Question 1:

Consider a firm with a production function $Q = \sqrt{K} + \sqrt{L}$. Assume that input price of L is \$1 and input price of K is \$2. Consider the following problem.

1.1 Does the production function exhibit constant return to scale? Explain.

1.2 In the short-run, K is fixed at 5 units, derive the short-run cost function.

1.3 Derive the long-run cost function. (Hint: you don't need to check the second-order derivative test.)

Question 2:

Joe's utility function depends on consumption and leisure, and it is given by:

$$U(c, l) = 8\sqrt{c} + l,$$

where *c* is consumption *l* is leisure. Each day Joe works *n* hours and spends the rest of the time on leisure; that is, $l + n = 24$. Suppose further that Joe's total income is the sum of a fixed non-wage income (Y_0) and a wage income (Y_n). The wage income (Y_n) is equal to wn, where *w* is the hourly wage rate. If Joe spends all of his income on the consumption, his budget constraint becomes:

$$p_c c = Y_0 + wn = Y_0 + w(24 - l),$$

which can also be written as:

$$p_c c + wl = Y_0 + 24w.$$

2.1. Use Lagrange method to derive the optimal values of c^* and l^* in terms of the parameters p_c, w, and Y_0. State the condition under which the consumer demands some leisure (i.e., $l^* > 0$).

2.2. Based on your answer in (2.1), derive the impact of an increase in the hourly wage (*w*) on the demand for leisure (l^*). Interpret its economic meaning.

2.3. Suppose now that the hourly wage rate (w) is \$24, the price of consumption (p_c) is \$12, and the fixed non-wage income (Y_0) is 384. Use Lagrange method to determine the levels of c^* and l^* that maximizes Joe's utility subject to the budget constraint, and determine the level of constrained maximum utility.

2.4. Verify your answers in (2.3) by using the second-order sufficient condition.