

$$P = a - cQ \quad \text{vs.} \quad \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$$

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

$$P = a - bQ$$

$$\frac{p-a}{-b} = Q$$

$$-\frac{p}{b} + \frac{a}{b} = Q$$

$$-\frac{1}{b}p + \frac{a}{b} = Q$$

$$PED = \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$$

$$PED = -\frac{1}{b} \times \left(\frac{P_0}{\frac{P_0-a}{-b}} \right)$$

$$P = a - cQ$$

$$\frac{p-a}{-c} = Q$$

$$-\frac{p}{c} + \frac{a}{c} = Q$$

$$-\frac{1}{c}p + \frac{a}{c} = Q$$

$$PED = \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$$

$$PED = -\frac{1}{c} \times \left(\frac{P_0}{\frac{P_0-a}{-c}} \right)$$

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that \uparrow Point PES

(i) elasticity of supply is always greater than 1 if $c > 0$,

(ii) elasticity of supply is always equal to 1 if $c = 0$,

(iii) elasticity of supply is always less than 1 if $c < 0$.

Hint: use $\frac{\% \Delta Q_s}{\% \Delta P}$

$$\frac{Q_1 - Q_0}{Q_0}$$

$$\frac{P_1 - P_0}{P_0}$$

i) $d = 1, c > 0; c = 2$

$$p = 2 + Q$$

$$Q = p - 2 \quad (4, 6)$$

$$4 = p - 2 \quad (8, 10)$$

$$6 = p$$

$$8 = p - 2$$

$$10 = p$$

$$PES = \frac{\frac{8-4}{4}}{\frac{10-6}{6}}$$

$$= 1.5$$

\therefore Elasticity of supply is greater than 1 ($1.5 > 1$) when $c > 0$

ii) $d = 2, c = 0$

$$p = 2Q$$

$$\frac{1}{2}p = Q \quad (2, 4)$$

$$p = 4$$

$$(6, 12)$$

$$\frac{1}{2}p = 6$$

$$p = 12$$

$$PES = \frac{\frac{6-2}{2}}{\frac{12-4}{4}}$$

$$= 1$$

\therefore Elasticity of supply is equal to 1 ($1=1$) when $c = 0$

iii) $d = 2, c = -4$

$$p = -4 + 2Q$$

$$p + 4 = 2Q$$

$$\frac{1}{2}p + 2 = Q$$

$$\frac{1}{2}p + 2 = 4 \quad (4, 4)$$

$$p = 4$$

$$\frac{1}{2}p + 2 = 6 \quad (6, 8)$$

$$p = 8$$

$$PES = \frac{6-4}{4}$$

$$\frac{8-4}{4}$$

$$= 0.5$$

\therefore Elasticity is less than 1 ($0.5 < 1$) when $c < 0$