HW

1. Find Cournot Equilibrium when there're 3 firms in the market.

$$P = a - bQ \; ; \; Q = q_1 + q_2 + q_3$$

$$c_1 = c_2 = c_3 = c$$

What is equilibrium price? P^*, what are firms' profit? $\pi_1 = \pi_2 = \pi_3 = ?$

2. If there are N Firm $q_i^* = f(N)$, $P = f(N)$, $\pi_i = f(N)$

3. From question 2, what happen if $N \to \infty$, what happen if $N = 1$

$$\pi_1 = (P - C)q_1 = [a - bQ - c]q_1$$
$$= [a - b(q_1 + q_2 + q_3) - c]\, q_1$$
$$= [a - bq_1 - bq_2 - bq_3 - c]q_1$$
$$\pi_1 = aq_1 - bq_1^2 - bq_1q_2 - bq_1q_3 - cq_1 \to \text{Firm}_1\text{'s } \pi$$

$$\text{Max } \pi_1 \to \frac{\partial \pi_1}{\partial q_1} = 0 \to a - 2bq_1 - bq_2 - bq_3 - c = 0$$
$$a - c - bq_2 - bq_3 = 2bq_1$$
$$\frac{a-c}{2b} - \frac{\cancel{b}q_2}{2\cancel{b}} - \frac{\cancel{b}q_3}{2\cancel{b}} = q_1^*$$
$$\frac{a-c}{2b} - \left[\frac{q_2 + q_3}{2}\right] = q_1^* \to \text{reaction f}^n \text{ of firm}_1$$

$$\pi_2 = (P - C)q_2 = [a - b(q_1 + q_2 + q_3) - c]q_2$$
$$= [a - bq_1 - bq_2 - bq_3 - c]q_2$$
$$\pi_2 = aq_2 - bq_1q_2 - bq_2^2 - bq_2q_3 - cq_2 \to \text{Firm}_2\text{'s } \pi$$
$$\text{Max } \pi_2 = \frac{\partial \pi_2}{\partial q_2} = 0 \to a - bq_1 - 2bq_2 - bq_3 - c = 0$$
$$a - c - bq_1 - bq_3 = 2bq_2$$
$$\frac{a-c}{2b} - \left[\frac{q_1 + q_3}{2}\right] = q_2^* \to \text{Firm}_2\text{'s reaction f}^n$$

$$\pi_3 = (P - C)q_3 = [a - b(q_1 + q_2 + q_3) - c]q_3$$
$$= [a - bq_1 - bq_2 - bq_3 - c]q_3$$
$$\pi_3 = aq_3 - bq_1q_3 - bq_2q_3 - bq_3^2 - cq_3 \to \text{Firm}_3\text{'s } \pi$$
$$\text{Max } \pi_3 \to \frac{\partial \pi_3}{\partial q_3} = 0 \to a - bq_1 - bq_2 - 2bq_3 - c = 0$$
$$a - c - bq_1 - bq_2 = 2bq_3$$
$$\frac{a-c}{2b} - \left[\frac{q_1 + q_2}{2}\right] = q_3^* \to \text{Firm}_3\text{'s reaction F}^n$$

Homogenous Product $\therefore q_1^* = q_2^* = q_3^*$ in equilibrium; $\frac{a-c}{2b} - \left[\frac{q_2 + q_3}{2}\right] = q_1^* \to \frac{a-c}{2b} - \left[\frac{q_1^* + q_1^*}{2}\right] = q_1^*$

$$\frac{a-c}{2b} - q_1^* = q_1^*$$
$$\frac{a-c}{2b} = 2q_1^*$$
$$q_1^* = \frac{a-c}{4b}$$

$$\boxed{NE : q_1^* = q_2^* = q_3^* = \frac{a-c}{4b}}$$

Total Production $q_1^* + q_2^* + q_3^* = \frac{3(a-c)}{4b}$; $P = a - bQ = a - b\left(\frac{3(a-c)}{4b}\right)$

$$P = a - b\left[\frac{3a - 3c}{4b}\right]$$
$$= a - \frac{3a\cancel{b}}{4\cancel{b}} - \frac{3\cancel{b}c}{4\cancel{b}}$$

$$P = a - \frac{3a - 3c}{4}$$
$$P^* = \frac{a - 3c}{4} \; *$$

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2. Assume $q_1 + q_2 + q_3 + q_n = A$

$P = a - b(q_1 + q_2 + q_3 + \ldots + q_n)$

$P = a - bq_1 - bq_2 - \ldots - bq_n$

$\pi_1 = (a - bq_1 - bq_2 - bq_3 - \ldots - bq_n) \cdot q_1 - C_1$

$\vdots$

$\pi_n = (a - bq_1 - bq_2 - bq_3 - \ldots - bq_n) \cdot q_1 - C_n$

$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 - \ldots - bq_n = 0$

$q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \ldots + q_n)$

$q_n = \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 \ldots + q_{n-1})$

$\therefore \; q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + q_3 + \ldots + q_n)$

$0.5q_1 = \frac{a}{2b} - 0.5A$

$q_1 = \frac{a}{b} - A$ ___ (1)

$q_2 = \frac{a}{b} - A$

$q_3 = \frac{a}{b} - A$

$\vdots$

$q_n = \frac{a}{b} - A$

Since $A = q_1 + q_2 + q_3 + \ldots q_n$, $\rightarrow A = n\left(\frac{a}{b} - A\right)$

$A = n\frac{a}{b} - nA$

$A + nA = n\left(\frac{a}{b}\right)$

$A(1+n) = n\left(\frac{a}{b}\right)$

$A = \frac{na}{(n+1)b}$ → sub. into (1) ; $q_2 = \frac{a}{(n+1)b}$

$\therefore \; q_i = \frac{a}{(n+1)b}$ *

Equilibrium price ; $P = a - b(A)$

$P = a - b\left(\frac{na}{(n+1)b}\right)$

$P = a - \left(\frac{n}{n+1}\right)a$

$P = \frac{a(n+1) - na}{n+1}$

$P = \frac{na + a - na}{n+1}$

$P = \frac{a}{n+1}$

$\pi_i = P \cdot q_i - C_i$

$\pi_i = \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - C_i$

$\pi_i = \frac{a^2}{(n+1)^2 b} - C_i$ *

3.) if $n \to \infty$, it will make $q_i = \frac{a}{(n+1)b}$ → nearly zero and each firm will sell at q nearly zero unit

; it'll make $A = nq_i$ → nearly zero. Q of every firm combinded will be nearly → ∞ unit

; it'll make $P = \frac{a}{n+1}$ → nearly zero. when supply increase, price will decrease → nearly zero

; it'll make $\pi_i = \frac{a^2}{(n+1)^2 b} - C_i \to -C_i$ Each Firm will lose the profit

If $n=1$; it will make $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ Since $Q = \frac{a}{2b} < Q = \frac{na}{(n-1)b}$, monopoly will sell less quantity.

; it will make $A = nq_i = Q$ since $n=1$, the firm will be a monopoly.

; it will make $P = \frac{a}{n+1} = \frac{a}{2}$ Since $P_m = \frac{a}{2} > P = \frac{a}{n+1}$ monopoly will set higher price.

; it will make $\pi_i = \frac{a^2}{(n+1)^2 b} - C_i = \frac{a^2}{4b} - C_i$ which higher than $\pi_i = \frac{a^2}{(n+1)^2 b}$ As a result, the monopoly will get higher profit.