the price change, at the <u>new</u> utility.

EV = income required to buy Bundle E – income required to buy Bundle C

Bundle E is the optimal bundle (at the old price) that gives the new utility as if the price had changed.

For CV calculation;

Income to buy A = 72

We know that $X_B = 6$ and $Y_B = 54$. This is the optimal bundle at Px = 9.

Income to buy B = 6(9) + 54(1) = 108

CV = 72 – 108 = – 24

For EV calculation;

Income to buy C = 72

We need to find Bundle E (we use the same approach when we find B)

We know that U(Bundle E) = U(Bundle C).

U(Bundle C) = $X_C Y_C$ = 4(36) = 144 = U(Bundle B)

Thus, $X_E Y_E = 144$.

We also know that MUx/MUy = Px/Py holds at Bundle E.

Thus, Y/X = 4/1 >>> $Y_E = 4X_E$.

Solving gives $4(X_E)^2 = 144$

$X_E = 6$ $Y_E = 4X_E = 24$

Income to buy E = 6(4) + 24(1) = 48

EV = 48 – 72 = – 24

i) This is the case of X being a normal good. The Slutsky Equation implies that both IE and SE reinforce each other and are "negative". The term "negative" here refers to the negative relationship between Px and X. Thus, as Px rises, X falls.

Question 3

Note that $MUx = 1/(\sqrt{X})$, $MUy = 1$, $Px = 0.2$, $Px' = 0.5$, $Py = 1$, $I = 10$.

a) To derive the demand, we use optimality condition.

$MUx/MUy = Px/Py$
$1/(\sqrt{X}) = Px/1$
$X = 1/(Px)^2$ (this is the demand for X)

(Here, we don't use the BL because there is no Y, i.e. the optimal bundle does not depend on Y.)

a) $dX/dI = 0$, so X is NEITHER normal NOR inferior, i.e. it does not depend on I.

b) $dX/dPx < 0$, so X is ordinary.

c) Find SE and IE, using the Hicksian Approach.

At Bundle A, $Px = 0.2$ and $I = 10$. We use the demand equation to find Bundle A.
$X_A = 1/(0.2)^2 = 1/0.04 = 25$
$Y_A = I - PxX_A = 10 - 0.2(25) = 5$

At Bundle C, $Px = 0.5$ and $I = 10$. We use the demand equation to find Bundle C.
$X_C = 1/(0.5)^2 = 1/0.25 = 4$
$Y_C = I - PxX_C = 10 - 0.5(4) = 8$

To find the Decomposition (Substitution) Bundle B:

We know that U(Bundle A) = U(Bundle B).

U(Bundle A) $= 2(\sqrt{X_A}) + Y_A = 2\sqrt{25} + 5 = 15 =$ U(Bundle B)

Thus, $2(\sqrt{X_B}) + Y_B = 15$.

We also know that $MUx/MUy = Px'/Py$ holds at Bundle B.

That is, we can substitute Px' into the demand equation to find X_B.

$X_B = 1/(Px')^2 = 1/(0.5)^2 = 4$

$Y_B = 15 - 2(\sqrt{X_B}) = 15 - 4 = 11$

$SE_{Hicks} = X_B - X_A = 4 - 25 = -21$.

$IE_{Hicks} = X_C - X_B = 4 - 4 = 0$.

d) Find SE and IE, using the Slutskian Approach.

Since this is the case of price rise, the consumer will get some money so that Bundle A can still be affordable after the price change.

Pivoted BL: $Px'X_A + PyY_A = I'$ (where $I' = 10$ + extra money)

$0.5(25) + 1(5) = 12.5 + 5 = 17.5 = I'$

Bundle B is the optimal bundle at Px' and I'. We can use the demand equation.

However, the demand equation does not depend on I':

$X_B = 1/(Px')^2 = 1/(0.5)^2 = 4$

$Y_B = 17.5 – Px'(X_B) = 17.5 – 0.5(4) = 15.5$

$SE_{Slutsky} = X_B – X_A = 4 – 25 = – 21.$

$IE_{Slutsky} = X_C – X_B = 4 – 4 = 0.$

e) The income required to compensate the consumer after the price change is $17.5 – 10 = 7.5$.

Alternatively, this can be obtained by $\Delta I = \Delta Px(X_A) = (0.5 – 0.2)(25) = 7.5$.

f) Find CV and EV. **(NOTE THAT we use Hicksian Bundle B)**

For CV calculation;

Income to buy A = 10

We know that $X_B = 4$ and $Y_B = 11$. This is the optimal bundle at $Px = 0.5$.

Income to buy B = $0.5(4) + 1(11) = 13$

$CV = 10 – 13 = – 3$

For EV calculation;

Income to buy C = 10

We need to find Bundle E (we use the same approach when we find B)

We know that U(Bundle E) = U(Bundle C).

$U(\text{Bundle C}) = 2(\sqrt{X_C}) + Y_C = 2\sqrt{4} + 8 = 12 = U(\text{Bundle E})$

Thus, $2(\sqrt{X_E}) + Y_E = 12.$

We also know that $MUx/MUy = Px/Py$ holds at Bundle E.

At Bundle E, $Px = 0.2$, we use the demand equation to find X_E.

$X_E = 1/(Px)^2 = 1/(0.2)^2 = 25$

$Y_E = 12 - 2(\sqrt{X_E}) = 12 - 10 = 2$

Income to buy E = 25(0.2) + 2(1) = 7

EV = 7 – 10 = – 3

i) Note that dX/dI = 0. The Slutsky Equation implies that IE = 0 and that TE = SE. The total effect (TE) is "negative" since the substitution effect is always negative. The term "negative" here refers to the negative relationship between Px and X. Thus, as Px rises, X falls.