Numerical Solution MA217 Midterm 2014

1. Mistake in Question sheet… it should be $x, y, z > \mathbf{0}$
   (a) The critical point is (6, 3, 6),
   $f_{xx} = 2, f_{yy} = 8, f_{xy=2}, D = 12$ (6, 3, 6) is a relative minimum
   (b) critical point in term of C is $\left(2\sqrt[3]{\frac{C}{4}},\ \sqrt[3]{\frac{C}{4}},\ 2\sqrt[3]{\frac{C}{4}}\right)$ and $f^* = 12\left(\frac{C}{4}\right)^{\frac{2}{3}}$
   (c) $C = 108$ and the critical point is (6, 3, 6)
   (d) $f^*_{new} = 107.73$

2. Critical point from $f(x, y)$ is (0,0)
   Critical points from boundary … substitute $x^2 = 4 - 4y^2$ get $f(y) = e^{-(4-3y^2)}$ are (2, 0) and (-2, 0) and end points are (0, -1) and (0, 1)
   Absolute maximum is (0, 0) and absolute minimum is (-2, 0)

3. Critical point from $f(x, y)$ is (0,0)
   Critical points from boundary $y = 0$ is (0,0); boundary $y = 2 - x$ is (1,1); and boundary $y = x$ is (0,0) with end points (corners) are (0,0), (2,0) and (1,1)
   Absolute maximum is (2, 0) and absolute minimum is (0, 0)

4. Mistake in Question sheet… it should not have the wording "~~Use information from Question 3 if possible and~~". Since the domain is unboned so this question we need to use the Lagrange Multiplier method.
   Case I $x + y = 4$ and $x + 3y = 9$
   Give invalid critical point $\left(\frac{3}{2}, \frac{5}{2}\right)$ because $\mu_2 = -1$
   Case II $x + y = 4$ and $x + 3y < 9$
   Give a critical point $(2,2)$, $\mu_1 = 4$

   Case III $x + y < 4$ and $x + 3y = 9$
   Give invalid critical point $\left(\frac{33}{10}, \frac{19}{10}\right)$ and $\mu_2 = -1$ but violate $x + y < 4$
   (A valid critical point needs to satisfy all starting questions.)
   Case IV $x + y < 4$ and $x + 3y < 9$
   Give invalid critical point $(4,4)$ but violate $x + y < 4$

   Hence, there is only 1 valid critical point $f^*(2,2) = -8$ but because there is only 1 point, we can only say that this is the critical point but cannot determine if it is the absolute maximum or minimum.