EE320 Introductory Mathematical Economics

Quiz 3

1. *(10 points total - 5 points each)*
   For the following functions, (i) determine the critical value of *x* and (ii) find the relative maxima and minima of *y* by the second-derivative test:

   a. $y = x^3 + 3x^2 + 9$
   $f'(x) = 3x^2 + 6x = 3x(x+2).$

   Critical values: x* = 0, -2

   $f''(x) = 6x + 6.$

   $f''(0) = 6 > 0.$ ➔ x* = 0 gives a relative local minimum *f(0) = 9.*

   $f''(-2) = -12 < 0$ ➔ x* = -2 gives a relative local maximum *f(-2) = 13.*

   [Alternative: for x* = -2, use the first-derivative test to show that *f'(-3) > 0 and f'(-1) < 0*.]

   b. $y = -\frac{1}{2}x^2 - 4x + 5$

   $f'(x) = -x - 4.$

   Critical value: x* = -4

   f''(x) = -1 < 0. ➔ x*-4 gives a relative local maximum *f(-4) = 13*

   [Alternative: use the first-derivative test to show that *f'(-5) > 0* and *f'(-3) < 0*.]

2. *(10 points total)* Given the total product function:

$$Q = aL + bL^2 - cL^3 \qquad (a, b, c > 0)$$

a) *(5 points)* Find the average product (AP) function, and determine whether the AP function is convex or concave.

$$AP = \frac{Q}{L} = a + bL - cL^2$$

To determine the curvature of the AP function, we need to evaluate its second derivative:

$$\frac{d(AP)}{dL} = b - 2cL$$

$$\frac{d^2(AP)}{dL^2} = -2c < 0$$

Thus, AP is a strictly concave function because AP" < 0.

b) *(5 points)* Find the marginal product (MP) function, and determine whether the MP function is convex or concave.

$$MP = \frac{dQ}{dL} = a + 2bL - 3cL^2$$

$$\frac{d(MP)}{dL} = 2b - 6cL$$

$$\frac{d^2(MP)}{dL^2} = -6c < 0$$

Thus, MP is a strictly concave function because MP" < 0.