Name ______ **Surname** ______ **Student ID.** ______
DUE DATE : Thursday 18^{th}, February 2016.

Assignment 2: (50 marks)

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

Student Signature: ______

1. (10 marks) Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2), & \text{if } -1 < x < 1. \\ 0, & \text{otherwise.} \end{cases} \tag{Eq.1}$$

(a) What is the value of c?

(b) P (-0.5< X<0.5)

2. (10 marks)Determine whether the following models are linear in the parameters, or the variables, or both. Which of these models are linear regression models?

2.1 Reciprocal

$$Y_i = \beta_1 + \beta_2(\frac{1}{X_i}) + u_i$$

2.2 Semilogarithmic

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

2.3 Inverse semilogarithmic

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i$$

2.4 Logarithmic or double logarithmic

$$\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i$$

2.5 Logarithmic reciprocal

$$\ln Y_i = \beta_1 - \beta_2(\frac{1}{X_i}) + u_i$$

3. (10 marks)Consider the following non-stochastic models (i.e., model without the stochastic error term). Are they linear regression models? If not, is it possible, by suitable algebraic manipulations, to convert them into linear models?

3.1

$$Y_i = \frac{1}{\beta_1 + \beta_2 X_i}$$

3.2

$$Y_i = \frac{X_i}{\beta_1 + \beta_2 X_i}$$

3.3

$$Y_i = \frac{1}{1 + exp(-\beta_1 - \beta_2 X_i)}$$

4. (10 marks)(Mid-term Exam 1/2015) Show that $\sum x_i^2 = \sum X_i^2 - n\bar{X}^2$ and that $\sum x_i y_i = \sum x_i Y_i$. In other words, show that:

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i Y_i}{\sum X_i^2 - n\bar{X}^2}$$

5.(10 marks) In the Table 1 X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

X	**Y**
63	2.8
72	3.4
78	3
81	3.5
87	3.6
75	3
75	2.7
90	3.7

Table 2. Raw Data Based on the Sample Data on Table 1

	(1) Y_i	(2) X_i	(3) Y_iX_i	(4) X_i^2	(5) $x_i = X_i - \bar{X}$	(6) $y_i = Y_i - \bar{Y}$	(7) x_i^2	(8) x_iy_i	(9) $\hat{Y}_i$	(10) $\hat{u}_i = Y_i - \hat{Y}_i$	(11) $\hat{Y}_i\hat{u}_i$
Sum											
Mean											

5.1 Now consider the two-variable model :

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

Based on Table 2, use the OLS to find the estimator of β_1 and β_2. Interpret the regression.

5.2 Show that $\sum \hat{u}_i = 0$