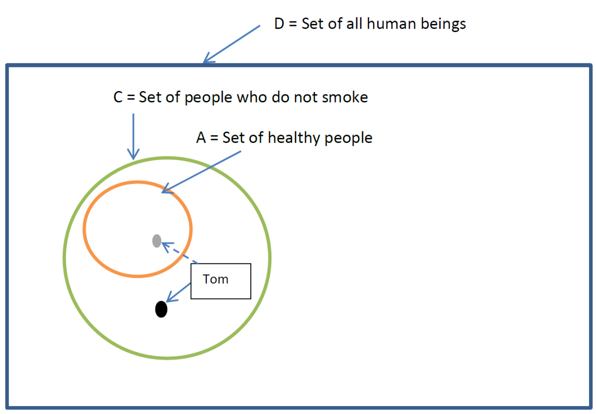
Solution: Quiz 2

1. Let the domain D be the set of all human beings. Suppose Tom $\in D$.
   Use the **diagram** to show that the following argument is valid or invalid.
   "No healthy people smoke ."
   "Tom does not smoke."
   $\therefore$ "Tom is healthy."

   Solution:

   To use the diagram,
   let A be the set of healthy people, and
   let C be the set of people who do **not** smoke.
   Then
   - the first premise "No healthy people smoke ." tells us that
   "**All healthy people do not smoke**" or "$A \subseteq C$."
   - the second premise "Tom does not smoke."
   tells us that "Tom is in the set C," or Tom $\in C$,
   - the conclusion tells us that "Tom is in the set A."

   From the diagram, since we only know that "Tom" is an element in C , it is possible that "Tom" is either an element in A or not an element in A. That is, the conclusion may not happen because it is possible that "Tom" does **not** in A or "Tom" is not healthy. Therefore the argument is **invalid**.



   To confirm this by using the universal rules of inferences.
   Let $P(x)$ be "x is healthy,"
   $Q(x)$ be "x does not smoke."
   Then we can transform the given argument in the quantified form of **converse error** as follows.

$$\forall x, P(x) \to Q(x)$$
$$Q(a) \text{ for a particular } a = \text{ Tom}$$
$$\therefore P(a)$$

   That is, this argument is **invalid** by the converse error. ■

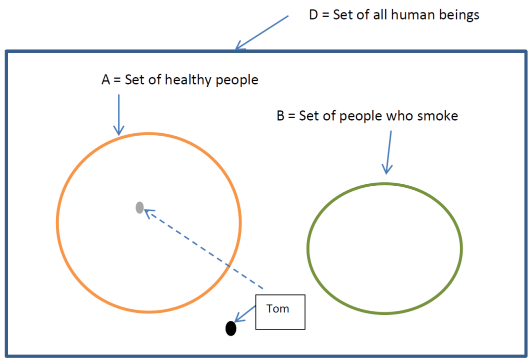
ALTERNATIVE ANSWER USING DIAGRAM:

Suppose we
let A be the set of healthy people, and
let B be the set of people who smoke.

Let B^c be the complement of B, i.e. B^c is the set of elements in D that are not in B. Note that $B \cap B^c = \emptyset$.

- The first premise "No healthy people smoke ." is equivalent to
"**All healthy people do not smoke**" or $A \subseteq B^c$, so A cannot have anything in common with B and this implies "$A \cap B = \emptyset$."
- The second premise implies "Tom"$\notin B$ or "Tom"$\in B^c$.
- The conclusion implies "Tom" $\in A$

Hence the following diagram is also **correct** for this argument and it gives the same answer "*invalid*".



From the diagram, since we only know that "Tom" is not an element in B , it is possible that "Tom" is either an element in A or not an element in A. That is, the conclusion may not happen when "Tom" does **not** in A or "Tom" is not healthy. Therefore the argument is **invalid**.

2. Let $D = \{(-1,1),(0,0),(0,2),(1,2)\}$ be the domain of variable (x,y) for the predicate $P(x,y)$ defined as

$$P(x,y): \ \exists a \in \mathbb{Z}^+, \quad a < xy.$$

Determine the truth set of the predicate $P(x,y)$.

Solution: Let T_p be the truth set of P. Recall that each element of the truth set T_p is the order pair $(x,y) \in D$ that makes $P(x,y)$ true.

- For $(-1,1) \in D$, we have $xy = -1$ and $\quad P(-1,1): \ \exists a \in \mathbb{Z}^+, \quad a < -1$
which is false because any $a \in \mathbb{Z}^+$ is positive and it cannot be less than -1. $\Rightarrow \quad (-1,1) \notin T_q$

- For $(0,0) \in D$, we have $xy = 0$ and $\quad P(0,0): \ \exists a \in \mathbb{Z}^+, \quad a < 0$
which is false because any $a \in \mathbb{Z}^+$ is positive and it cannot be less than 0. $\Rightarrow \quad (0,0) \notin T_q$

- For $(0,2) \in D$, we have $xy = 0$ and $\quad P(0,2): \ \exists a \in \mathbb{Z}^+, \quad a < 0$
which is false because any $a \in \mathbb{Z}^+$ is positive and it cannot be less than 0. $\Rightarrow \quad (0,2) \notin T_q$

- For $(1,2) \in D$, we have $xy = 2$ and $\quad P(1,2): \ \exists a \in \mathbb{Z}^+, \quad a < 2$
which is true because we can find $a \in \mathbb{Z}^+$, for example $a = 1 \in \mathbb{Z}^+$ that is less than 2. $\Rightarrow \quad (1,2) \in T_q$

From above, we have considered all elements (x,y) in D and $P(x,y)$ is true only when $(x,y) = (1,2)$. Therefore the truth set is $T_q = \{(1,2)\}$. ■